ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

Post Graduate Program: Business Mathematics

"MEASURING THE SYSTEMIC RISK IN GREEK BANKING SECTOR USING THE CONTINGENT CLAIM ANALYSIS"

Master thesis

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Abstract

Classical financial decision-making is based on the assumptions of 'efficient market', in an economy with many restrictions that cannot capture all the necessary aspects of modern needs. For instance, once we add some real-life sources of funding for corporation traditional models of valuation, in respect to which many popular risk metrics are calculated are no-longer suitable for use.

During the early 70's a strong engagement started between economic problems and more mathematical and sophisticated valuation and modelling techniques. Option Pricing Theory showed the pathway, according to which corporate liabilities can be treated as a combination of simple option contracts, allowing us to model many economic instances and securities with non-linear equations, that were defined based on an underlying asset and an assumed risk-free rate.

This thesis highlights the importance of Contingent Claim Analysis approach into calculating key features of economic entities like market value of asset and market value of debt. By calculating these we are able then to extract suitable risk metrics allowing us to both compare different sectors and evaluate the progress of a specific issuer of debt in time.

At the end of the methodology presentation, we apply the CCA method to extract risk-metrics for a systemic Greek bank from the first quarter of 2001 to the third quarter of 2020, a time-period of high interest as it included many critical points, like the Greek debt and financial crisis and the 2015 referendum. We investigate whether Distance-to-Distress and Probability-of-Default are good predictors of vulnerabilities related to the Greek Banking Sector.

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1. Introduction

All parts of the global economic system have nowadays realized the essence of models that are able to quantify the risks taken from financial corporations, firms and individuals. Following the early 21st century financial crisis, analyzing credit risks became critical for every decision globally.

The technique presented here, Contingent Claim Analysis (CCA), is used for determining the price of a security whose payoffs depend upon the prices of one or more other securities [4]. CCA roots are present in the Option Pricing Theory as developed by Black and Scholes (1973), Merton (1974) and Vasicek (1977) and have been developed rapidly since then.

Here, our main focus is using CCA to get the market value of assets of a debt issuer and through that exploiting some very useful and important risk metrics.

Before we dive further into the CCA approach, it would be beneficial to make a brief introduction both into Black and Scholes and Merton work combined with some more widely-used and influential models on Option Pricing Theory as it would allow us to deeply understand the concepts and methodology of Contingent Claim Analysis.

1.1 The Black-Scholes-Merton Model

Black-Scholes-Merton model is a mathematical model of financial derivative market from which the Black-Scholes formula can be derived. With the use of this formula, we can estimate prices of options during their whole life-time and not strictly at their maturity [7]. Black and Scholes model had an incredible influence in modern financial pricing and marked the beginning of a significant increase in option trading, which has nowadays adopted more scientific methods. The formula itself is used even today for pricing, with some customizations from the practitioners.

In their initial work, Black and Scholes came up with a Partial Differential equation named as “the Black-Scholes equation”, and later Robert Merton (1973) contributed to the mathematical understanding of their equation (using Stochastic Calculus) that lead to what later became known as the “Black-Scholes-Merton formula”.

Assumptions of the Model

- There exists in the market a constant risk-free rate.
- The price of the underlying asset in our option, follows a geometric Brownian motion, where we initially assume its drift and volatility are constant.
- The above security, does not distribute any dividend during the option’s life-time.
- Arbitrage opportunities do not occur in the under-investigation market.
- We can instantly borrow and lend money at the risk-free rate.
- Even fractions of security amounts can be traded.

- Transactions do not require any additional cost to be delivered.

With these assumptions holding, we can determine the price of an option traded in the previously described market and written on the above security; at any time until the option's maturity, using the Black-Scholes formula.

The Black-Scholes Equation

As already described, this is a Partial Differential Equation (PDE) describing the price movement of European Options under Black-Scholes model. For a European call or put option, with respect to the previous market-assumptions this equation is denoted as:

$$\frac{\partial V_{S,t}}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V_{S,t}}{\partial S_t^2} + rS_t \frac{\partial V_{S,t}}{\partial S_t} - rV_{S,t} = 0 \tag{1.1.1}$$

Where $V_{S,t}$ is the option price with respect to time and underlying's price. The risk-free rate is denoted as r, the volatility of the underlying asset is σ and the underlying's price with respect to time is denoted as S_t.

Black-Scholes Formula

This formula can calculate the value of European calls or puts. It is obtained by solving the equation (1.1.1) with respect to the conditions for a European call option:

- $c_{0,t} = 0, \forall\, t \in [0, T]$
- $c_{S,t} \to S_t,\ for\ S_t \gg 0$
- $c_{S,t} = (S_T - K, 0)$

Where $c_{S,t}$ is the call price with respect to time and underlying's price. Strike price is denoted as K, and T represents the time of the option's maturity.

The above conditions are basically describing the main characteristics of the option, like what happens to the option's price when the underlying security completely loses value or how its value is calculated at maturity.

Note that for European puts, the above conditions are slightly different, to capture the information of put options unique characteristics.

Finally, the previously described process gives us the formula for the price of a European Call option with respect to time and value of the underlying security:

$$c_{S,t} = F(d_1)S_t - F(d_2)Ke^{-r(T-t)} \tag{1.1.2}$$

Where the parameters d_1, d_2 are defined as:

$$d_1 = \frac{ln\left(\frac{S_t}{K}\right) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{1.1.3}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{1.1.4}$$

For better intuition regarding how a European call's price changes with respect to the value of underlying security changes and for different time to maturity values we performed some experimental calculation on an IPython notebook and we ended up with the following three-dimensional plot:

1: European call prices for different values of underlying security and time to maturity.

Note that, throughout our calculations we assumed Strike Price (K) and volatility (σ) to be constant.

After defining the call price for every time-point up to maturity with equation (1.1.2), we can leverage put-call parity to extract the value of the corresponding put option.

Put-call parity, is a static mathematical formula that implies a relationship between two identical options (a put and a call option with same strike prices and maturities) based on the simple idea that a portfolio containing at time t a call option and the discounted at time t strike price value in cash, should have identical value with a portfolio formed by the, corresponding to previous call, put option and the underlying security of both options. Hence, we can describe this assumption via the mathematical formula:

$$c_t + Ke^{-r(T-t)} = p_t + S_t \tag{1.1.5}$$

Note, at this point that put call parity only stands for European options, as American options can be exercised at any time during time interval $[0, T]$ and not strictly at maturity. Also, traders very often use put-call parity to investigate arbitrage opportunities in markets of low liquidity.

Lastly, by combining equations (1.1.5) and (1.1.2) we can calculate the value of a European put option with strike price and maturity identical to those of (1.1.2) equation's call as:

$$p_{S,t} = Ke^{-r(T-t)} - S_t - F(d_1)S_t + F(d_2)Ke^{-r(T-t)}$$

$$p_{S,t} = F(-d_2)Ke^{-r(T-t)} - F(-d_1)S_t \qquad (1.1.6)$$

1.2 Alternative Models for Option Pricing

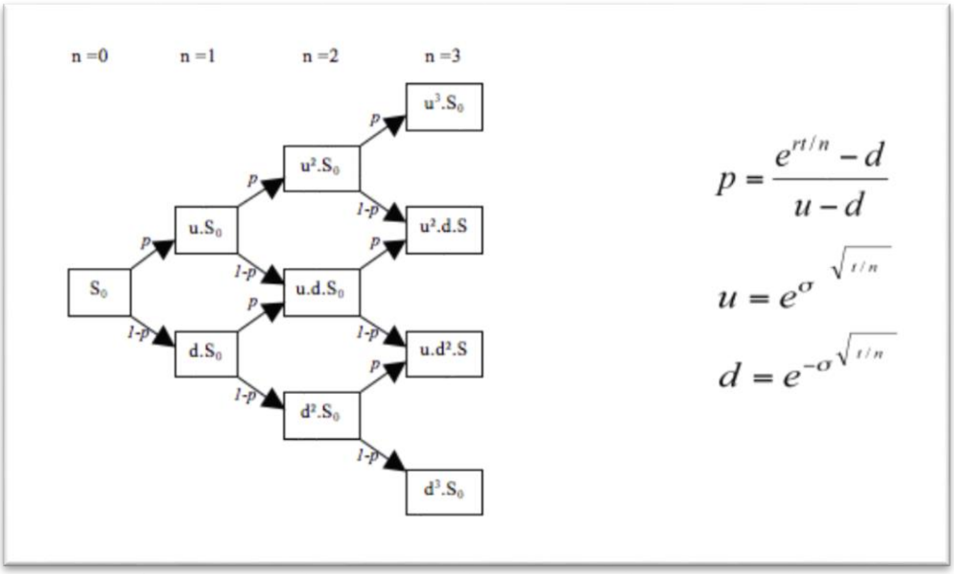
Apart from Black-Scholes there are some other very interesting models trying to estimate what the price of an option should be, given the input characteristics for that option like the Strike Price, Time to Maturity etc. Combined with Black-Scholes-Merton model, these approaches are heavily used by traders and other professionals as a very handful way of estimating the fair price for an instrument so that they can build portfolios and strategies.

The Cox-Ross-Rubinstein Binomial Option-Pricing Model

Binomial Option-Pricing Model is a risk-neutral model used to price instrument such as vanilla options. It is heavily used for path-dependent options like American Put Options and American Call Options.

In this model, after defining the number of periods we want to investigate, we build a decision tree for the evolution of the underlying security over these periods. At every node of our decision tree exist three possible paths/scenarios (markets will go up, down or stay neutral) each one associated with a probability. After building our tree of the underlying's price possible evolutions we can then define the pay-off of our option at each node.

To help our understanding with an example, imagine the following example, where we want to build a three-period model to price a Call Option that has a stock as the underlying Asset. At every period, markets can go up or down increasing or decreasing the security's value by a pre-define factor u or d with a probability of p or $1-p$ respectively.



2: Presentation of binomial model. (Source: https://en.wikipedia.org/wiki/Binomial_options_pricing_model)

After constructing our tree containing underlying security values, an algorithmic process can be followed to price the Call Option:

1. Calculate the price of option at each final node. For the example of Call Option, use the formula: $\max\left(S_3^j - K, 0\right), with\, j \in \{u, d\}$.
2. Use backpropagation approach to define the option value at each one of the "earlier" nodes up to the $n = 0$ node that represents option's value for the present time.
3. For every node $i \in [0, 3)$ use the formula:

$$c_i = e^{-r}[pc_{i+1}^u + (1 - p)c_{i+1}^d].$$

Note that, in the last formula, the option's value at period i is calculated as the discounted value (using the assumed risk-free rate r) of the expected value in the next period.

Monte Carlo Methods for Option Pricing

Monte Carlo Simulation is a technique to generate multiple possible future outcomes of a random variable [16] that are also commonly called realizations of a stochastic process; that is followed by the quantity modelized by that random variable. In Mathematical Finance, a Monte Carlo approach for option pricing aims into estimating the price of an option that is exposed into some forms of uncertainty and the valuation process relies on risk-neutral valuation as the current value of the examined option is just the discounted value (using a pre-defined risk-free rate) of the value that was calculated for the option in the end of the simulated process.

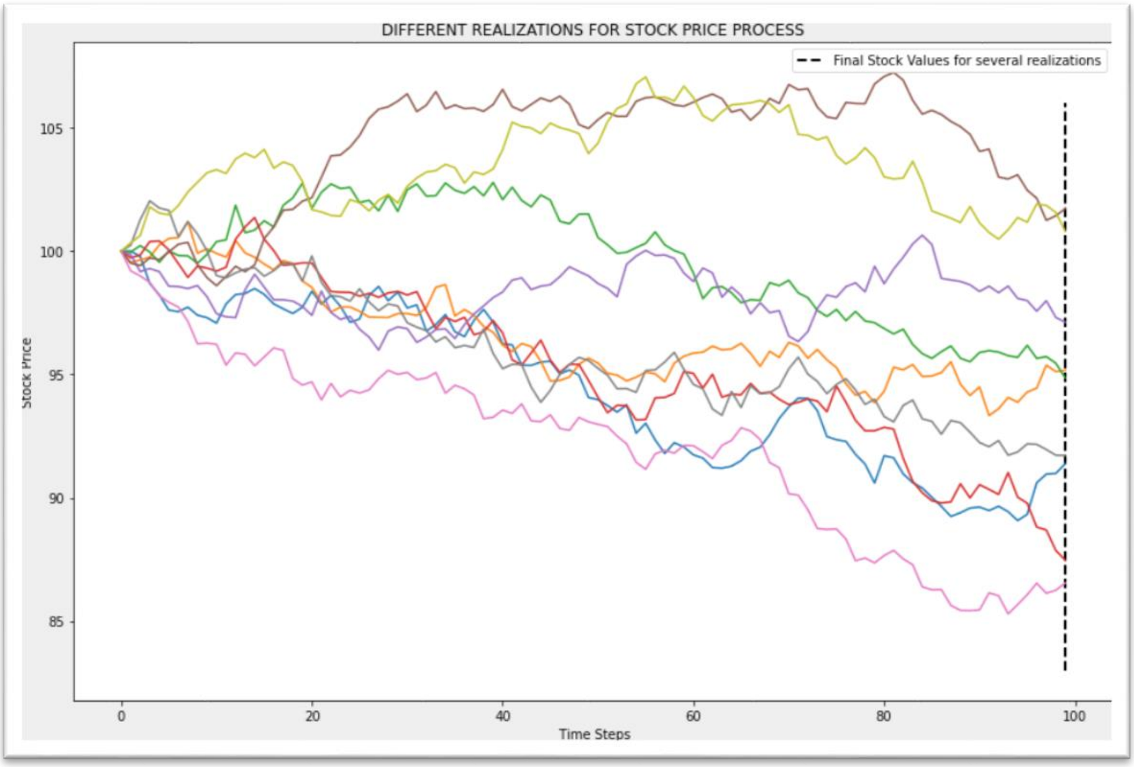
In practice, most of the times we consider that for a Call Option with a Stock as the underlying security, the stock price to be the main source of uncertainty. Hence, we can estimate the option price if we simulate the process that the value of stock (let it be denoted as S_t) follows. In our analysis, we consider that the stock price follows a Brownian motion like:

$$dS_t = \mu\, dt + \sigma\, S_t\, dW_t \qquad (1.1.7)$$

where the coefficients of both deterministic and stochastic term try to replicate the conditions in the market.

After conducting several realizations of process described by equation (1.1.7), we can take an average forecast of the stock price in the future. Therefore, we can calculate the value of option then via its pay-off function. After completing these steps, we can price the option by discounting its future price into the present.

For better intuition, this approach can be demonstrated via an example. In an IPython notebook, ten realizations of the process followed by the stock price were constructed with $\mu = 0.05$ standing for the risk-free term and a volatility of 5%. For the Call option we assumed a Strike Price of 100. For every one of ten different final outcomes of the stock price the Call payoff was calculated.



3: Monte Carlo simulation example.

The price of the option today, was just the discounted average of the ten payoffs. The formula we used can be summarized in the following equation:

$$c_0 = e^{-00.5T}[\frac{1}{10}(\max(S_{T,1} - K, 0) + \max(S_{T,2} - K, 0) + \cdots + \max(S_{T,10} - K, 0)\ )]$$

where c_0 stands for the current price of option, T is the final time-point for each of the ten realizations, K denotes option's Strike Price and each of $S_{T,i}\ , i\ \in \{1,2,3, \dots ,10\}$ representing the final stock price in i-th realization.

1.3 Description of the CCA Model

The main objective of CCA is to use Merton's (1974) model to investigate if the debt issuer that can be a bank, an economic sector, a Non-Financial Corporation or even a country's households will have enough assets to honor their obligations at maturity [1]. The issuer will be able to honor its obligations at maturity, if the market value of his assets is greater than the debt to be repaid. If that is not the case, the issuer declares bankruptcy and all the assets now belong to the creditors. The loss of creditors in that case, can also be quantified via the difference between assets and total liabilities.

At this point, we should get an intuition for the reason behind the CCA and Option Theory association. As we already know a call option gives the holder the right-not the obligation-to buy the underlying asset at a predetermined price, known as the "strike price" at option's maturity or during the option's lifespan for European and American options respectively. On the other hand, a put option provides the owner with the right to sell the underlying security at a predetermined strike price. For European options, we can summarize the above in the following non-linear functions:

$$c = (S_T - K, 0) \tag{1.2.1}$$

$$p = max(K - S_T, 0) \tag{1.2.2}$$

Where Kstands for the strike price of option and S_t stands for the underlying security market value at a specific time t, with T representing maturity.

Hence, we can associate the decision of a debt issuer with whether he will honor his obligations at maturity with a decision or whether a call option will be exercised at maturity. Supposing that he would exercise the call option if underlying's market value exceeded strike price, the same way he will pay his liabilities if the market value of assets at maturity exceeds his nominal debt. As, in the simplest approach, a firm's equity market value equals the difference between assets and debt, we can handle the issuer's equity market value as a call option with assets representing the underlying security and strike price equal to the nominal debt.

An important aspect of our analysis is that debt repayments are not certain and whether they will happen heavily relates on assets value. Therefore, we cannot discount values based on a risk-free term and we need to take uncertainty under consideration. This uncertainty is described in CCA with the use of put options.

Our goal is to create a formula for debt for the investor , that could be discounted using a risk-free rate, and could also contain the information associated with the case of non-repayment for debt. This is achieved through a put option with the firm's total assets as the underlying security and strike price to be the debt amount to be paid at maturity [1]. What the put option represents here, is exactly what happens in case the issuer of debt is unable to repay that amount at maturity. Then that amount's value (strike price of put option) will be greater than the assets' value (underlying security of put option), hence put option will be exercised returning to the holder (the investor) a recovery in case of default from the debt issuer.

So now, we can describe the issuer's risky-debt as the summation of default-free amount (B) to be paid at maturity plus the previously described put option. If, at maturity, the assets of the issuer are greater than that amount ; creditors will be paid normally and the put option's value will be equal to zero, otherwise the investor suffers a loss, which is the equivalent of liquidating issuer's assets in case of no-repayment (guarantee against default). The risky-debt, in that case, is described by the following formula:

$$D_t = Be^{-r(T-t)} - p_t \qquad (1.2.3)$$

where:

The risky-debt at a specific time point is denoted as D_t, value of the previously described put option is denoted as p_t and the default-free amount to be paid at maturity is denoted as B. Also, as usual, t refers to time and belongs in the interval $[0, T]$, T represents the maturity time and r is the assumed risk-free rate in our analysis.

1.4 Distress Barrier

In order to deploy the CCA for quantifying the risks associated with an economic entity, we use the concept of Distress Barrier (B_t). Distress barrier represents what we previously described as default-free debt amount and is the amount to be repaid by the issuer to the creditors at maturity. Then, if assets are found to be less than the distress barrier the issuer is in distress or default.

Distress barrier is described in a mathematical formula, as the weighted sum of long-term debt (D_L) and short-term debt (D_S). In fact, we consider the whole short-term debt and a fraction of the long-term one.

$$B_t = D_{S,t} + aD_{L,t} \qquad (1.3.1)$$

where:

$$a \in [0,1]$$

We should note here that, by changing the a value, we get different distress barriers corresponding to different economic entities and fitting their unique features.

1.5 Approximation for Equity Market Value

As we stated earlier, whether the issuer will be able to repay his debt at maturity is described as a call option with underlying security to be the assets and strike price to be the amount to be repaid. The value of that option, with respect to notation we set before, is:

$$S_t \equiv c_t = F(d_1)A_t - F(d_2)Be^{-r(T-t)}, \forall t \in [0,T) \qquad (1.4.1)$$

Where A_t is the market value of assets at time t, $Be^{-r(T-t)}$, is the distress barrier discounted at time t and r is the risk-free rate we use throughout our analysis.

Although at maturity $(t \equiv T)$ the above function describes exactly what explained above, during the whole time-spectrum ($t \in [0,T]$) and assuming that assets are greater than liabilities, this function is also used as an approximation for the equity market value of the entity issuing the debt. Hence, we can write that:

$$S_t = A_t - D_t, \forall t \in [0,T] \qquad (1.4.2)$$

1.6 Applications of CCA

In the framework described earlier, where market values of assets for the debt issuer are extracted with CCA in order to calculate insightful risk metrics, we are going to present the methodology used for different sectors:

- Standard CCA approach
- Variation of methodology for Non-Financial Corporations
- Variation of methodology for Household (HH) sector
- CDS valuation for the Government Sector (GVT)

2. The Standard CCA Approach

2.1 Formalization of Model

As already mentioned before, the market value of equity can be described as a junior claim on the assets, and hence as a call option on them with liabilities as strike price [2]. With that in mind, we can describe it as in equation (1.3.2): $S_t = A_t - D_t, \forall t \in [0, T]$. Where S_t, A_t, D_t represent equity market value, assets value and risky debt at every time point from start to maturity respectively.

We assume that the assets follow a stochastic process with a drift of μ_A in the deterministic term and their volatility σ_A affecting the stochastic term.

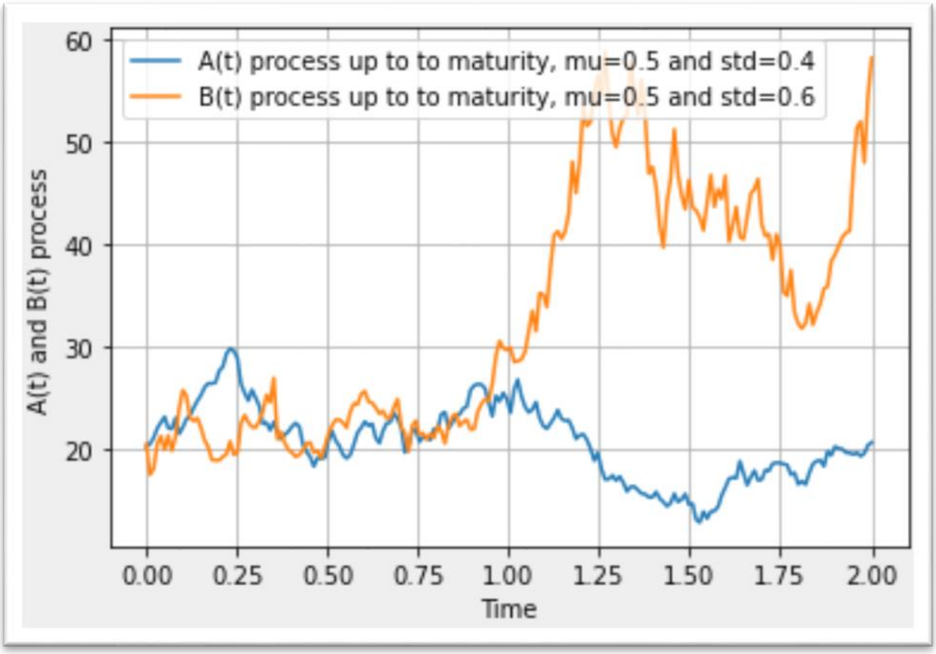
$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dW_t \tag{2.1.1}$$

Here, W_t represents the random component of the process modeled as a standard Brownian motion. In discrete time, the random part follows a normal distribution of zero mean and variance equal to $\sqrt{dt}$. In continuous time, the above equation is referred to as a geometric Brownian motion [15], where W_t is a Wiener-process with $dW_t \sim N(0, dt)$ [2].

Keeping in mind the assumptions above and defying an initial value for assets (A_0), assets value for any time-point in the interval from start to maturity is given from the following mathematical formula:

$$A_t = A_0 e^{\left(\mu_A - \frac{\sigma_A^2}{2}\right)t + \sigma_A W_t}, \forall\, t \in [0, T] \tag{2.1.2}$$

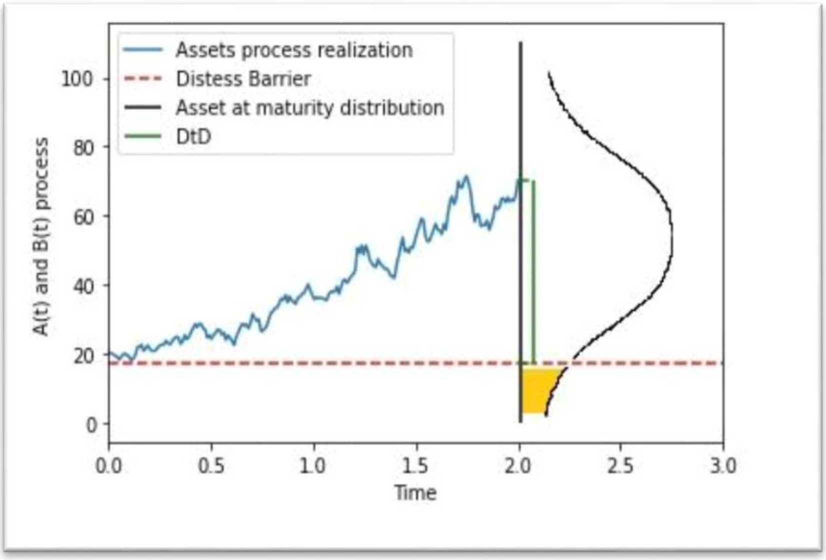
For better understanding, we created some realizations of stochastic processes to demonstrate.



4: Realizations of stochastic moves, in the same time interval but with different drifts and volatilities. The graph was generated in Jupyter notebook for IPython environment.

In the above graph we can observe exactly what the previous equations describe. The sequential values that A_t gets in the interval $[0, T]$ are derived from the summation of a deterministic variable with already known values through the interval and a random variable whose values are randomly picked from a distribution.

As we already explained, the debt issuer will declare bankruptcy or will be in distress if at maturity $(t \equiv T)$ assets value A_T is less than the distress barrier B. This is visualized into the following graph:



5: CCA approach visualization with respect to assets stochastic process realization. The graph was generated in Jupyter Notebook for IPython environment.

Here apart from the methodology intuition part, we can also visualize some very important risk metrics, associated with CCA that will be calculated later on. The first metric is the Distance-to-Distress (DtD) denoted here as the green line. This metric basically represents the distance between the final value of the assets and the distress barrier and the higher value it gets the safer the issuer is from distress. The second noticeable metric in the above graph is Probability-of-Default visualized here as the orange area under A_T probability density function. Basically, this metric provides us with the probability that assets of debt will fall below the distress barrier at maturity.

2.2 Calculation of Assets Value

Treating issuer's equity market value as a call option on assets, we can retrieve the following equation using Black and Scholes (1973) methodology:

$$S_t = A_t\, F(d_1) - Be^{-r(T-t)}\, F(d_2) \qquad (2.2.1)$$

Also, by using equation (1.3.2) we can also write:

$$D_t = A_t - A_t\, F(d_1) + Be^{-r(T-t)}\, F(d_2) \qquad (2.2.2)$$

Where:

$$d_1 = \frac{ln\left(\frac{A_t}{B}\right) + (r + \frac{\sigma_A^2}{2})(T-t)}{\sigma_A\sqrt{T-t}},\ d_2 = d_1 - \sigma_A\sqrt{T-t} \qquad (2.2.3)$$

And F denotes the cumulative normal distribution, r is the chosen risk-free rate and σ_A is the assets market value volatility of returns.

Note that, in order for the above formulas to be accurate they must describe an economic system with no transaction costs, arbitrage opportunities and dividend distribution. Another important assumption of this analysis is that bankruptcy can only occur at at maturity and by no means in the meantime $[0, T)$.

Our final objective is to calculate assets value and volatility as with the knowledge of them we can calculate both the value of our debt and some very informative risk metrics. On this direction and leveraging the fact that we can retrieve equity value and volatility of equity from market, we apply Itô's lemma on S [15]:

$$dS_t = \left(\frac{\partial S_t}{\partial A_t}\mu A_t + \frac{\partial S_t}{\partial t} + \frac{1}{2}\frac{\partial^2 S_t}{\partial A_t^2}\sigma^2 A_t^2\right)dt + \frac{\partial S_t}{\partial A_t}\sigma A_t\, dW_t \qquad (2.2.4)$$

From which equation, we can retrieve:

$$S_t\sigma_S = A_t\sigma_A\frac{\partial S_t}{\partial A_t} = A_t\sigma_A F(d_1) \qquad (2.2.5)$$

Equations (2.2.5) combined with equation (2.2.1) form a two-by-two system of equations, which let us calculate the two unknown entities that are assets and their volatility. After solving that system, with respect to A_t, σ_A we obtain a time-series for both assets and the volatility of their returns in the under-investigation time interval.

An important notice should be made here, as it is also highlighted in the literature. The above system is highly-sensitive to initial values for equity market value and volatility of equity returns. In practice, we observe large deviations in the final solutions for assets and their volatilities, even for small changes in initial conditions [2].

Note that, the above approach, where in order to obtain values of Assets and their volatility we solve the system of equations (2.2.5) and (2.2.1), allows changes in the volatility of Assets (σ_A) value. However, to be consistent with initial model's theory we should treat σ_A as a constant value [14] and only allow Asset value to change over time. In this framework and to keep up with mathematical strictness, the best way of obtaining A_t values and their "risk" σ_A would be via an equation that tries to minimize at every time-point in the interval $[0, T]$ the deviation from what equations (2.2.5) and (2.2.1) state [1]. Hence, this objective can be described as the following optimization problem:

$$\min_{A_t,\sigma_A} \sum\nolimits_{t\in[0,T]}[A_t\, F(d_1) - Be^{-r(T-t)}\, F(d_2)]^2 + \sum\nolimits_{t\in[0,T]}[\,A_t\sigma_A - S_t\sigma_S]^2 \qquad (2.2.6)$$

Where, we try to ensure that at every time-spot in the interval of interest, we pick the value for A_t, σ_A that make equations that formed the initial system stand or in the worst case have the least deviation possible.

2.3 Risk Metrics Calculation

In the present approach, as we already stated before, we use Contingent Claim Analysis to quantify credit risk. After solving the system of equations described in section 2.2, we can now easily calculate the value of debt, via equation (2.2.2) and various interesting risk metrics, leveraging the newly acquired knowledge of A_t and σ_A values. The metrics used for calculating credit risk are presented below:

2.3.1 Distance to Distress (DtD)

Distance to Distress (DtD) stands for the distance between assets value and the distress barrier assumed for the specific debt issuer at the start of our analysis. Obviously, the higher value this metric gets the more distant the default or distress scenario is, as assets value would be significantly over the distress barrier in that case.

In CCA, DtD equals to d_2 and statistical terms, it describes the number of standard deviations final assets value is away from distress barrier. We use the following formula for this metric:

$$DtD = d_2 = \frac{ln\left(\frac{A_t}{B}\right) + (r - \frac{\sigma_A^2}{2})(T-t)}{\sigma_A\sqrt{T-t}} \tag{2.3.1}$$

In literature, a more naïve approach on calculation of this metric is often used [2]:

$$DtD = \frac{A_t - B}{A_t \sigma_A} \tag{2.3.2}$$

2.3.2 Probability of Default (PD)

Probability of Default (PD) is a metric that provides us with the probability, that at a specific time t, the value of assets fall below the distress barrier threshold. For known value of the distress barrier at time t, B_t we can calculate the previously described metric as:

$$PD = Pr\ Pr\ (A_t \leq B_t)\ = F\left(-\frac{ln\left(\frac{A_0}{B_t}\right) + \left(\mu_A - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}\right) \tag{2.3.3}$$

In the above equation, the argument of the normal cumulative function has great similarities with the previously described risk metric, Distance-to-Default. In fact, the absolute values are exactly the same, if we use the risk-free rate r instead of the above drift term μ_A.

Calculating probability with the exact (2.3.3) formula, leads to the calculation of the "actual" Probability of Default [2]. Throughout our analysis, we will stay in "risk-neutral world" for our calculations and we will instead use the "risk-neutral" Probability of Default that is calculated with (2.3.3) equation by substituting drift term of assets with the risk-free rate.

Note that, because risk-free rate in practice tends to be lower than the drift term of assets, the "actual" Probabilities of Default tend to be lower than the risk-free ones.

2.3.3 Ex-ante expected loss

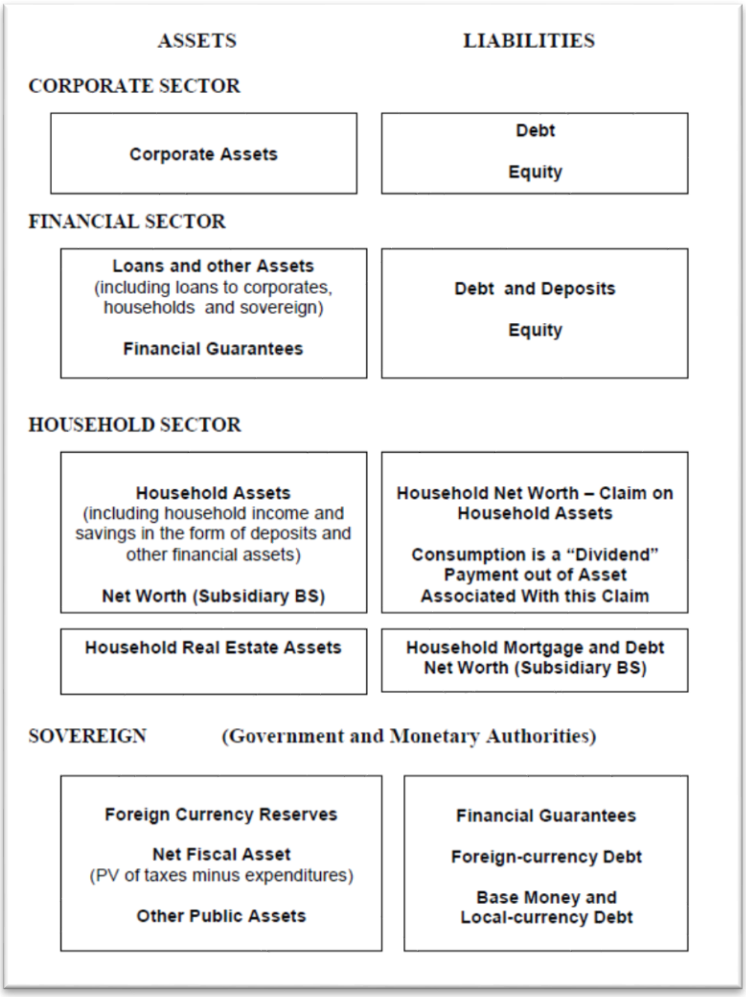
This metric, basically represents the loss the investor would suffer if the issuer of debt declares bankruptcy at maturity. In CCA terminology, it is the value of the previously described put option and can be calculated via the following formula:

$$max(0, D_T - A_T) \tag{2.3.4}$$

3. CCA Extension to Other Sectors

The analysis described in the previous sections can change its direction and start focusing on whole sectors of specific characteristics and not on pre-determined debt and instances, as it was initially designed to. In order to apply CCA to various sectors like households of a country (HH) or Non-Financial Corporations (NFC) we should re-adjust our model and its parameters to fit the available data for the sector the best possible way. In the sections that follow, we are going to describe the methodology that leads to the calculation for each specific sector of interest, exploiting the knowledge provided by data we have available.

As stated from the beginning of the Contigent Claim Analysis description, the whole process is based on the construction of a risk-adjusted balance sheet for the under-examination instance. In the following graph, we present a highly informative categorization of how we model the various quantities in each CCA application case in order to continue our analysis.



6: 1 Balance-Sheet construction guide for CCA Analysis. Source: Dale F. Gray, Robert C. Merton, Zvi Bodie. New Framework for Measuring and Managing Macrofinancial Risk and Financial Stability [12].

1

3.1 Non-Financial Corporations (NFC)

Here, we treat the whole under-investigation sector as it was one entity according to Gray and Malone (2008) approach [14]. To follow the methodology analyzed in chapter 2, we need to first make some assumptions on how we are going to "construct" the model's parameters from the available data. The main focus of this process is what our equity market value and volatility would stand for and how we will define the sector's distress barrier. Setting these up, would allow us to calculate assets and their volatility and then exploit the desired risk-metrics.

The equity market value of under examination sector (S_t) is approximated as the sum of the market capitalization of all members of that sector. For example, if we try to analyze the banking sector of a country, the equity market value in our equations would be the summation of market capitalization for every bank in that country.

A building-block for Contingent Claim Analysis is the distress barrier assumed at the start of every analysis. Distress barrier is given by equation (1.2.1) and as we stated at section 1.2 the value of a parameter is chosen with respect to each sector's individual characteristics. For NFC's, the most common practical approach is to set a equal to 0.5 [2]. This decision leads to a distress barrier that is the sum of short-term debt and half of the long-term debt.

Last but not least, we should also define the equity volatility of the sector with respect to the available data. We consider the volatility of NFC sector's equity a combination of sector's corporations' volatilities. In practice, there are two popular approaches for calculating the whole sector volatility, that although their differences they produce similar outputs. The first technique treats sector volatility as the average of sector's "members" volatilities, with each volatility weighted with respect to each market capitalization contribution to the whole sector's market capitalization. The second approach, focuses again on the weighted average of corporation volatilities but with respect to correlation between equities. Note that most of the time estimating NFC sector volatility with the second approach leads to higher DtD values, as the corresponding volatility of the sector tends to be lower due to the existence of negative correlations among the sector's equities.

3.2 Households Sector (HH)

In order to apply our model as defined in chapter 2 for a country's household (HH) sector we have, again, to redefine the way we extract our important parameters from our available data. One of the trickiest things about this process is that households do not issue equity [2].

Although very serious attempts have been made in this direction and balance sheets with respect to household characteristics have been created, it is very difficult to "construct" this kind of balance sheets as the different items that they consist of are sometimes impossible to quantify. Another important drawback we have to deal with, is that as there has never been reported a massive default from the HH sector, our application of CCA results in extremely high DtD values and close to zero PD values.

Applying the approach made by Castrén and Kavonius (2009), leveraging the knowledge provided from our dataset we approximate S_t as the net financial worth of the whole HH sector. Also, the distress barrier in that case is modeled through the total debt of the household that mainly contains mortgages. Finally, the HH equity volatility through the volatility of the ten -year government bond for every country's households.

The main reason behind that last assumption about volatility is the common belief that the government bond volatility is a very good estimation for HH sector volatility. This thought can be justified as a government bond reflects information about critical features of a domestic economy, as growth and inflation.

3.3 The Government Sector (GVT)

In order to analyze the Government Sector (GVT) we have to change our approach of constructing balance sheets and model the features of interest via options. Also, the GVT sector does not issue any equity to capture some information about S_t. Lastly, if we continued in the same direction for extracting risk metrics for the GVT sector, we would face the practical problem of market value of debt constantly exceeding the distress barrier [2].

Following the methodology firstly presented by Gray et al. (2013) will extract risk-neutral PD and DtD through risky debt valuation. Here, we are going to use the market traded Credit Default Swaps (CDS) and their spread ($s = \frac{CDS_t}{10000}$), that are financial derivatives or contracts that allow investors to interchange their credit risks [5].

In the first step of our analysis, we define the Expected Loss Ratio (ELR), that is the Expected Loss (EL) per unit of riskless debt. Hence, we calculate ELR via the following formula:

$$ELR = \frac{Be^{-r(T-t)} - D_t}{Be^{-r(T-t)}} = 1 - \frac{D_t}{Be^{-r(T-t)}} \tag{3.3.1}$$

The spread s relates to ELR via the following equation [2]:

$$s = -\frac{1}{T} log(1 - ELR) \tag{3.3.2}$$

Thus, using equations (3.3.1) and (3.3.2) we can solve for debt value and get as a result the formula:

$$D_t = Be^{-(s+r)(T-t)} \tag{3.3.3}$$

Now, given the CDS spread s, we can calculate the risk-neutral PD through the formula [6]:

$$PD = 1 - e^{-\frac{s}{1-\rho}} \tag{3.3.4}$$

Where ρ is used to denote the recovery rate.

Finally, working in "risk-neutral world" where we use the risk-free rate for every deterministic value change, we can calculate Distance-to-Distress metric according to what we discussed in chapter 2. There, we defined Probability of default as:

$$PD = F(-d_2) \tag{3.3.5}$$

Since we already have mentioned that $DtD = d_2$, we can now calculate DtD with the formula below:

$$DtD = -F^{-1}(PD) = -F^{-1}(1 - e^{-\frac{s}{1-\rho}}) \tag{3.3.6}$$

4. Application for Greek Banking Sector

In the present part of the project, we try to apply the previously described technique to a Greek Bank's case in a time range from first 2001 quarter to third quarter of 2020. Our main goal is to apply the Contingent Claim Analysis to extract from our available data the values for both risky-debt and Assets and continue with the calculation of Distance-to-Distress and Probability of Default risk metrics.

4.1 Available Data

For modeling essential features of the under-examination bank we used information from the bank's balance sheets. Leveraging the knowledge from them, we extracted the time-series for short-term and long-term debts that helped us define the distress barrier by setting our a value equal to 0.5.

To define the equity market value, we used the public data for the bank's equity market capitalization. On the other hand, we used daily data from the bank's stock price closing value to extract the volatility of equity returns with a 30-days' time lag.

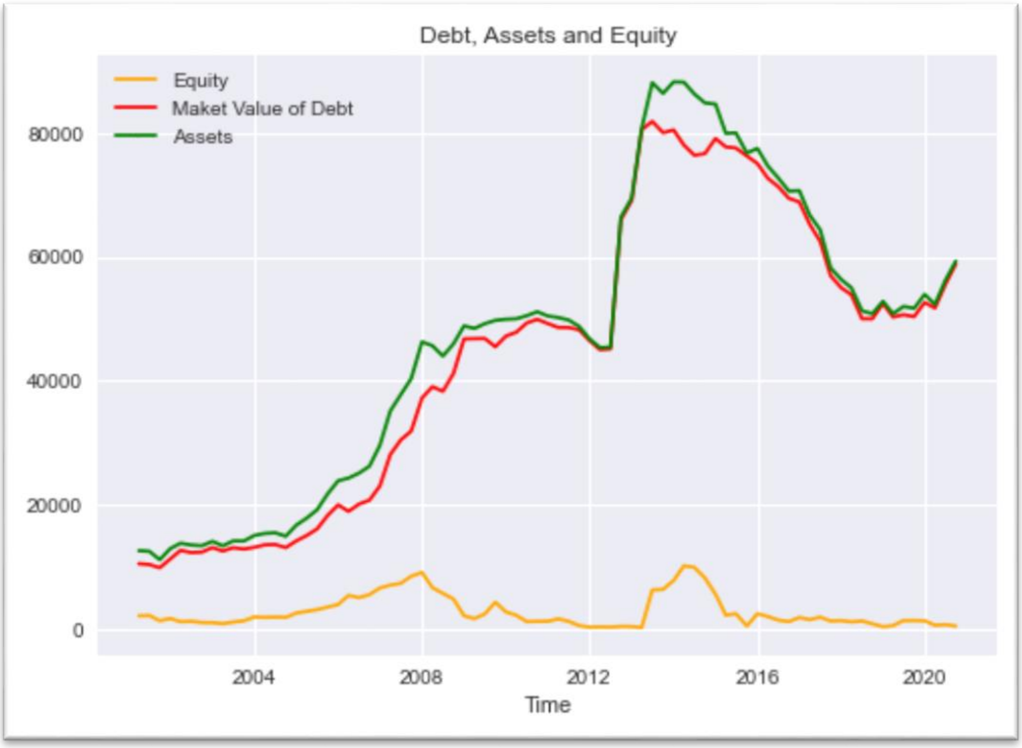
The used risk-free rate r, was constructed using the yield of the 3-month Greek Government Bond.

After defying the Assets market value via CCA we calculated the market value of risky debt in our final data, by subtracting equity from them following equation (1.3.2):

$$S_t = A_t - D_t, \forall t \ \in [0, T].$$

4.2 Visualization of Results

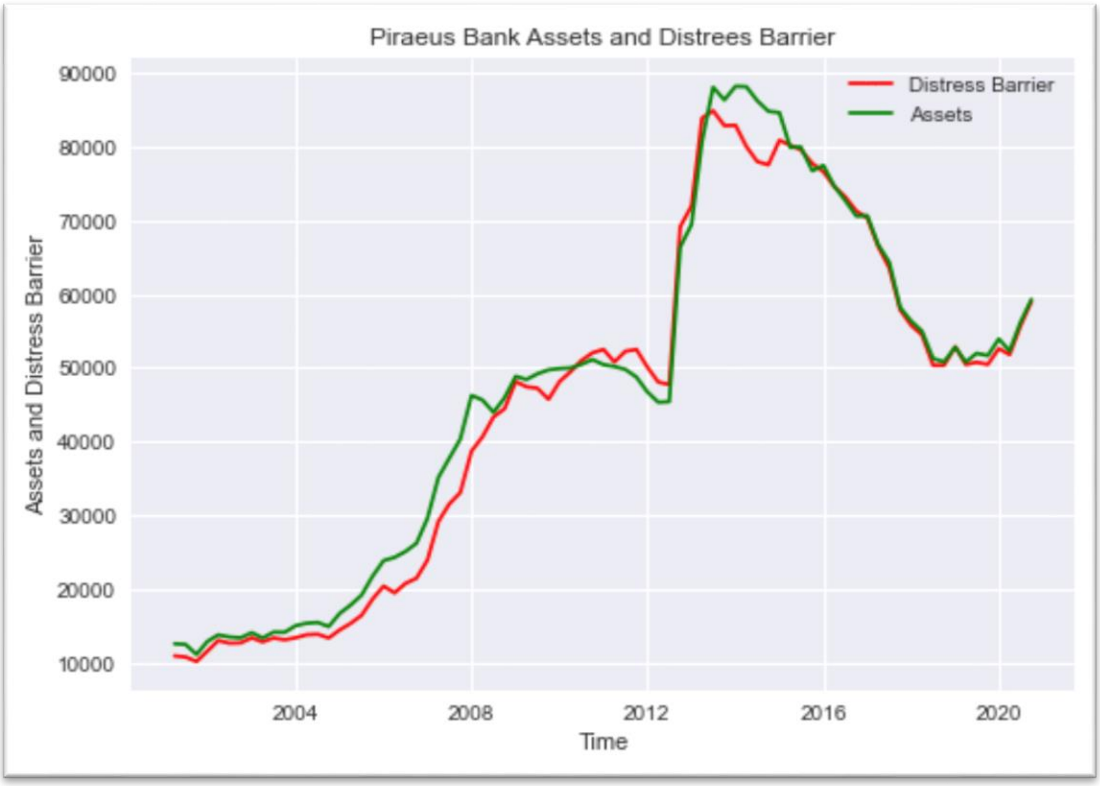
After importing the data into our IPython notebook, we began our data processing. Performing our analysis, we came up with values for Assets, Debt, Distress Barrier by solving the defined two by two system of equations with respect to A_t, σ_A and more important DtD and PD metrics throughout the time interval of interest (Q1-2001 to Q3 2020). In the next lines will present our findings in the form of graphs:



7: Assets, Equity and Debt market values, calculated from available data. The graph was generated in Jupyter Notebook for IPython environment.

In the above graph, we can observe the evolution of Equity, Debt and Assets values from early 2001 to late 2020. Debt and assets seem to have a similar movement, while equity is in very lower magnitudes. An interesting fact here, is the significant and continuing rise in debt's value from the beginning of Greek Crisis until approximately the early 2015, when we have a turning point, from where debt starting to slightly decrease in value. The above results seem to approximately satisfy the assumed equation (1.3.2) that is a building block of our analysis.

The next graph to be presented is the one of both Distress Barrier and Assets with respect to time. As we stated from the start of this project the debt issuer will be in distress if the value of assets falls below the distress barrier. Combined with the graph where we plot the values of risk-metrics, the next is one of the most interesting and informative graphs that reflect the condition of the under-examination institute in the long-term, with respect to its liabilities.



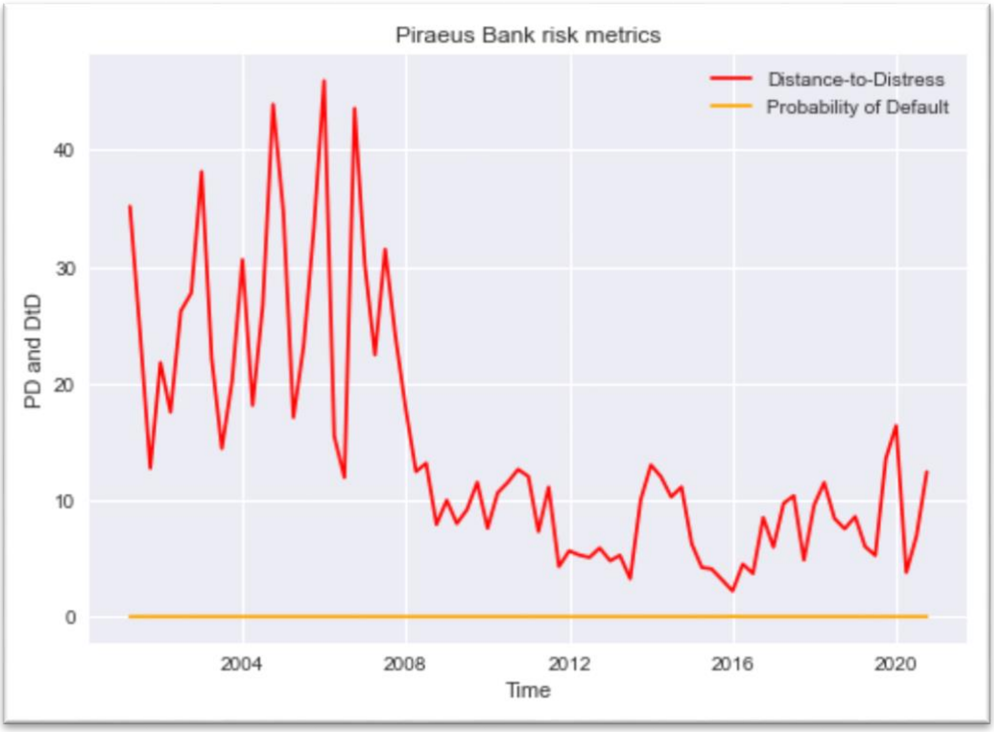
*8: Distress Barrier and Assets results from our analysis. The graph was generated in Jupyter Notebook for IPython environment *.*

Here it is obvious that the difference between Assets and Distress Barrier reached its maximum values before the year 2009. That was a breaking point, from which mainly due to Greek financial crisis things started to rapidly change.

Beyond that time-point, we can observe a severe decrease in the difference between Assets and Distress Barrier values, a fact that reflects the proximity to distress for the Greek banking sector at that period. Indeed, during this time-period, Greek Banks were recapitalized several times.

Although there was an improvement in the Distress Barrier-Assets difference approximately around 2014, things started to get worse again from year 2015 and then, reflecting a period of high uncertainty about the Greek Economy.

In the final graph, we present the values of risk-metrics we calculated over the time period of 2001-2020. This is probably the most important and informative graph of our analysis, as the risk-metrics presented can be used as benchmarks that directly associate the condition of the examined bank with the value of a metric designed explicitly to describe this kind of scenarios.



9: Distance-to-Distress and Probability of Default as calculated through our analysis. The graph was generated using Jupyter Notebook for IPython environment.

A stand-out result here, is that Distance-to-Distress was at significantly higher levels before the Greek financial crisis, but this changed drastically from late 2007 combined with the global crisis started from the U.S.A at that year. Since then, DtD was never restored to its initial high scale, with a slight improvement observed the years after 2016. This impressive decline in Distance-to-Distress metric indicates that the examined instance was constantly closer to the default the years after 2008 and reflects the global concerns about the path of Greek Economy at that time-period.

On the other hand, Probability of Default results are not that insightful. The reason behind that, is that it is almost stable in extremely low-near zero values. This is justified as the above are the "risk-neutral" PD, where the risk-free rate is used and they tend to be lower than the "actual" ones [2]. As we already mentioned, this is the reason we mainly compare DtD's among sectors or over time in these kinds of analysis.

In conclusion, DtD metric seems to describe accurate enough the potential problems associated with the Banking Sector and can be used to produce efficient predictions for the condition of Banking Institutions.

4.3 Analytical Results of Analysis

The final results of the analysis, were exported from our IPython notebook to an excel workbook in order to be presented in a more analytical way. These results are presented in the following table, where each row represents a specific day when analysis was made and each column representing the values of a specific variable of the model.

Dates	Distress Barrier	Assets	Asset Volatility	Market Value of Debt	DtD	PD
2001- Q1	10922.39	12575.06	0.00518459	10478.90926	35.16915	2.9629E-271
2001-Q2	10771.03	12484.95	0.00772657	10348.68775	24.28435	1.4346E-130
2001-Q3	10156.58	11121.28	0.00980813	9814.113259	12.7436	1.6921E-37
2001-Q4	11576.32	12900.54	0.00635267	11232.31099	21.79464	1.3041E-105
2002-Q1	13016.55	13781.4	0.00500883	12621.33437	17.5528	2.8311E-69
2002-Q2	12649.7	13494.86	0.00361569	12274.61556	26.21054	1.0077E-151
2002-Q3	12694.83	13383.77	0.0029016	12347.39524	27.77544	4.295E-170
2002-Q4	13387.52	14084.17	0.00194253	13077.88795	38.16003	0
2003-Q1	12826.05	13359.35	0.00293821	12516.87834	22.16798	3.4989E-109
2003-Q2	13364.38	14159.09	0.00559887	13059.19614	14.44013	1.44638E-47
2003-Q3	13117.79	14132.05	0.00475347	12834.91611	20.25153	1.72171E-91
2003-Q4	13390.36	15032.08	0.00444346	13119.5265	30.62369	2.9612E-206
2004-Q1	13793.4	15360.7	0.00705774	13515.31933	18.13094	9.08147E-74
2004-Q2	13874.04	15469.57	0.00489196	13572.82103	26.73643	8.8803E-158
2004-Q3	13359.23	14913.91	0.00300699	13068.53368	43.92521	0
2004-Q4	14459.1	16703.73	0.0047544	14162.87524	34.70398	3.4308E-264
2005-Q1	15338.74	17826.59	0.01000323	15022.99303	17.10041	7.36817E-66
2005-Q2	16430.9	19173.41	0.00755246	16089.44791	23.21523	1.5977E-119
2005-Q3	18649.88	21749.53	0.00523835	18265.0566	33.32895	7.3523E-244
2005-Q4	20402.86	23854.04	0.00387517	19964.88624	45.92598	0
2006-Q1	19475.16	24291.19	0.01611995	18919.43391	15.49598	1.8465E-54
2006-Q2	20764.05	25091.79	0.01856039	20096.05015	11.9526	3.14608E-33
2006-Q3	21465.64	26204.84	0.0053958	20714.9032	43.56652	0
2006-Q4	23926.01	29581.29	0.00829393	23022.36606	30.21984	6.4999E-201
2007-Q1	29159.94	35129.56	0.00993241	28104.48569	22.45821	5.3189E-112
2007-Q2	31546.49	37756.7	0.00683158	30445.22049	31.50231	4.0385E-218
2007-Q3	33083.22	40342.98	0.00976218	31879.11877	24.11535	8.6265E-129
2007-Q4	38728.86	46268.74	0.01224522	37212.13645	17.78298	4.78735E-71
2008-Q1	40661.82	45635.6	0.01254374	39028.00841	12.46278	5.95725E-36
2008-Q2	43340.76	43969.3	0.01051006	38281.37471	13.17526	6.09022E-40
2008-Q3	44459.07	45949.17	0.01383313	41170.78971	7.931038	1.08661E-15
2008-Q4	48136.84	48844.29	0.00440864	46735.20801	10.00987	6.89645E-24
2009-Q1	47437.51	48425.73	0.00431717	46778.01336	8.016516	5.43935E-16
2009-Q2	47245.39	49195.58	0.00540795	46808.04675	9.196472	1.84948E-20
2009-Q3	45733.13	49732.41	0.00775686	45471.83307	11.5425	4.0274E-31

2009-Q4	48151.15	49911.06	0.00736411	47190.61379	7.607228	1.40019E-14
2010-Q1	49465.98	49990	0.00417997	47814.31427	10.6434	9.35837E-27
2010-Q2	50990.34	50477.15	0.00203741	49303.55546	11.54527	3.89975E-31
2010-Q3	52037.58	51119.43	0.00191028	49898.76193	12.65083	5.5352E-37
2010-Q4	52499.75	50440.91	0.00204533	49213.518	12.04315	1.05371E-33
2011-Q1	50797.45	50190.58	0.00442298	48589.92526	7.325701	1.18827E-13
2011-Q2	52248.92	49794.49	0.00223309	48571.12678	11.13815	4.09004E-29
2011-Q3	52501.15	48755.01	0.00256251	48217.64339	4.323741	7.67028E-06
2011-Q4	50104.03	46746.34	0.00109481	46457.08172	5.669065	7.17893E-09
2012-Q1	48066.11	45329.59	0.00139506	44994.59694	5.316371	5.29287E-08
2012-Q2	47749.15	45422.35	0.00128047	45126.23205	5.107351	1.63353E-07
2012-Q3	69148.89	66383.75	0.00103932	65976.72168	5.917045	1.63888E-09
2012-Q4	71981.88	69461.55	0.0011566	69075.10402	4.822998	7.07084E-07
2013-Q1	83880.8	80699.17	0.00052281	80476.22379	5.291406	6.06899E-08
2013-Q2	84878.1	88073.29	0.02226464	81834.02813	3.288862	0.000502966
2013-Q3	82852.95	86345.56	0.00758965	80004.85316	10.04543	4.81141E-24
2013-Q4	82919.35	88199.98	0.00706996	80438.94772	13.02457	4.43473E-39
2014-Q1	80037.28	88146.98	0.0101744	78001.84612	12.01265	1.52467E-33
2014-Q2	77986.34	86239.45	0.01182233	76354.24439	10.2919	3.83235E-25
2014-Q3	77543.02	84806.98	0.00909763	76630.32644	11.13955	4.02614E-29
2014-Q4	80871.84	84621.39	0.0108335	79068.58925	6.259538	1.9306E-10
2015-Q1	80206.71	79900.08	0.0065356	77709.47435	4.250315	1.06735E-05
2015-Q2	79583.67	79980.79	0.00751082	77539.99793	4.122631	1.87284E-05
2015-Q3	77734.21	76728.06	0.00196174	76252.10677	3.170714	0.000760323
2015-Q4	76610.95	77463.03	0.01418259	75035.20493	2.233736	0.012750226
2016-Q1	74585.69	74634.26	0.00592962	72651.83096	4.537139	2.85112E-06
2016-Q2	73198.88	72696.32	0.00522809	71290.27676	3.733107	9.45663E-05
2016-Q3	71263.46	70584.61	0.00195987	69414.35988	8.529334	7.35965E-18
2016-Q4	70326.09	70660.28	0.00436535	68835.03996	5.992894	1.03069E-09
2017-Q1	66433.87	66701.43	0.0023112	65216.7916	9.738154	1.03643E-22
2017-Q2	63560.85	64348.96	0.00284863	62471.32746	10.39414	1.31827E-25
2017-Q3	57853.81	58196.71	0.00449999	56930.39471	4.886511	5.13192E-07
2017-Q4	55784.5	56346.75	0.0025085	55006.20773	9.597508	4.09503E-22
2018-Q1	54470.44	54979.25	0.00178914	53857.0381	11.52574	4.89364E-31
2018-Q2	50360.99	51275.24	0.00298058	50000.19582	8.446883	1.49592E-17
2018-Q3	50352.28	50806.47	0.00215439	49985.54726	7.5601	2.0138E-14
2018-Q4	52873.5	52809.03	0.00080962	52442.234	8.608434	3.70326E-18
2019-Q1	50462	50794.92	0.00174172	50263.06828	6.042446	7.58975E-10
2019-Q2	50758	51976.78	0.00493515	50638.85827	5.28164	6.40163E-08
2019-Q3	50465	51673.33	0.00194432	50328.42439	13.56254	3.33922E-42
2019-Q4	52623	53928.61	0.00149457	52623.00014	16.39717	1.00184E-60
2020-Q1	51798	52306.07	0.00296846	51715.70434	3.822325	6.60997E-05
2020-Q2	55681.5	56188.05	0.00176612	55502.49369	6.950025	1.82611E-12
2020-Q3	59013.5	59288.82	0.00063291	58823.78265	12.44158	7.7701E-36

Where the observations related with the two important risk-metrics are similar to what we already described. Probability-of-Default, due to the reasons described earlier, remains in extremely low values that are practically zero and is not suitable for further analysis.

In contrast, the Distance-to-Distress metric has a significant fall around 2008 and continues reflecting the bad condition of the Banking Sector until 2016, when a slight improvement appears.

From the analysis above, appears that Distance-to-Distress is a good predictor of potential problems in the Greek Banking sector.

Appendix

In this section will be presented the code used to extract figures throughout the project. On the other hand, code used for CCA analysis and results visualization would be available upon request.

Figure 1:

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.cm as cm
from scipy.stats import norm
from math import sqrt, exp
from mpl_toolkits.mplot3d import Axes3D


class BS:
    '''
   create BS class so that we can instantly create Black-Sholes valu
es objects!
    '''

    def __init__(self, call, stock, strike, maturity, interest, vola
tility, dividend):
        self.call = call
        self.stock = stock
        self.strike = strike
        self.maturity = maturity
        self.interest = interest
        self.volatility = volatility
        self.dividend = dividend
        self.d1 = (self.volatility * sqrt(self.maturity)) ** (-1) *
(np.log(self.stock / self.strike) + (self.interest-self.dividend + s
elf.volatility ** 2 / 2) * self.maturity)
        self.d2 = self.d1 - self.volatility * sqrt(self.maturity)
   #define methods to our class to price the different options-attri
butes
    def price(self):
        if self.call:
            return exp(-self.dividend * self.maturity) * norm.cdf(se
lf.d1) * self.stock - norm.cdf(self.d2) * self.strike * exp(-self.in
terest * self.maturity)
        else:
            return norm.cdf(-self.d2) * self.strike * exp(-self.inte
rest * self.maturity) - norm.cdf(-self.d1) * self.stock * exp(-self.
dividend * self.maturity)

#Create arrays with the different input values for each variable
S = np.linspace(0, 200, 50) #stock price
T = np.linspace(0.001, 3, 50) #time to maturity
```

```
#s = np.linspace(0.001, 0.8, 50) #volatility
s=np.ones((50,1))/4

#Calculate call price for different price combinations
ct = np.array([])
for i in range(0, len(T)):
  ct = np.append(ct, BS(True, S, 80, T[i], 0.05, 0.3, 0.02).price(),
axis=0)
ct = ct.reshape(len(S), len(T))

#Calculate call price for volatility-price combinations
cs = np.array([])
for i in range(0, len(s)):
  cs = np.append(cs, BS(True, S, 80, 3, 0.05, s[i], 0.02).price(), a
xis=0)
cs = cs.reshape(len(S), len(s))

#Generate 3D graph
X1, Y1 = np.meshgrid(S, T)

fight = plt.figure()
ax = Axes3D(figct)
ax.plot_surface(X1, Y1, ct, rstride=1, cstride=1, cmap=cm.coolwarm,
shade='interp')
ax.view_init(27,-125)
plt.title('Call Option Price')
ax.set_xlabel('S')
ax.set_ylabel('T')
ax.set_zlabel('c')


X2, Y2 = np.meshgrid(S, s)

figcs = plt.figure(figsize=(12,8))
ax = Axes3D(figcs)
ax.plot_surface(X2, Y2, cs, rstride=1, cstride=1, cmap=cm.coolwarm,
shade='interp')
ax.view_init(27,-125)
plt.title('Call Option Price wrt Volatility')
ax.set_xlabel('Stock Price')
ax.set_ylabel('Volatility')
ax.set_zlabel('Call price')

plt.show()
```

Figure 3:

```
import numpy as np
from matplotlib import pyplot as plt


S0 = 100 #initial stock price
K = 100 #strike price
r = 0.05 #risk-free interest rate
sigma = 0.50 #volatility in market
T = 1 #time in years
N = 100 #number of steps within each simulation
deltat = T/N #time step
i = 10 #number of simulations
discount_factor = np.exp(-r*T) #discount factor

S = np.zeros([i,N])
t = range(0,N,1)


plt.figure(figsize=(12,8))
for y in range(0,i-1):
    S[y,0]=S0
    for x in range(0,N-1):
        S[y,x+1] = S[y,x]*(np.exp((r-(sigma**2)/2)*deltat + sigma*de
ltat*np.random.normal(0,1)))
    plt.plot(t,S[y])

    plt.title("DIFFERENT REALIZATIONS FOR STOCK PRICE PROCESS")
plt.xlabel(' Time Steps')
plt.ylabel('Stock Price')
plt.tight_layout()
plt.vlines(x=99,ymax=106,ymin=83, linestyle='--',color='black',linew
idth=2.0,label="Final Stock Values for several realizations")
plt.legend()
plt.show()


C = np.zeros((i-1,1), dtype=np.float16)
for y in range(0,i-1):
    C[y]=np.maximum(S[y,N-1]-K,0)

CallPayoffAverage = np.average(C)
CallPayoff = discount_factor*CallPayoffAverage
print(CallPayoff)
```

Figure 4:

```
import matplotlib.pyplot as plt
import numpy as np

T = 2
mu = 0.5
sigma = 0.4
S0 = 20
dt = 0.01
N = round(T/dt)
t = np.linspace(0, T, N)
W = np.random.standard_normal(size = N)
W = np.cumsum(W)*np.sqrt(dt) ### standard brownian motion
X = (mu+0.5*sigma**2)*t + sigma*W
S = S0*np.exp(X) ### geometric brownian motion


W2 = np.random.standard_normal(size = N)
W2 = np.cumsum(W2)*np.sqrt(dt)
X2 = (0.5+0.5*0.6**2)*t + 0.6*W2
S2 = S0*np.exp(X2)

plt.plot(t, S, label='A(t) process up to to maturity, mu=0.5 and 
std=0.4')
plt.plot(t, S2, label='B(t) process up to to maturity, mu=0.5 and 
std=0.6')
plt.xlabel("Time")
plt.ylabel('A(t) and B(t) process')
plt.legend()
plt.grid()
```

Figure 5:

```
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as stats
import math

T = 2
mu = 0.5
sigma = 0.4
S0 = 20
dt = 0.01
N = round(T/dt)
t = np.linspace(0, T, N)
W = np.random.standard_normal(size = N)
W = np.cumsum(W)*np.sqrt(dt) ### standard brownian motion
X = (mu+0.5*sigma**2)*t + sigma*W
S = S0*np.exp(X) ### geometric brownian motion

#plt.style.use('seaborn-bright')


plt.plot(t, S, label='Assets process realization')
plt.hlines(S0-3,xmin=0,xmax=T+1, color='red', label='Distess Barrier
',linestyle='--')
plt.vlines(T+0.01,ymin=0,ymax=S[-1]+40, color='black', label='Asset
at maturity distribution')
plt.vlines(T+0.08,ymin=S0-3,ymax=S[-1], color='green', label='DtD')
plt.hlines(S[-1],xmin=2,xmax=T+0.08,linestyle='--', color='green')
plt.hlines(S0-3,xmin=2,xmax=T+0.08,linestyle='--', color='green')
plt.xlabel("Time")
plt.xlim((0.00,3.00))
plt.ylabel('A(t) and B(t) process')
plt.legend(loc='upper left')
#plt.grid()
plt.show()
```

Figures 7, 8, 9 and the Summary table of Results:

Code available upon request.

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