



Master of Science in Business Mathematics

Non-Performing Loans and the Greek economy

An approach in modelling, forecasting and credit risk analysis



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Abstract

The main objective of the study is to model the evolution of non-performing loans in the Greek banking system with the purpose of first estimating the impact of various macroeconomic & bank specific determinants on the rate of bad loans, and second forecast its future path.

In the first chapter we provide an introduction on non-performing loans and a descriptive analysis of their characteristics in the European Union, as well as in the Greek banking system. According to the growing literature, we identify the macroeconomic, bank specific and debt determinants of the NPLs rate. Furthermore, we describe the policy regulations undertaken by the European Union regarding the steps of management and the resolution methods of NPLs.

In the following two chapters we analyze two short-run forecasting methods: Autoregressive Integrated Moving Average models (ARIMA) and Vector Autoregressive models (VAR) in order to estimate the future rate of NPLs. The method of ARIMA concerns the univariate forecast technique as applied to the NPLs time series. The second method provides forecast results by using a number of variables such as the GDP growth, Unemployment rate and long-term interest rate. The dataset of NPLs were sourced from the Bank of Greece and concern the period from 2002 to 2019.

In chapters 3 & 4 we study the effect of credit risk metric on the NPLs rate, including the z-score metric and the Tier 1 ratio. The z-score captures the bank's probability of default and the Tier 1 ratio estimates the bank's financial strength. Besides that, we include bank-specific variables according to the NPLs literature. Considering data availability, we use a Panel approach of 16 European Union countries and apply panel data econometric techniques to study the behavior of NPLs. The datasets cover the period from 2008 to 2017.

Finally, the evidences of the empirical models show that the short-run forecasting through the VAR model leads to more reliable results when comparing the values with the out of sample rate of the Bank of Greece for the next period. Moreover, considering the panel approach, the bank-specific determinants are negatively associated with the NPLs ratio except of government debt which is positively linked to the rise of NPLs. As for the z-score, the analysis underlines the importance to maintain a low level of NPLs in order for banks to be credit risk sustainable.

Keywords: Non-performing loans, NPLs, Greece, Europe, Forecast, ARIMA, Vector Autoregressive models (VAR), Panel data, Econometric analysis, Credit risk



CHAPTER 1: An introduction to Non-Performing Loans

1.1 General background

One of the major issues of the European banking systems is the high ratio of non-performing loans that has been observed after the 2008 global financial crisis. As a result, the consequences on the core banks as well as for the whole economy have been extremely serious. The European authorities implement many initiatives in order to reduce the stock of these loans, especially in the period of debt crisis. The ratio of non-performing loans presents significant differences across the European Union. Especially countries located in the European periphery such as Greece, Portugal, and Ireland seem to face the biggest challenges.

There are many definitions for Non-Performing loans in the literature and they are different in each country. These differences for example may be related to the number of days from the last payment or the developed criteria according to which each country classifies the loans. In 2014, the European Central Bank (ECB) performed a comprehensive assessment and determined criteria to define loans as Non-Performing if they are:

1. More than 90 days past due, without the borrower being able to pay the agreed instalments
2. Impaired¹ with respect to the accounting specifics for Generally Accepted Accounting Principles (GAAP²) and International Financial Reporting Standards (IFRS³) of banks
3. In default according to the Capital Requirements Regulation which reflects Basel III⁴ rules on capital measurements and capital standards ((EU) No. 575/2013).

The International Monetary Fund (IMF) summarized the definition of Non-Performing Loans in its 18th Meeting of the IMF Committee on Balance of Payments Statistics Washington, D.C.(2005).

According to IMF a loan is classified as Non-Performing when:

1. Payments of instalments (interest and/or principal) are past due by 90 days or more,

¹ A loan is considered to be impaired when it is probable that not all of the related principal and interest payments will be collected.

² GAAP refers to a common set of accepted accounting principles, standards, and procedures that companies and their accountants must follow when they compile their financial statements.

³ IFRS set common rules so that financial statements can be consistent, transparent and comparable around the world.

⁴ "Basel III" is a comprehensive set of reform measures in banking prudential regulation developed by the Basel Committee on Banking Supervision, to strengthen the regulation, supervision and risk management of the banking sector. <https://eba.europa.eu/regulation-and-policy/implementing-basel-iii-europe>



2. Interest payments equal to 90 days or more have been capitalized, refinanced, or delayed by agreement, or
3. Payments are less than 90 days overdue, but there are other good reasons —such as a debtor filing for bankruptcy—to doubt that payments will be made in full.

1.2 The evolution of NPLs in Europe and Greece

In Europe there was a great increase of Non-performing loans due to the European sovereign debt crisis that started after the outbreak of the global financial crisis of 2008.

In 2013 Non-Performing loans peaked at 8% of total gross loans (almost €1 trillion) across the Euro Area and have decreased only gradually in some countries since then.

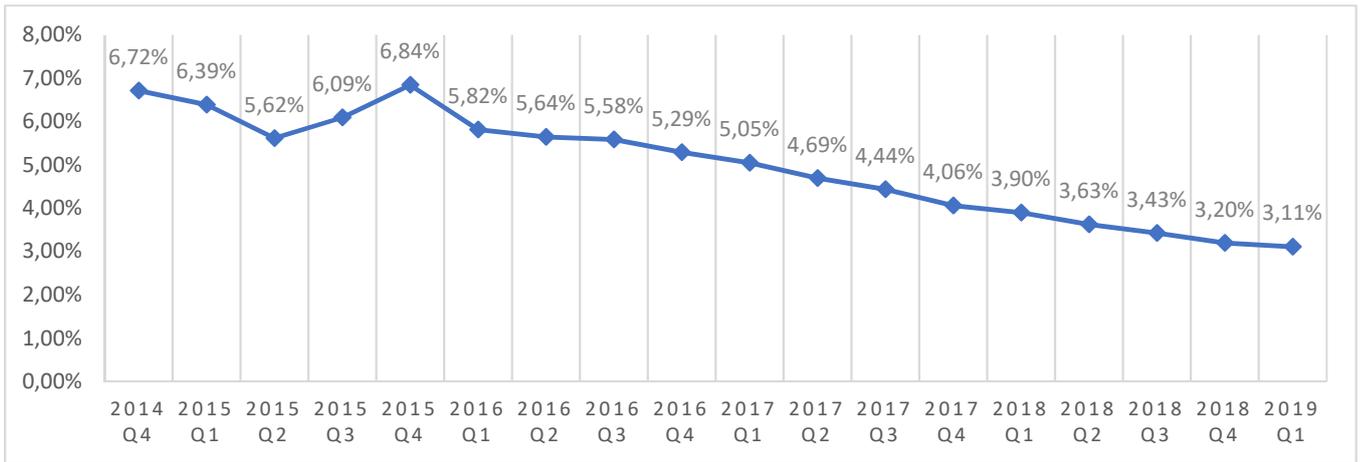
Figure 1.1. NPLs ratio in Europe



Source: KPMG report (May 2017)

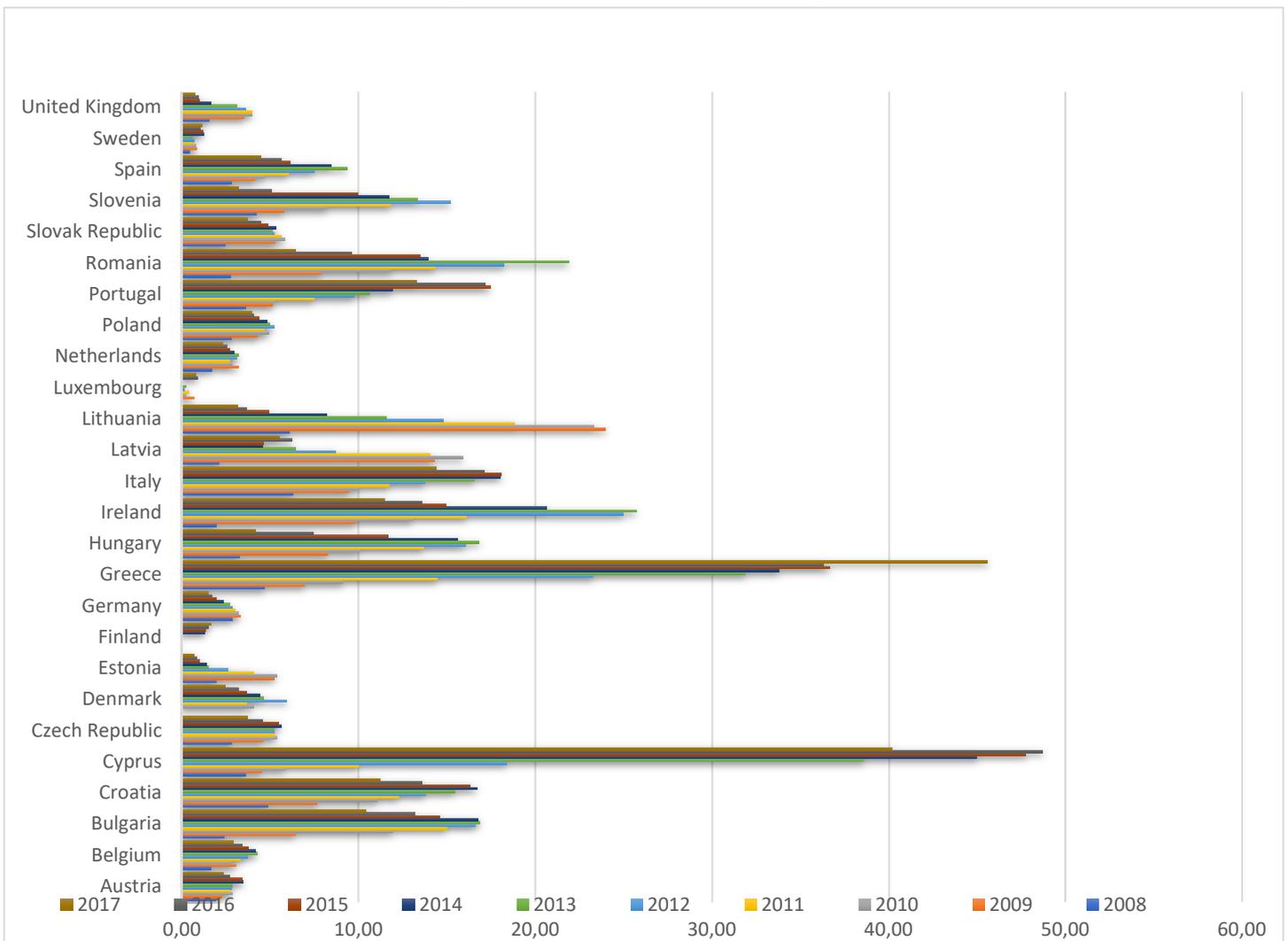
The reduction of Non-Performing loans was an important goal for the European Central Bank and with significant efforts and appropriate policies the percentage has almost halved (3.11%), and now stands at €580 billion (June 2019).

Figure 1.2. NPLs percentage in European Union 2014-2019



Source: European Central Bank- Statistical Data Warehouse

Figure 1.3 NPLs in each EU-country



Source: European Central Bank- Statistical Data Warehouse

Significant differences of Non-performing loans ratio can be observed within the Euro Area. We still notice that the exposure of Greece and Cyprus to Non-Performing loans is the highest in Europe. This amount peaked at 45.57% in Greece (2017) and 48.68% in Cyprus (2016). Countries such as Italy and Portugal have an important ratio of Non-Performing loans. It is noteworthy that, the depth of debt crisis in these countries was greater in comparison with the others European members. Furthermore, during 2013 there was a remarkable increase of Non-Performing loans ratio in all European countries.

Based on the above data source and in the quarterly report on the Euro area of the European Commission, non-performing loans proportion of the European members summarized in the following three categories:

Level 1: Non-performing loans proportion less than 10% of total loan portfolio at the last 15 years

BE, DK, FI, DE, LU, NL, FR, SE, UK

Level 2: Non-performing loans proportion less than 10% of total loan portfolio but increased in the period of debt crisis (2008-2013)

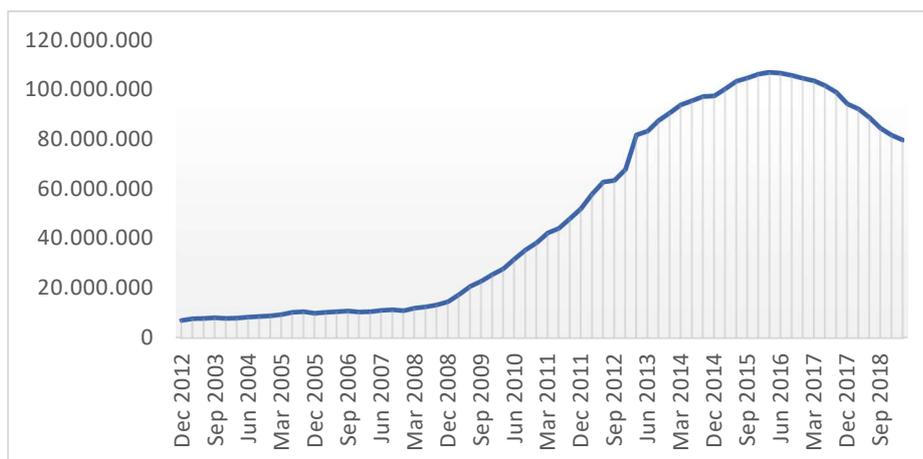
AT, EE, CZ, PL, HU, SK, ES, LV, LT

Level 3: Non-performing loans proportion more than 10% of total loan portfolio

GR, CY, HR, BG, IE, IT, MT, PT, SO, SI

In more detail, the rate of Non-Performing Loans for all Greek commercial and cooperative banks had a steep increase during the period of debt crisis. Greek banks made significant progress during these years in reducing the rate of non-performing loans and the percentage started to decline in 2017 and in March of 2019 stands at 45.12% (about 79.917.569,59 thousand €).

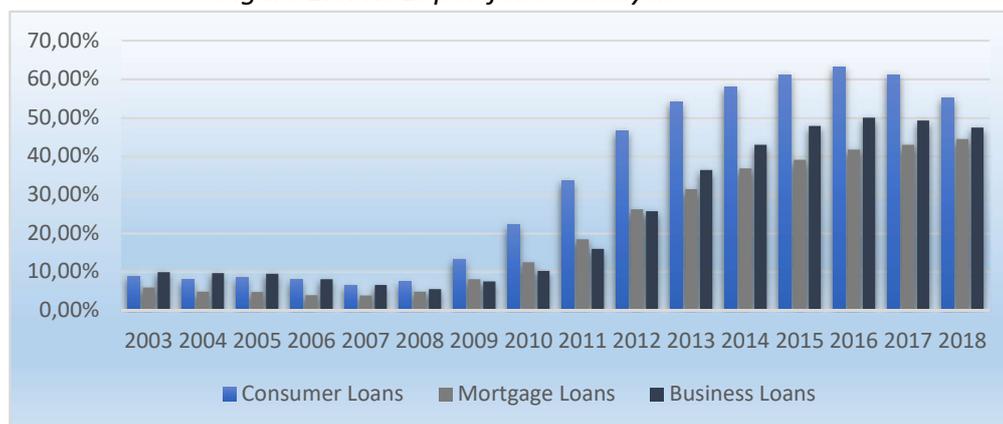
Figure 1.4 NPLs ratio in Greece



Source: Bank of Greece

More specifically, the portfolio of Non-Performing Loans consists of consumer, mortgage, and business loans. Each category has different evolution through the years. What is remarkable is that consumer loans have an important increase, in comparison to the residential loans, which note a small amount of Non-Performing loans. It is obvious that this difference is more evident in 2009 and continues in the next years.

Figure 1.5. NPLs portfolio's analysis



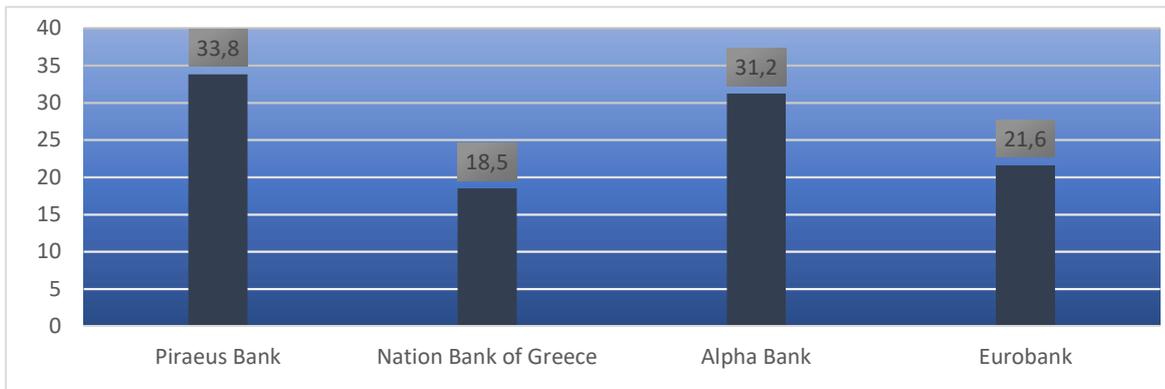
Source: Bank of Greece

There are four systemic banks⁵ today in the country: Alpha Bank, Eurobank, National Bank of Greece, and Piraeus Bank. All these have absorbed all the smaller banks that existed in the country in the last years. Each of these banks faced problems with the management of bad loans. Under the

⁵ A systemically important financial institution (SIFI) or systemically important bank (SIB) is a bank, insurance company, or other financial institution whose failure might trigger a financial crisis.

pressure of their stability and the forces of the European central Bank they made big steps to reduce the ratio of Non-Performing Loans.

Figure 1.6. NPLs of Greek systemic banks in billions € for quarter ended 30 .09. 2017



Source: Greek Bank's Press releases

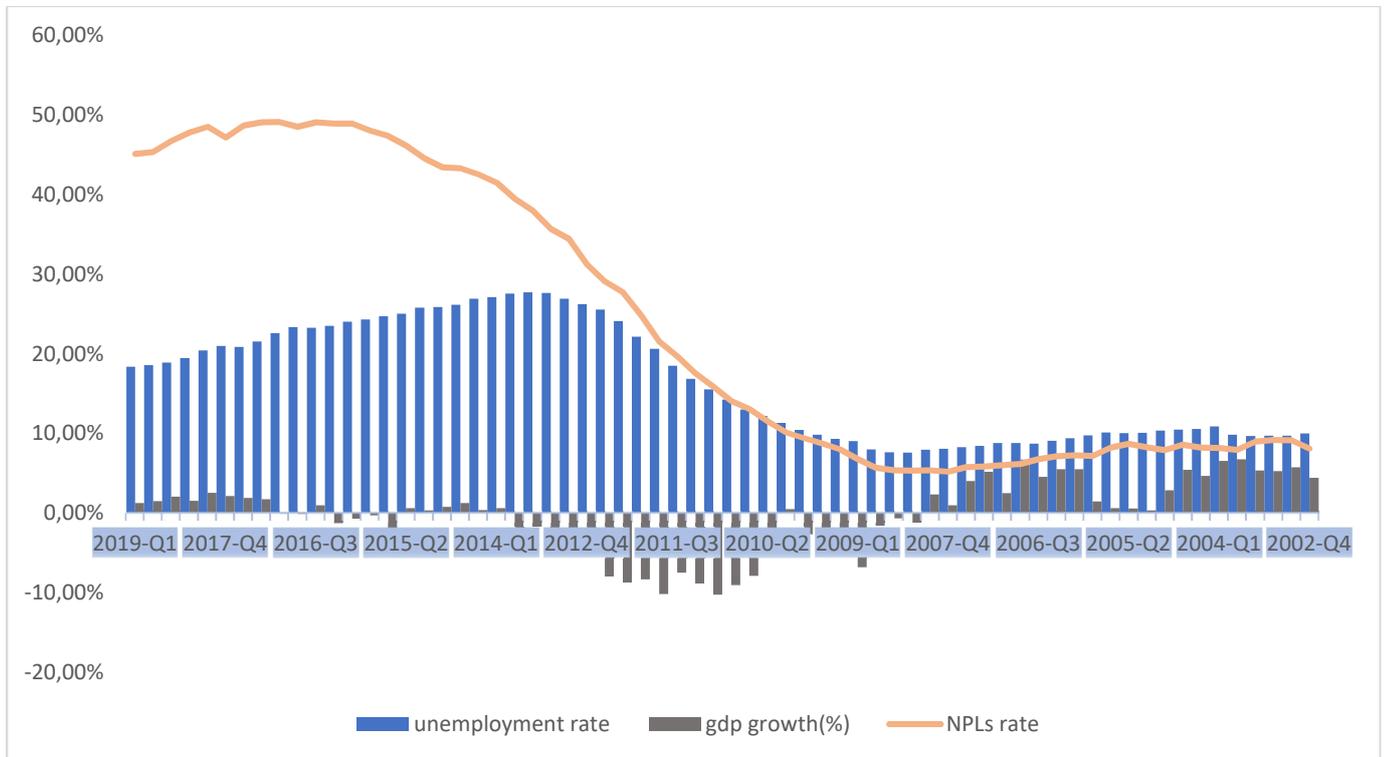
1.3 Determinants of NPLS and the sovereign debt crisis

According to the growing literature on non-performing loans such as Louzis, Vouldis, Metaxas(2012) and Anastasiou, Louri, Tsionas(2016), the determinants of problem loans could be based on a broad set of macroeconomic conditions, public policies effects on debt ratios and further bank-specific factors.

-Macroeconomics determinants:

One of the most important macroeconomics factors is the GDP growth rate. Countries which have severe contraction of GDP tend to have an increase of bad loans (Louzis, Vouldis,2012). According to Anastasiou, Louri, Tsionas (2016) ,another factor that is related to Non-Performing loans growth is the unemployment indicator. An increase in unemployment may affect the borrower's ability to fulfill any debt obligations.

Figure 1.7. Unemployment, NPLs ratio and annual growth rate in Greece



Source: ELSTAT (Hellenic Statistical Authority) and Bank of Greece

Other empirical studies indicate that the increase of lending rate has an important relation to Non-Performing Loans ratio. The lending interest rate is set in a way to ensure bank's future profit, therefore commercial banks usually charge higher interest rate in riskier borrowers in anticipation of default. Moreover, the liberalization and the flexibility of lending rate allowed banks to modify the level of interest rate according to the maturity of loan and the special term of loans agreement.

According to the study of Nkusu (2011) the ratio of bad loans can be also related to the inflation. Higher inflation can reduce the real value of loan and also affect the real income, therefore reduce the ability of borrowers to fulfill loans agreement.

Also, in a study by ECB (2013) an increase of non-performing loans can be closely linked with the exchange rate. Countries with lending obligation in foreign currencies can be affected by currency mismatches. An example in Greece, where borrowers (about 70.000) who took out loans in Swiss franc tend to rise default rate.

It is also worth mentioning the empirical analysis of Louzis, Vouldis and *Metaxas (2012)*. The results of this study came to the conclusion that the macroeconomics determinants have a differential impact of each category of loans (mortgage, business, consumer). For example, the GDP growth rate has more influence in consumer and business loans, as well as the unemployment rate. On the contrary, mortgage type of loans is less sensitive in unemployment rate, which can be explained by the fact that in Greece mortgage loans are mostly extended to civil servants and high-skilled workers of the private sector, who are less likely to get unemployed (Mitrakos et al., 2005; Mitrakos and Simigiannis, 2009). Besides that, the mortgage loans are also less sensitive in interest rate due to the fact that usually business and consumer loans' agreements tend to use floating rates.

-Banks-specific determinants:

Several investigations and inquiries note that there are many specific banking sector's hypotheses, which are related to the increase of bad loans. These hypotheses subsume the following categories:

A. Operating efficiency

Initially, the empirical analysis of Berger and DeYoung (1997) concluded in three hypotheses that affect the rate of non-performing loans. These hypotheses are strictly interrelated. Bad management and bad luck hypothesis are related to the reduction of banks efficiency. Contrary to the skimping hypothesis, which reflects the increase of bank efficiency only by reducing near-term cost but in the long-term creates problems with high proportion of past due arrangements.

- **Bad management hypothesis:** The increase of non-performing loans ratio is related to the cost efficiency⁶. Banks are cost efficient if they increase their profits by minimizing their cost as far as possible. One factor that reduces the cost efficiency is the 'bad' managers' inability to choose a portfolio of loans with positive present values. Moreover, they underestimate the value of collaterals and they have difficulty to control if a loan agreement

⁶ Bank cost inefficiency is defined as the difference between observed costs and predicted minimum costs for a given scale and mix of outputs, factor prices and other country-level variables.

is fulfilled. They provide loans covenants with low effectiveness and they have a delay in classifying borrowers according to their economic status. These facts lead to an increase of non-performing loans in the future.

- **Skimping hypothesis:** The amount of resources that will be spent to issue and monitor bank's loans affects both cost efficiency and the ratio of non-performing loans. Banks sometimes aim to increase their short-term profit by reducing their short-term costs. This reduction of short-term costs is closely linked to low quality underwriting methods and inefficient loans monitoring. This fact will provisionally reduce bank's cost efficiency, but in the future it will lead to a higher proportion of non-performing loans.
- **Bad luck hypothesis:** When the bank is faced with an exogenous determinant (for example financial crisis or other macroeconomics determinants) it is obvious that it will have an important growth of bank costs. The further monitoring of these loans, the need of economic analysis, the underwriting of loans arrangements, the management of collaterals will cause a further increase of bank's expenses. Furthermore, this exogenous determinant will cause imbalances in bank's account, which will create additional costs to maintain bank's stability. This extra operating effort leads to extra expenses, which are highly related to the increase of non-performing loans.

B. Capitalization

Banks are required to be sufficiently capitalized, meaning they must have enough assets that can be readily converted to cash to meet short-term and long-term financial obligations. Hypotheses that are related to the banking sector's capitalization is the fourth Berger and DeYoung hypothesis of moral hazard and also the analysis of Keeton(1999) in credit growth.

- **Moral hazard hypothesis:** The low rate of bank's capitalization increases the amount of bad loans. Many times, managers decide to increase the riskiness of their loans-portfolio when the banks are low capitalized. This policy in low-capital banks relies to the fact that high risk strategies lead to lower potential loss in terms of capital. Thus, the increase of riskiness

accumulate loans with high risk of default and many agreements on average will be overdue in the future. On the other hand, highly capitalized banks under the argument that they are “too big to fail”, they decide to follow a liberal lending policy, which implies a positive relationship between capitalization and non-performing loans.

- Furthermore, Keeton and Morris (1987), analyze the credit growth. Banks in order to increase their capital, increase the portfolio of loans, which leads to the reduction of interest rates and credit standards. Such discounts increase the portfolio riskiness and the chances of borrowers’ default.

C. Non- interest income

Banks earnings include non-interest incomes and interest incomes. Interest incomes are the earnings from loans and investment securities, while non-interest incomes are the bank’s assets management, trading, derivatives and investment banking.

- Studies refer to the impact of bank diversification. Amit Ghosh’s (2015) study concluded that the proportion of non- interest income⁷ as a share of total income reduces total credit risk and improves loan quality. Also, Hu, Yang and Yung-ho in their study argue that an increase of non-interest incomes reduces credit riskiness and as a result causes a better loan underwriting policy.
- On the contrary, Stiroh’s (2004) study rejects the above view. According to his study, he finds a positive relationship between diversification and non-performing loans. Diversification lowers credit risk because the banks relies on other types of income and the profits from loan agreements have low priority. As a result, managers take high risk strategies that lead to lower loans standards.

⁷As non-interest incomes we cover investment banking, asset management and insurance underwriting, fee-paying and commission-paying services, trading and derivatives.

D. Size

According to literature, there are two views of relation between non-performing loans and bank's size.

- Diversification hypothesis: According to Salas and Saurina's (2002) study, they find a negative relation between non-performing loans and the bank's size. This evidenced based on the fact that banks with large size tend to have more diversified portfolios.
- Too big to fail hypothesis: However Stern and Feldman's (2004) concluded that countries with developed banking sector have more riskiness in their policies because they expect the government to refinance in the case of failure. Thus, they tend to accumulate a high amount of non-performing loans.

E. Profitability

Banks with high level of profitability have fewer probability to high-risk activities, thus fewer rates of non-performing loans in the future.

- Louzis' hypothesis of "Bad management II" endorses the above view. In other words, managers with low quality of skills offer loans to high-risk borrowers, in order to increase bank profitability. The short-term profits may be increased, but in the future banks will be faced with a higher proportion of bad loans.
- Louiz's hypothesis of Procyclical credit policy (2012). This hypothesis tests the influence of past Greek banking performance on non-performing loans ratio. Thus, a negative net present value (NPV) may lead to liberal lending policies, because the banking managers want to convince the investors for their future profitability. Consequently, this strategy reflects a positive relationship with the increase of non-performing loans.

-Debt impacts:

The sovereign debt crisis causes major changes in economy and banking sector stability. First of all, banks display liquidity shortage, as a result of the public sector's inability to refinance the banks. Moreover, an increase of public debt's ratio may lead to austerity measures and drastic cut in social services and household's incomes. An example of this is the European sovereign debt crisis, which especially in Greece affects the majority of economic rates. It is obvious that in 2009 the ratio of non-performing loans starts to increase rapidly.

By and large, in recent years Greece needs to sign three bailout programs so as to manipulate the imbalances in macroeconomics rates. These policies had important consequences for the Greek economy, as GDP growth, unemployment and inflation rate increased rapidly. Furthermore, the banking sector had inabilities to manage this high increase of non-performing loans. Thus, the borrower's income experienced a severe contraction and many loans agreements resulted past due.

1.4 The impact of NPLS on the banking sector and the overall economy

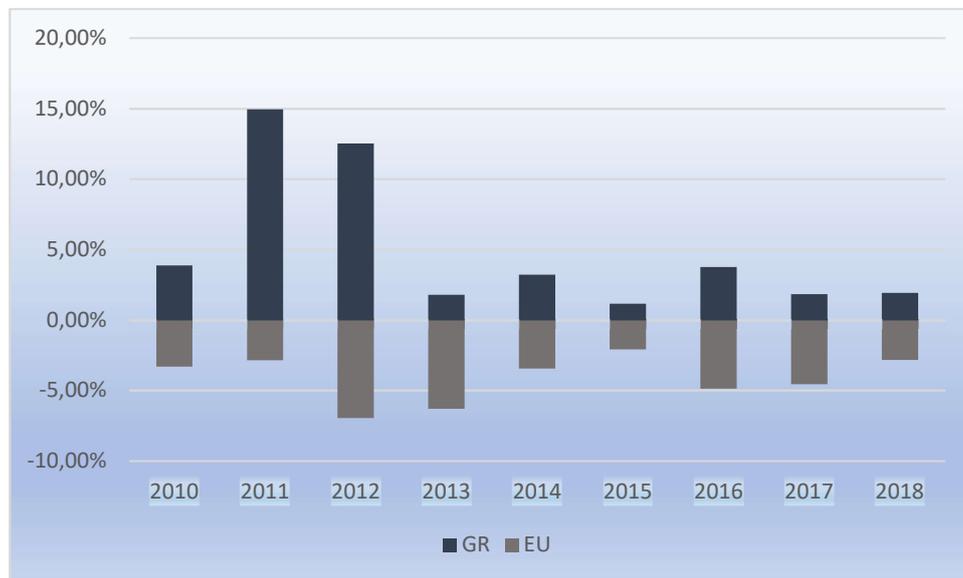
The significant increase of non-performing loans, especially in the period of the European sovereign crisis, had negatively affected bank's balance sheet and, by extension, the balance of the whole economy. The impacts on banking sector concern bank's profitability, efficiency, allocation of resources and lending, as well as they may lead banks to be unviable.

In particular, non-performing loans accumulation is related to bank's low profitability and liquidity. The management of these loans needs specific handling, improvement of information systems and bank employees' training. Therefore, a bank increases its expenses and needs additional capital to manage these procedures. As regards market and investors, a bank with high rate of non-performing loans leads investors to a lack of confidence, which results in low liquidity and increase in funding costs.

Moreover, a sizeable amount of non-performing loans increases the loan loss provision. A loan loss provision is an expense that is reserved for defaulted loans or credits. It is an amount set aside in the event that the loan defaults. From a balance sheet perspective, a loss on a loan is still a loss of an asset. However, on an operating basis, because of the loan loss provision, cash flow remains

available. The loan loss provision ensures that banks will have sufficient funds to provide services to its depositors and it's an indicator for investors for the bank's lending policy stability.

Figure 1.8. Banks provisions (% of total assets) in Greece and 28 EU members



Source: European Central Bank

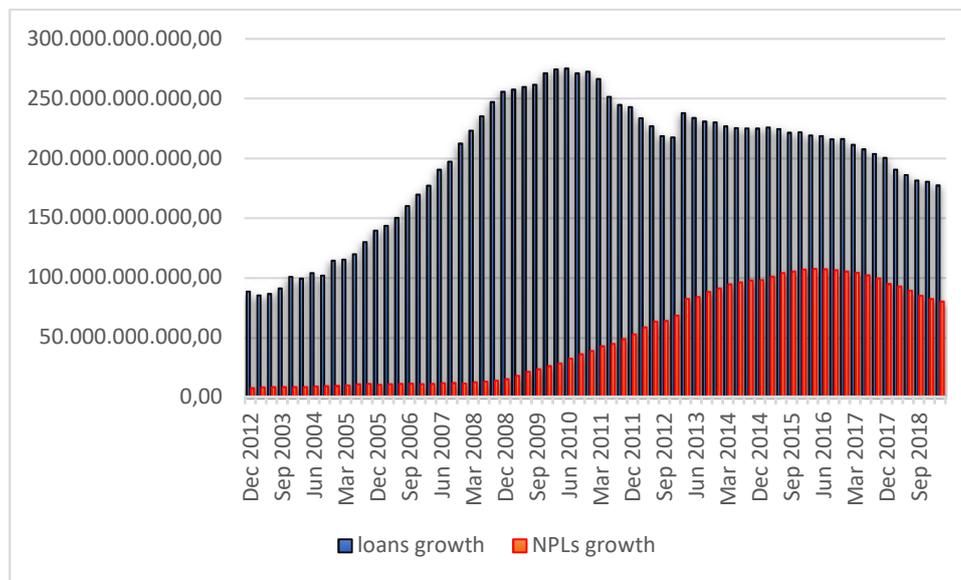
Bank's efficiency is highly related to non-performing loans⁸. Managers and bank strategies focus on reducing this rate and for this reason they are really distracted from core business. Furthermore, the increase of bad loans has negative consequences on new lending. The study of Leon and Traceys (2011) specifies a reasoned analysis of the link between lending policy and non-performing rates. They concluded that banks lend less when non-performing loans rate rises above a certain threshold.

Further studies confirm that poor loan portfolio quality affects bank allocation strategies, leading to a lower loan growth rate and a lower total loans over total asset ratio (Brunella and Marino, 2016). A higher NPL ratio is also associated to a greater amount of resources allocated to government bonds, which results to a flight to quality effect. One indication of a flight to quality is a dramatic fall of the yield on government securities, which is a result of the increased demand for them.

⁸ Cucinellia, Credit risk in European banks: The bright side of the internal ratings based approach

It is remarkable to notice that the amount of loans declines in comparison with the non-performing loans growth, especially in the period of debt crisis (2009).

Figure 1.9. Total loans growth in relation to NPLs growth in the Greek economy



Source: Bank of Greece

Another consequence of a high stock of non-performing loans is the low rate and quality of investment planning. For instance, non-financial corporations are unable to invest in a new plan since their debt is non-performing. This low rate of liquidity leads corporation to share their profits in order to achieve its goals. Moreover, an increase of non-performing loans is related to a high level of lending cost. This causes unwillingness to corporations to finance their operations from external resources, such as lending. Especially, high profitable corporations finance their functions with internal resources, while weaker non-financial corporation need more external banking financing. Therefore, due to the high cost of lending, low profitable corporations are obligated to avoid an investment, which needs capitals from the banking sector.

Finally, high proportion of bad loans increases the investment's riskiness since it's difficult for banks to have flexibility in case of difficulties in lend performing. Corporations are usually risk-averse and they want to be profitable in term of low risk. The rising difficulties in investments lead to problems in the real economy and affect a country's macroeconomic rates such as GDP growth, unemployment etc.

In the *Quarterly report on the Euro Area (Marco Butti, volume 16, 2017)* reference is made to the impacts on cross-border lending. Countries in Europe Area are really interrelated and the financial environment of a Eu-member affects the others European economies. A high stock of non-performing loans in a Eu-member reduces the cross-border lending and the credit supply among European countries, while domestic banks may raise lending problems concerning foreign relation with foreign banks. For example, the British banks seem to be exposed to Ireland, the German one to Italy, the Romanian to Greece.

1.5 Non-performing loans management and resolution methods

In order to address the high volume of non-performing loans, banks should develop strategies and management tools. Thus, loan portfolio management is a key function that directly leads to the decrease of the defaulted loan's recover amount. Moreover, the resolution methods aim to decrease the ratio of NPLs and improve bank's creditworthiness.

The reduction methods and the management of non-performing loans is an unstoppable challenge for the banking sector. The resolution methods have different characteristics in each country, which depend on the specific country's parameters and on the level of the problem that has arisen. The European banking authorities have classified the non-performing loans resolution as one of the top priorities of the European economy and have made important agreement to address this sizeable stock of non-performing loans.

Initially, it is remarkable to refer to the Basel II (2004), which is a set of international banking regulations by the Basel committee of Bank Supervision. Basel II regulations differ from Basel I (1988) since they incorporate the credit risk of the assets as a determinant of the minimum capital requirement⁹. The Basel II contains three pillars: minimal capital requirements, regulatory supervision and market discipline. The second pillar "regulatory supervision" establishes the framework for banks to manage the different types of risk including systemic risk, liquidity risk, and legal risks. Moreover, the context of second Pillar has underlined the importance of fair control of asset quality and rules for non-performing loans categorization and provisioning analysis.

The European Banking Authority (EBA) guidelines have identified the importance to establish a common strategy across the European banking sector for non-performing loans classification,

⁹ Minimum capital requirement is the minimum capital that a bank needs to be able to cope with potential loss from doubtful loans

valuation and credit risk measurement since 2014. The European Central Bank (ECB) and the National Competent Authorities (NCA) demonstrated the “Asset Quality Review” (AQR) in November 2014. The AQR is a manual, which contains the methodology for valuating bank’s assets from prudential perspective. These methodologies contain detailed indicators for loan classification and cash flows models.

Finally, the European Central Bank provides a “Guidance to banks on non-performing loans”. This guidance to banks published in March 2017 refers to the following sections:

- i. the NPLs strategies
- ii. NPLs governance and operation
- iii. Forbearance solutions¹⁰
- iv. NPLs recognition
- v. NPLs impairment measurements and write-offs
- vi. collateral valuation for immovable property.

First of all, banks should elaborate a strategy based on the specific characteristics of their portfolio. The developed strategy should estimate both external and internal determinants. Furthermore, banks should develop strategies and operating plans to reduce non-performing loans in short and long-term projects. This includes all the necessary steps including changes in the structure of banks functions and management’s perspective. The supervision of the reduction of non-performing loans should be regular and independent.

The European Central Bank refers to the governance and the operations which are associated with the non-performing loans reduction. The governance and operational strategy are of particular importance for addressing non-performing loans. The management of these loans should contain the following pillars:

- Annual and regular review of non-performing loans operational plan
- Define targets and incentives for the reduction-project of non-performing loans
- Periodical reports and effective supervision of the different projects
- Management approval for high rate non-performing exposure

¹⁰ Forbearance, is a special agreement between the lender and the borrower to delay a foreclosure. The literal meaning of forbearance is “holding back.” When mortgage borrowers are unable to meet their repayment terms, lenders may opt to foreclose.

- The formulation of the new strategies and the assurance that have been efficient understood by the staff
- Creating strategy framework for internal procedures, which aims to loan classification, provisioning, collateral valuation and forbearance solutions.

Moreover, the efficient operating model indicates a dedicated and separated non-performing loan workout units (WUs). This aims to the reduction of the potential conflict of interest¹¹ and the ability of the staff to be more dedicated and experienced on non-performing loan management. This separation of the unit has a positive influence on the management procedures (e.g. client negotiations) and on the decision-making process.

Another determinant that provides an effective management of non-performing loan is the forbearance measures. Forbearance should aim to turn borrowers from non-performing to performing status and provide the right policies in order to maintain their viability. The forbearance measures could be distinguished in short-term and long-term.

Short-term measures:

- i. Interest only: During a short-term period, borrowers only pay their interest and the principal remains stable. At the end of the interest payment period banks re calculate the terms of the loan repayment. This measure should only be provided when borrowers are in a temporary financial difficulty.
- ii. Reduced payments: Decreasing the loan installments, the borrowers gain more time to regain their revenue and after this they continue to comply their obligations.
- iii. Grace period/payment moratorium: It is an agreement between lender and borrowers, which allows borrowers to delay their repayments (principal and interest).
- iv. Arrears/interest capitalization¹²: Arrears and/or accrued interest arrears are added to the outstanding principal balance for repayment under an effective rescheduled program. Arrears capitalization should only be applied in cases when the borrowers are unable to align their interest or payments. Moreover, banks according to the European Banking Authority

¹¹ Conflict of interest: Is a situation in which an individual has competing interests or loyalties. Conflicts of interest involve a person who has two relationships that might compete with each other for the person's loyalties.

¹² Arrears (or arrearage) is a legal term for the part of a debt that is overdue after missing one or more required payments. The amount of the arrears is the amount accrued from the date on which the first missed payment was due.

should avoid providing this method more than once and should be applied in a predefined percentage of arrears interest according to the bank forbearance policy.

Long-term measures:

- i. Interest rate reduction: Borrowers cash flow sometimes may be reduced and the country's financial situation may be upset. An effective solution may be the temporary or permanent reduce of interest to a new one, according to the reassessed financial determinants. However, banks should estimate that the credit risk could be sufficiently covered by a reformed interest rate.
- ii. Extension of maturity: The extension of loan payments lead to the extension of loan over a longer period and as a result installments amount is reduced. This method could be effective only if this extension is linked to a viable repayment from the borrower in the future.
- iii. Additional security¹³: Banks should obtain additional liens on unencumbered assets from the borrowers in order to mitigate the risk. This measure is linked to the improvement of loan-to-value (LTV)¹⁴ ratio.
- iv. Dispose agreements: When a lender and a borrower agree to sale a secured asset in order to partially or fully repay the loan. The assisted sale should be linked to a rescheduled payments plan. In addition, banks should consider the future value of the dispose asset and implement the appropriate procedures in an appropriate time.
- v. Rescheduled payments: Under the reassessed financial status of the borrower, the plan of installments payment should be rescheduled. Some examples of these methods are the partial payment (the repayment program rescheduled from a sale of asset), the balloon or bullet payment (large payment before loan maturity) and the step-up payment (smaller payments in the beginning of your loan for an assigned time period, with payments increasing over the life of your loan).
- vi. Conversion of currency: When the currency of the loan is exchanged in another currency.
- vii. New credit facilities: When the lender provides new proposal to the borrower in order to align the debt repayment.

¹³ A secured loan is a loan in which the borrower pledges some asset as collateral for the loan, which then becomes a secured debt owed to the creditor who gives the loan. The debt is thus secured against the collateral, and if the borrower defaults, the creditor takes possession of the asset used as collateral and may sell it to regain some or all of the amount originally loaned to the borrower.

¹⁴ The loan-to-value (LTV) ratio is a term used by lenders to express the ratio of a loan to the value of an asset purchased.

- viii. Debt consolidation: When the borrower has many loans especially cards, banks should combine all these loans into a single loan or in a few manageable loan-arrangements. This option is very effective in cases when the loans are combined with collaterals and the loans are more secured.
- ix. Debt forgiveness: It is an agreement between the bank and the borrower to pay a reduced principal in an agreed timeline. Banks should be careful with the debt forgiveness because it can lead to strategic defaults.

Another problem that appeared, is the definition issues with the term of Non-Performing Loans (NPLs). The European Banking Authority submitted the definition Non-Performing Exposures (NPEs). This new definition is strictly used for reporting issues (public reporting, internal risk control). Moreover, the NPEs term is based on two categories past-due and unlikely-to-pay:

- past-due: Exposures with installments more than 90 days are categorized as non-performing and there is a legal obligation for a payment to be made and the payment is compulsory.
- unlikely-to-pay: Exposures which are unlikely to be paid through a pre-defined event. Banks should collect all the necessary financial status from their customers and comply with the right processes for the debtor's creditworthiness.

The above definition is very important for the Non-performing Loans management especially for reporting issues and the necessary lay down of the future policy options.

The European Banking Authority also has also mentioned provisioning and write-off procedures. Provisioning is very important for banks due to the fact that it estimate the bank's credit risk, which is highly important for asset quality reviews (AQRs) and stress tests (STs). Provisioning on non-performing loans should be with measures that are clearly specified, especially the procedures for each provision and the timeliness should be linked with every type of exposure. As regard the write-offs, the IMF underlines the timely policy of uncollectable loans and the banks should elaborate their write-off criteria.

The final strategy of the NPLs management is the valuation of the collaterals for immovable property. The procedures classified as internal or external based on the bank's size and the business model. The main principles are:

- The procedures should be carried out from an unit that is independent of loans management, monitoring and underwriting process.
- The independence of the external appraiser
- An appropriate similar sample of internal and external valuations should be

compared on a regular basis

- Back-testing of both collateral valuation's methods should be carried out on a regular basis;
- The quality assurance process should be based on an appropriate sample size.

Moreover, there are two types of valuations : individual and indexed valuation. The first is based on a non-automatic process through property-specific appraisals and the second one through an automatic process such as empirical analyses. The minimum valuation frequency should be annual for commercial immovable property (such as premises, offices, etc.) and every three years for residential (such as house, flat, etc.) immovable property. All the above processes should be based on the Article 208(3) of Regulation (EU) No 575/2013.

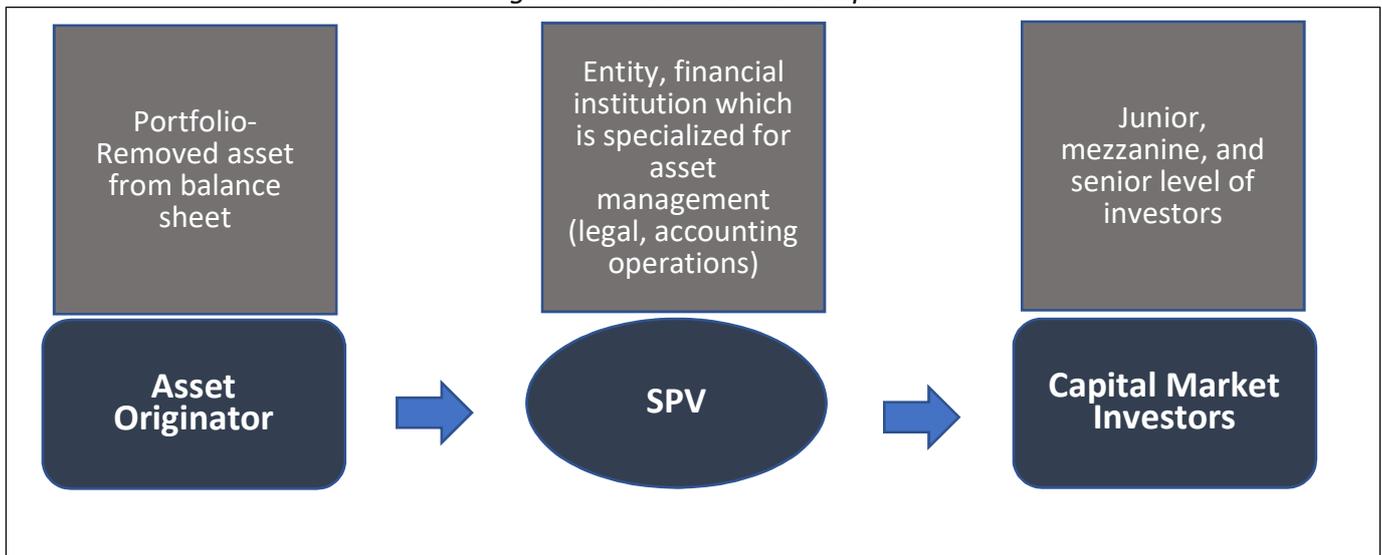
Besides the management of NPLs, the main resolution methods for the bad asset followed by the European banking sector containing on-balance and off-balance strategies, are briefly analyzed in the following paragraphs.

➤ **Securitisation:** Securitisation is the financial practice of gathering various types of debt (such as, mortgage, commercial, business loans or credit card debt obligations or other non-debt assets which generate claims) and selling their awaited cash flows to third party investors as securities, such as bonds, pass-through securities, or collateralized debt obligations (CDOs). Considering the fact that the financial regulators and standards securitized assets are less costly for the banks than other assets (for example loans), they are preferred by the banking industry. Investors are repaid from the principal and interest cash flows collected from the debtors and redistributed through the capital structure of the new financing. Furthermore, securitisation can be described as a transfer of risk for impaired assets through the special-purpose vehicles/entity (SPV or SPE)¹⁵. The bank that owned the non-performing loans sells them to the SPV for a price which is less than the face value of the loans and the cash flow is collected on behalf of the SPV. The cash flows are distributed to the cost of the securitisation structure and the repayments of the investors. These cash inputs are collected through a third party which is appointed by the SPV and when the non-performing loans are unable to generate enough cash flows they active structural

¹⁵ SPV is a subsidiary created by a parent company to isolate financial risk. Its legal status as a separate company makes its obligations secure even if the parent company goes bankrupt. SPVs issue securities with guarantee the delegated receivables, which sell to the investors.

methods used such as collateral liquidity, credit enhancement¹⁶ or hedging¹⁷. The investors are categorized by rating agencies in three tranches (junior, mezzanine, and senior) depending on the level of risk exposure. The junior, or first loss position, which is usually the smallest of the tranches but the one that tolerates most of the credit risk, receives the highest cash flows.

Figure 1.10. Secutisation steps



➤ **Asset management companies (AMCs):** These companies obtain non-performing loans portfolio from a systemic bank for a specific time horizon and their aim is to manage the banking sector's bad debt. The AMCs offer many advantages to banks to reduce their stocks of bad assets. The most important advantage is that they achieve more effective results than individual banks through the economies of scale¹⁸. Moreover, they achieve professional recovery management, while individual banks have not the required skills and knowledge for managing bad assets. On the contrary, a disadvantage of AMCs is the cost of establishment. This cost sometimes may be very high and as a result unattractive to investors and banks. For example, if the state liabilities are high then the requirements for private participation are reduced.

¹⁶ Credit Enhancement is a method whereby a borrower attempts to improve its debt or credit worthiness. Through credit enhancement, the lender is provided with reassurance that the borrower will honor its repayment through an additional collateral, insurance, or a third-party guarantee.

¹⁷ Hedging is similar to undertake a insurance policy, which leads to reducing the potential risk and it also undermining the potential gains.

¹⁸ Economies of scale are cost advantages reaped by companies when production becomes efficient. Companies can achieve economies of scale by increasing production and lowering costs

➤ **Asset protection schemes:** Developed in the Euro Area in order to manage non-performing loans. In these scheme certain assets are limited within the bank's balance sheet (in comparison to the AMCs, which are an off-balance method) and guaranteed by the government. Italy had established the GACs scheme in 2016, which is an asset protection scheme with the Italian state in the role of the guarantor. Afterwards, the EU commission approved the HERCULES scheme proposed by the Greek government which is similar to Italy's GACs model. The role of the guarantor aims to ensure the losses of this asset and as a result to maintain the price in the market of the securitized non-performing loans.

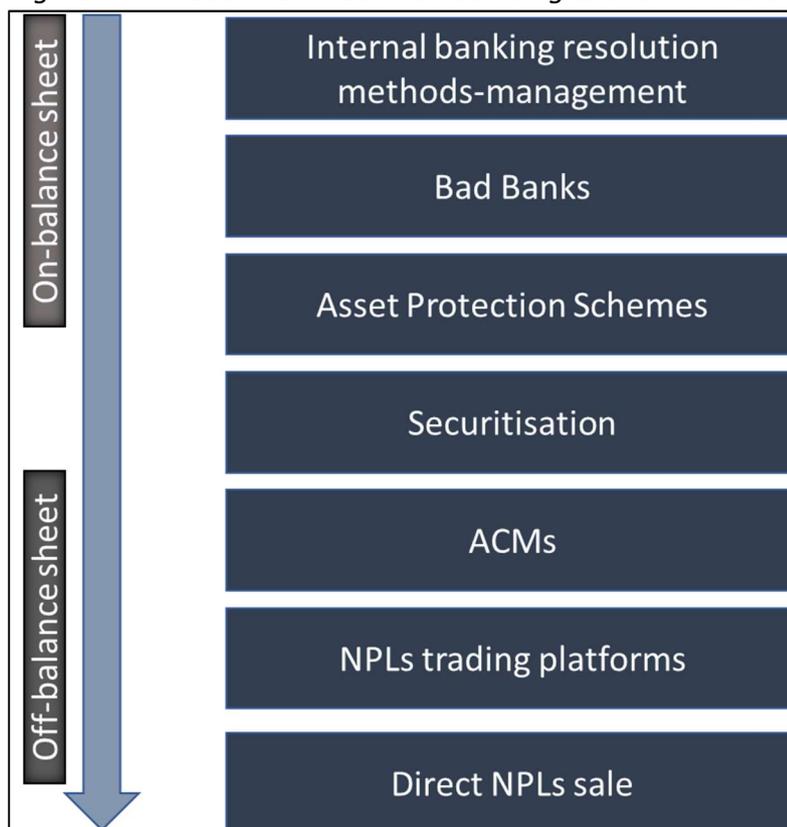
➤ **Direct sale of non-performing loans to investors:** According to Xu(2005) direct sale takes two forms: "sales of individual assets and bulk sales, including negotiated sales and auctions". The most common type of sale is the sale of debt rights, followed by settled assets and to a lesser extent, equity rights. The difference between the purchase price and their ultimate recovery price¹⁹, typically through either negotiated settlements with the debtor or sales/transfers to a third party, leads to the profits of the investor. Some investors manage to foreclose on assets backing the loans, although this is much more difficult due to legal and bureaucratic restrictions. The main advantage of the direct sales method is immediate liquidity to the bank or the AMC, somewhat at the expense of lower recovery value.

➤ **Bad Banks:** When banks are in crucial situation and there is a great need to reduce the non-performing loans, then bad banks should be created in order to accumulate these loans. The purpose of this strategy is to isolate the bad asset from a financial institution to a new entity. The bad banks could be public or private companies and many countries established one bad bank in order to accumulate all the bad asset of the systematic banking industry. The first bad bank was created by Mellon Bank in 1989 to hold \$1.4 billion of bad loans (USA). Many countries all over Europe have established bad banks like Germany, Austria, Spain, Greece etc. The main critics for these banks are that they encourage banks to take high risk policies which they otherwise would not.

¹⁹Recovery valuer is the projected value of an asset that can be recovered in the event of liquidation or winding down

- **The debt-to-equity swaps which creditors become shareholders:** This method is usually associated with companies in order to address their obligations while they continue to operate. Creditors take some of the responsibility for operational restructuring of the borrower, rescheduling the debt repayment, and taking advantage of the future company's profits. However, the main disadvantages of this method is that debt-to-equity swaps are time-consuming, expensive and often are not able to combine both creditor and company's purposes (Xu,2005).
- **NPLs trading platforms:** The European Central Bank proposed the establishment of a NPLs trading platform. These platforms will contain a network of nation AMCs and will act as an NPLs data warehouse in order to facilitate the dad asset transactions.

Figure 1.11. Resolution methods according to the balance sheet



Additionally, the European Systemic Risk Board (ESRB) provides a guidance to the European banking sector for resolving the high stock of these loans. The main purpose for the bank is to establish objective actions in order to access the large stock of non-performing loans in the balance sheet. The approach of the resolution of these problems should be macroprudential and the policies should aim to maximize the net present value (NPV)²⁰ of non-performing loans. The ESRM underlines five principles in order to achieve the minimization of the economy losses and the increase of NPV.

Principle 1: The bank should act directly to resolve non-performing loans. The strategy of pending the future economy growth and the recovery of assets prices should be avoided. Many European countries follow this policy with negative results in their liquidity and financial growth. Besides that, banks should act carefully in order to avoid the periodically low prices in the assets which lead to the decrease of banks capital needs.

Principle 2: Shareholders and investors should first undertake the losses and the risk in order to avoid moral hazard. Banks should not expect the public sector or other shareholders (healthy banks) to support them and the losses should preliminary concern owners-investors.

Principle 3: Resolution methods should comply to European Union legal framework. The strategy of the resolution of non-performing loans should be based on the European Union available policies and be adjusted under the countries' special circumstances.

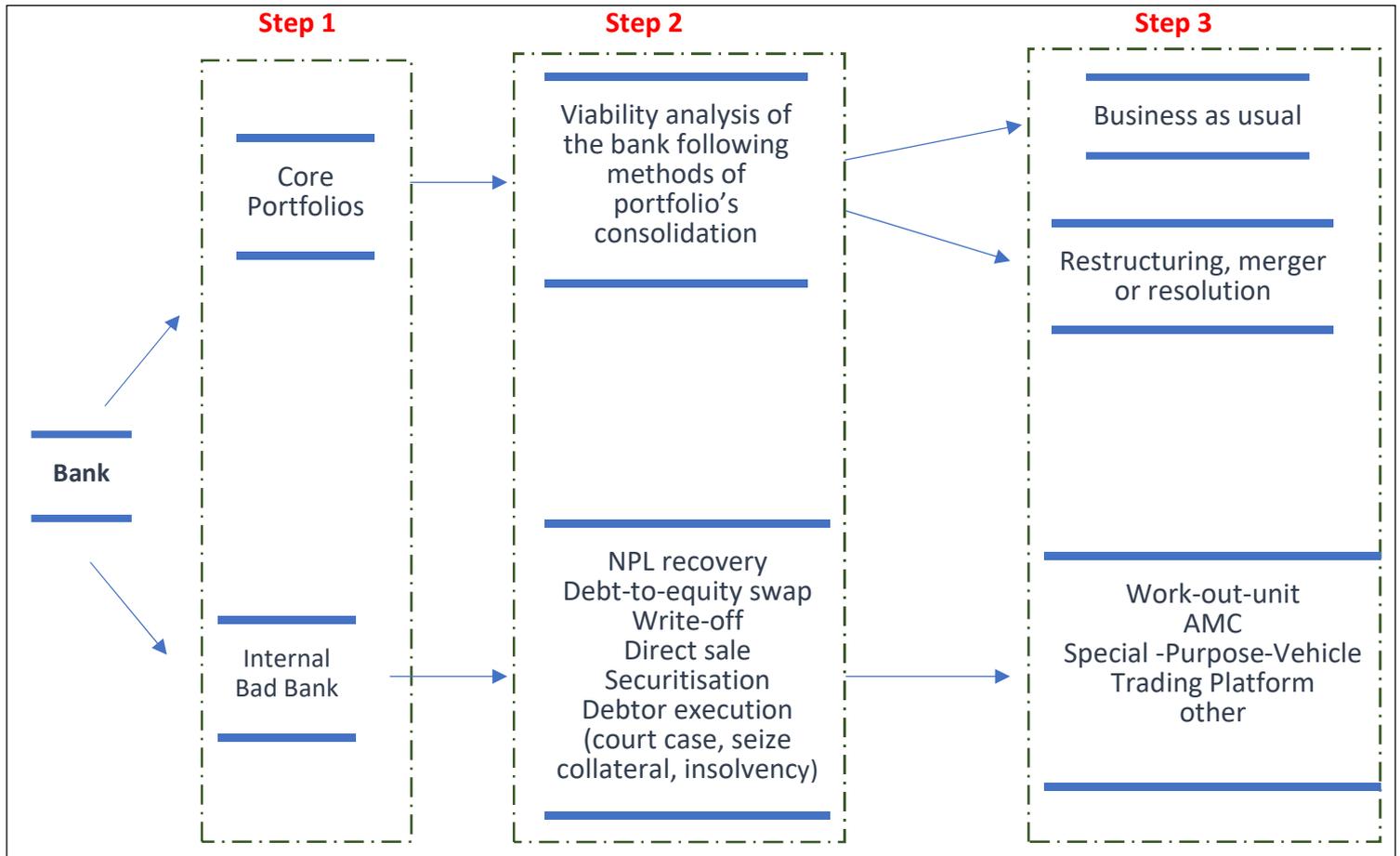
Principle 4: The non-performing loans resolution should be aligned with the bank's whole operational adjustment. The efforts of this resolution relate to many bank's procedures and is a very useful way to increase the long-term viability of the banking sector's growth.

Principle 5: The sizeable stock of non-performing loans should be addressed in a multidimensional package of policies (tax, legal, supervision etc.)

The strategy of the non-performing loans requires an organized and methodological approach. The following is the decision trees of the strategy as it is displayed in the ESRB report "the Resolving non-performing loans in Europe July 2017".

²⁰ Net present value (NPV) is the difference between the present value of cash inflows and the present value of cash outflows over a period of time.

Figure 1.12. Decision tree for the resolving of non-performing loans in Europe²¹



The step 1 of non-performing loans reduction includes the separation and the valuation of loans portfolio/collaterals. This valuation contains collateral valuation, impairments, cash flows (for going concern cases) or any other information which is necessary for the establishment of the banks strategy. This strategy may aim to recognize the amount of non-performing loans on the balance sheet, transfer the stock of non-performing loans to an asset management company or in an independent party without any additional credit risk. When the portfolio is valued, the non-performing loans are separated from the other bank's assets and then policymakers decide to transfer non-performing loans to an internal bad bank or to a dedicated unit for this portfolio.

The step 2 of non-performing loans reduction is the viability analysis (2a) and loan-by-loan analysis (2b). The long-term viability analysis aims to determine if the bank is able to operate as an independent entity. The core portfolio of the bank should ensure a long-term profit in order to be able to cope with the future operational costs. The second scenario of step 2 includes loan-by-loans

²¹ European Systemic Risk Board: Resolving non-performing loans in Europe July 2017

analysis from the internal bank entity and the debtors are separated into non-viable and viable.

These cases of debtors who are not viable the non-performing loan should be liquidated. The tools for this procedure are:

-the seizure and sale of the collateral: This tool is used when the debtor is in default and unable to return in viability in the future. However, the sale of collateral is not a very easy solution since it is costly and time-consuming, so the bank should consider other alternatives.

-the bankruptcy and the sale of proceeds: This solution relates to debtors who will not be viable in the future and leads to the loss of company's goodwill.

Concerning the debtors who will be viable in the future, but they deal with periodical economic difficulties the reduction methods are:

-the restructuring of non-performing loan: This restructuring involves the modification of pay-terms that offer the debtor the potential to fulfill the loans obligations on a sustainable basis.

- The debt-to-equity swaps which creditors become shareholders: This method is usually associated with companies in order to address their obligations while they continue to operate.

Moreover, other resolution methods for both viable and no-viable borrowers could be performed, such as Direct sale, securitisation, write-offs.

The step 3 of the decision tree is the final restructuring form which enables:

- Some resolution methods and policies in order to improve the viability (merge, sell etc.), if the analysis of the step results in long-term viability of bank
- Prerequisite structures, which will support the loan-by-loan analysis of step (2b).

1.6 Future evolution

According to the ECB Annual Report on supervisory activities (2019), the stock of non-performing loans has reduced about 8% in the first half of 2019, although it remains a high level within the European Union. The NPLs strategies submitted in 2019 provide a gross reduction in the NPL proportion of roughly 50% from the end of 2018 to the end of 2021. The European banks continue the efforts on reducing the ratio of NPLs, according to the ECB's Guidance to banks on non-performing loans (hereinafter referred to as the "NPL Guidance") in March 2017. Many banks of the European Union target on reduction strategies concerning older non-performing loans. ECB mentions that non-performing exposures for more than five years are estimated to be reduced at a significantly faster time horizon than that of exposures that have been non-performing for less than five years. Another ECB's goal of reduction is to prevent the build-up of new non-performing loans.

For Greece, according to the Bank of Greece *Monetary Policy Report 2018-2019, 01/07/2019 - Press Releases*²², Greek banks made progress on the reduction of the NPLs ratio, especially at the end of March 2019 and NPLs amounted to €80 billion reduced €27.2 billion from their peak at March 2016. For the Bank of Greece, the aim is to reduce the average NPL ratio down to below 20% by end-2021, although it is impracticable to reach the average 3.2% of the European Union (Dec 2018). Finally, prerequisites for an investment- and export-led recovery should target on a more significant reduction in NPLs in corporation with the Bank of Greece and the Ministry of Finance, which is necessary in order to increase lending by the bank and to recover the investment.

The non-performing loans issue gave rise to the setting-up of the secondary market and new investments in Europe. Over €205bn in completed NPL sales were tracked by Debtwire²³ during the period and over €45bn were in the pipeline at the end of January 2019. According to Debtwire²⁴, in the second quarter of 2018 the biggest rate of NPLs transactions occurred in Italy (€103.6bin GBV²⁵), Spain (€43.2bin GBV), Ireland (€14.3bin GBV), Greece (€13.9bin GBV), Portugal (€8bin GBV).

²² <https://www.bankofgreece.gr/en/news-and-media/press-office/news-list/news?announcement=ba4e1b6b-410d-4976-a5ec-8ca224c13950>

²³ <http://www.debtwire.com/pdf/EuropeanNPLFY18!.pdf>

²⁴ European NPLs - FY18: An overview of the non-performing loan market

²⁵ GBV; Gross Book Value



Chapter 2: Modelling and forecasting non-performing loans in the Greek economy

2.1 Introduction

In this chapter a theoretical overview of the univariate and multivariate forecasting methods is provided. The methodology assumptions, tests and forecast techniques are analyzed both for univariate and multivariate cases. The univariate modelling uses the non-performing loans time series and its own lags whereas the multivariate modelling besides the NPLs time series and its lags includes macroeconomics variables such as GDP growth, unemployment rate and interest rate. The datasets cover the period from the 12/2002 to 09/2009. Finally, the forecast results for the next three quarters along with the forecast errors are displayed and in the last unit the results of the above methods are briefly compared.

2.2 Definition and analysis of ARIMA univariate modelling

In this chapter we adopt the Box Jenkins methodology of ARIMA modelling in order to forecast the future amount of non-performing loans in Greece using quarterly data of the Bank of Greece²⁶. The stock of non-performing loans is one of the most remarkable issues of the Greek banking sector with a great increase over the last years that started growing during the period of the sovereign debt crisis of the country.

The first model concerns the univariate forecast, in which future values of a time series are assumed to be based only on the past values of the time-series itself. Historical data are analyzed in an attempt to create a pattern, and a similar historical pattern is assumed to continue in the future. For this method we will focus on the Auto Regressive Integrated Moving Average (ARIMA) forecast. According to Box and Jenkins (1970), the ARIMA forecast is a model based on its own values, lags and error lags, so it can be used to predict the future values of the time series. A time series Y_t is said to follow an integrated autoregressive moving average model if the d^{th} difference $W_t = \nabla^d Y_t$ as a stationary ARMA process. This forecast model is denoted by ARIMA(p,d,q) and characterized by three parameters:

²⁶ <https://www.bankofgreece.gr/en/statistics/evolution-of-loans-and-non-performing-loans>

- i) p is the order of the AR (autoregressive) term, and it refers to the number of lags of NPLs time series to be used as predictors
- ii) q is the order of the MA (moving average) term and it refers to the number of lags of the forecast errors
- iii) d is the number of differencing required to make the time series stationary.

The general form of the AR and MA models are the following:

-AR: $Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p}$

-MA: $Y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q}$

The general form of the ARIMA(p,q,d) is the following:

$$\Phi(L)Y_t = A(L)(1 - L)^d Y_t = \delta + \theta(L)e_t$$

where :

$$\Phi(L) = A(L)(1 - L)^d$$

$$A(L) = 1 - a_1 L - a_2 L^2 - \dots - a_p L^p$$

$$\Theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \dots - \theta_q L^q$$

$$\Delta Y_t = (1 - L)Y_t = Y_t - Y_{t-1}$$

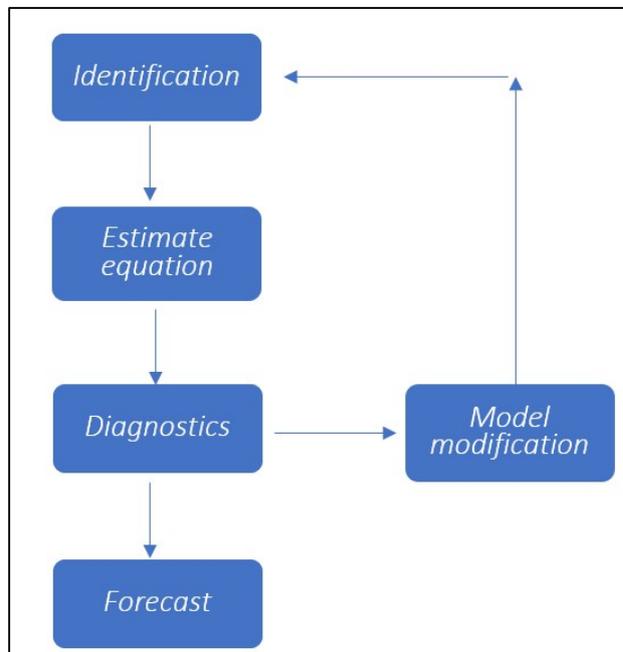
- If $a(z) \neq 0$ for $|z| \leq 1$ then Y_t is stable and stationary
- If $\theta(z) \neq 0$ for $|z| \leq 1$ then Y_t is invertible

The spectral density of the ARIMA process is:
$$f_y(\lambda) = \frac{1}{2\pi} \sigma_e^2 \left(\frac{\theta(e^{i\lambda})}{\alpha(e^{i\lambda})} \right)^2$$

Stationarity is crucial in time series analysis. In a stationary time series, the statistical properties such as mean, variance and autocorrelation are constant over time and the forecast of the future values is more precise to the real data. To achieve the stationarity, we can use many mathematical transformations. For instance, in the ARIMA methodology stationarity can be achieved through differencing the original series.

The Box and Jenkins in 1976 developed a practical approach to fit the data in an ARIMA(p,q,d) process. This approach consists of the following main steps:

Figure 2.1. The Box-Jenkins steps



Identification:

The most common techniques of model identification is the sample autocorrelation (ACF) and partial correlation functions (PCF). The autocorrelation and partial correlation functions are determined as follows:

- Autocorrelation function (ACF):

Considering a stationary series Y_t , the correlation between Y_t and Y_{t-k} is called the lag-k autocorrelation of Y_t . In particular is defined as :

$$\rho_k = \frac{Cov(y_k, y_{t-k})}{\sqrt{Var(y_k)Var(y_{t-k})}} = \frac{Cov(y_k, y_{t-k})}{Var(y_t)} = \frac{\gamma_k}{\gamma_0}$$

For the weakly stationary series, $Var(y_k) = Var(y_{k-t})$ is used. Moreover, from the above definitions arise that $\rho_0=1$, $\rho_k=\rho_{-k}$ and $-1 \leq \rho_k \leq 1$.

If we have a sample of T observations, the estimator $\hat{\rho}_k$ of ρ_k is defined as:

$$\hat{\rho}_k = \frac{\sum_{t=k+1}^T (y_t - \bar{y})(y_{t-1} - \bar{y})}{\sum_{t=1}^T (y_t - \bar{y})^2}, \quad 0 \leq k < T-1$$

- Partial autocorrelation function (PACF):

The $\rho_{11}, \rho_{22}, \dots, \rho_{kk}$ are the partial autocorrelation measures which are obtained by fitting a sequence of autoregressions:

$$y_t = \rho_{11}y_{t-1} + u_t$$

$$y_t = \rho_{21}y_{t-1} + \rho_{22}y_{t-2} + u_t$$

.....

$$y_t = \rho_{k1}y_{t-1} + \rho_{k2}y_{t-2} + \dots + \rho_{kk}y_{t-k} + u_t$$

The ρ_{kk} measures the correlation between y_t and y_{t-k} after the effect of $y_{t-1}, y_{t-2}, \dots, y_{t-k+1}$ has been netted out.

The ACF and PACF are very important in order to determine the value of p, q and d of an ARIMA process.

Differencing d: The level of differencing is estimated by considering the autocorrelation plots.

When the autocorrelations die out quickly, the appropriate value of d is found.

Value of p: The value of p is determined from the partial autocorrelations of the appropriately differenced series. If the partial autocorrelations are cut off after a few lags, the last lag with a large value would be the estimated value of p. If the partial autocorrelations are not cut off, you either have a moving average model (p=0) or an ARIMA model with positive p and q.

Value of q: The value of q is found from the autocorrelations of the appropriately differenced series. If the autocorrelations are cut off after a few lags, the last lag with a large value would be the estimated value of q. If the autocorrelations are not cut off, you either have an autoregressive model (q=0) or an ARIMA model with a positive p and q.

In the following sections we analyze the next steps of Box and Jenkins approach.

2.3 Model Testing

The ARIMA forecast model should satisfy some criteria in order to be utilized for future values predictions and for more reliable interpretation of the dataset. According to Dimelis (2013)²⁷, the following criteria should be met:

a. Parsimony

This principle focuses on the simplest possible method which should be chosen. The estimated forecast model should enable as fewer parameters as possible. With the increase of model parameters, the risk of overfitting subsequently increases. An over fitted time series model may describe the dataset better, but it may not be suitable for future forecasting values because every sample has its own quirks which increase the difficulty of the out-of-sample forecast. One method to detect an overfitted model is to increase the parameters of the estimated model and check if they are statistically significant.

b. Stationarity

A time series are stationary if:

$$E(\mathbf{y}_t) = \boldsymbol{\mu}_y \quad \forall t$$

$$Var(\mathbf{y}_t) = \boldsymbol{\sigma}_y^2 \quad \forall t$$

$$Cov(\mathbf{y}_t, \mathbf{y}_{t+k}) = \boldsymbol{\gamma}_k \quad \forall t, k$$

The first two conditions imply that the means and the variance are independent of time and the third relation means that the covariance is also independent of the time and it is just depended on the distance (h) in time.

²⁷ ΔΗΜΕΛΗ ΣΟΦΙΑ (2013) ΣΥΓΧΡΟΝΕΣ ΜΕΘΟΔΟΙ ΑΝΑΛΥΣΗΣ ΧΡΟΝΟΛΟΓΙΚΩΝ ΣΕΙΡΩΝ

c. Diagnostic criteria of the model's fit

A measure that shows how well the chosen model fits the data is the statistical R^2 /adjusted R^2 , which represents the proportion of the variance for a dependent variable that is explained by an independent variable or variables in a forecast model. R-squared coefficients range from 0 to 1 and can also be expressed as percentages in a scale of 1% to 100%. If this percentage is close enough to 1 then the estimated model has good explanatory power. The difference between the R^2 and the adjusted R^2 is that the adjusted R^2 increases only when the new added parameters are statically significant.

Another indication of the adaptability is the S.E. of Regression (standard error of regression) measure and therefore the selection of our model should be based on the minimum percentage of SER. Furthermore, when we have a great number of parameters, it is very useful to use the information criteria in order to test the model adaptability with the data. The information criteria compensate for the reduction of residuals from the increase of the parameters and they are based on the likelihood function values. The most well-known criteria are Akaike information criterion (AIC) and Schwartz information criterion (SIC) or Bayesian information criterion (BIC).

These criteria are defined as:

- $AIC = \ln(s^2) + n \frac{2}{N}$
- $SIC = \ln(s^2) + n \frac{\ln(N)}{N}$

Where:

- s^2 is the Maximum likelihood estimation of residuals variance,

$$s^2 = \frac{1}{N} \sum_{i=1}^N e^2_i \quad \text{and } N=p+q+1 \text{ of an ARMA}(p,q) \text{ model}$$

- n is the number of the model's estimated parameters
- N is the sample size

Another information criterion is the Hannan and Quinn criterion (HQC), which is defined as:

$$HQC = \ln(s^2) + n \frac{2 \ln(\ln N)}{N}$$

The above information criteria penalize a candidate forecast model by the number of parameters used and each information criterion uses its own penalty function. The following relations among the information criteria are applicable even in small samples ($N \geq 16$):

$$\hat{p}(SIC) \leq \hat{p}(HQC) \leq \hat{p}(AIC) \text{ }^{28}$$

where \hat{p} is the order estimator of p .

To conclude, the SIC results in more parsimonious parameters than the other two criteria.

d. White noise residuals

The residual diagnostics are of great importance when it comes to modifying the ARIMA model, as they express the difference between the real and the fitted data. A reliable forecast method enables the following properties of residuals diagnostics:

- i. The residuals are uncorrelated. If there are correlations between residuals, then there is an amount of information left in the residuals which has not been computed in the forecast model.

$$\gamma_k = Cov(e_t, e_{t-k}) = 0 \quad \forall t \quad \text{and} \quad k \neq 0$$

- ii. The residuals have zero mean. If the residuals have a mean other than zero, then the forecasts are biased.

$$E(e_t) = 0 \quad \forall t$$

- iii. The residuals have constant variance.

$$\gamma_0 = Cov(e_t) = s_e^2 \quad \forall t$$

- iv. The residuals are normally distributed.

$$e_t \sim N(0, s_e^2) \quad \forall t$$

²⁸ Lütkepohl (2005)

e. Stability of parameters

Another important principle for the forecast modelling is the stability of the parameters over time. Sometimes a structural change may occur in our dataset and the values of parameters do not remain stable through the time period due to external factors (such as debt crisis). For the ARIMA model the most common stability test is the maximum likelihood (ML) ratio test.

f. Accuracy of forecasts

In the final step of fitting the data in the most suitable model, we test our forecast efficiency. When our data have the same scale as the errors the comparison among for the accuracy of our forecast models is based on the following measures:

- mean absolute error: $MAE = \frac{1}{h} \sum_{t=N+1}^{N+h} |e_t^{\wedge}|$
- mean squared error: $MSE = \frac{1}{h} \sum_{t=N+1}^{N+h} e_t^{\wedge 2}$
- root mean squared error: $RMSE = \sqrt{MSE}$
- mean absolute percentage error: $MAPE = \frac{100}{h} \sum_{t=N+1}^{N+h} \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right|$
- Theil Inequality Coefficient :
$$U = \frac{\sqrt{\frac{1}{h} \sum_{t=N+1}^{N+h} (Y_t - \hat{Y}_t)^2}}{\sqrt{\frac{1}{h} \sum_{t=N+1}^{N+h} Y_t^2 + \frac{1}{h} \sum_{t=N+1}^{N+h} \hat{Y}_t^2}}$$

Where, Y_t is the observed value
 \hat{Y}_t is the predicted value
 $e_t = Y_t - \hat{Y}_t$ is the error of the forecast

Furthermore, the MSE measure focuses on greater errors due to the fact they are raised to the square contrary to MAE and MAPE.

2.4 Forecasting techniques

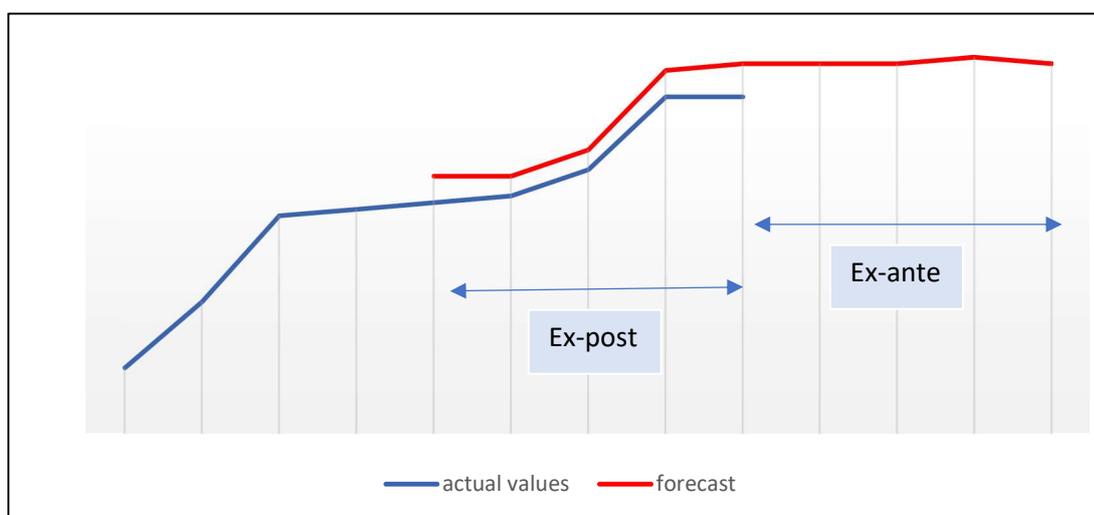
There are two types of forecasts, point forecasts and interval forecasts. Point forecasts concern a specific predicted value of a future time and the interval forecasts lead to an interval of values in which it is expected to be the real value in the future. It is more useful to estimate an interval rather than a point, since predicting a specific(point) value enables a greater forecast failure.

According to Dimelis(2013) and Jenkins & Reinsel (1994) the main reasons of the forecast errors that need mentioning are the following:

- the type of the random factor even if our forecast model is well identified and estimated
- errors of the estimated parameters, since these parameters are random variables and have deviations from real values
- errors resulting from the wrong choice of model
- errors that occur when different forecast models are used for some explanatory variables.

The forecasts results are distinguished in **ex-post** and **ex-ante** forecast. The ex-post forecast concerns forecasting in the sample period and is run in past periods for which actual demand history is also available. The ex-ante forecast uses all the observations in the sample period for forecasting future values out of the sample period. In ex-post forecast the actual values are known and the difference between actual and ex-post values(estimated) determine the accuracy of the model through specific measurements.

Figure 2.2. ex-post and ex-ante forecast plot



Based on the available values of the time series up to time t , Y_1, Y_2, \dots, Y_t , the forecast value of the Y_k in k time units into future is the estimated value \hat{Y}_t . In order to select the most accurate (optimum) forecast model, we optimize the mean squared error (MSE) function.

$$MSE(\hat{Y}_{t+1}) = E(Y_{t+1} - \hat{Y}_{t+1})^2 = E(e_{t+1})^2$$

The forecast which minimizes the MSE function is the conditional expectation of the time series,

namely:
$$\hat{Y}_{t+1} = E(Y_{t+1} | Y_t, Y_{t-1}, \dots, Y_1) = E_t(Y_{t+1})$$

In the conditional expectation value the restriction of the Y_t, Y_{t-1}, \dots, Y_1 is estimated below.

Moreover, in our forecast results it is necessary to have an interval forecast. In order to set up this interval we make an assumption about y_t and e_t distributions. Consider that $y_t, y_{t+1}, \dots, y_{t+h}$ have a multivariate normal distribution for any t and h . Furthermore, the e_t are also normally distributed. Under these assumptions the forecast errors are following normal distribution, which result in the following transformation:

$$y_{t+h} - y_t = \sum_{i=0}^{h-1} \psi_i e_{t+h-i} \square N(0, \sigma_\kappa(h))$$

$$\frac{y_{t+h} - \hat{y}_{t+h}}{\sqrt{\hat{Var}(e_{t+h})}} \square N(0, 1)$$

If z_a is the upper $a/2$ critical value of normal distribution $N(0,1)$ the forecast interval is estimated as follows:

$$1 - \alpha = \Pr \left\{ -z_{a/2} \leq \frac{y_{t+h} - \hat{y}_{t+h}}{\sqrt{\hat{Var}(e_{t+h})}} \leq z_{a/2} \right\}$$

$$= \Pr \left\{ \hat{y}_{t+h} - z_{a/2} \sqrt{\hat{Var}(e_{t+h})} \leq y_{t+h} \leq \hat{y}_{t+h} + z_{a/2} \sqrt{\hat{Var}(e_{t+h})} \right\}$$

Hence, the point forecast is the \hat{y}_{t+h} and the interval forecast is estimated as:

$$[\hat{y}_{t+h} - z_{a/2} \sqrt{\hat{Var}(e_{t+h})}, \hat{y}_{t+h} + z_{a/2} \sqrt{\hat{Var}(e_{t+h})}]$$

In the ARIMA models the forecast is calculated through two procedures. The first way is to forecast the differences and the second one is the direct forecast through the estimated equation.

➤ Forecast Y_t through forecasting the differences $w_t = \Delta^d Y_t$:

The ARIMA(p,d,q) process is an ARMA(p,q) in the d^{th} differences. If $w_t = (1-L)^d Y_t = \Delta^d Y_t$ we can make forecasts using the ARIMA model by forecasting the differences w_t .

For the ARIMA(p,1,q), which is a stationary ARMA(p,q) : $w_t = (1-L)Y_t = Y_t - Y_{t-1}$ or $Y_t = w_t + Y_{t-1}$

For stationary ARMA(p,q) it applies that:

$$\begin{aligned} \hat{Y}_{t+1} &= \delta + \alpha \hat{Y}_t - \theta e_t \\ \hat{Y}_{t+2} &= \delta + \alpha \hat{Y}_{t+1} \\ &\cdot \\ &\cdot \\ \hat{Y}_{t+h} &= \delta(1 + \alpha + \dots + \alpha^{h-1}) + \alpha^h \hat{Y}_t - \alpha^{h-1} \theta e_t \end{aligned}$$

Following the steps above we estimate the w_t and then we reach in the forecast of ARIMA(p,1,q) for h future steps in time:

$$\hat{Y}_{t+h} = \hat{w}_{t+h} + \hat{Y}_{t+h-1} = \hat{w}_{t+h} + \hat{w}_{t+h-1} + \dots + \hat{w}_{t+1} + Y_t$$

For the ARIMA(p,2,q) entails that : $w_t = (1-L)^2 Y_t = Y_{t-2} - 2Y_{t-1} + Y_t$ and the forecast equation of the ARIMA(p,2,q) for h future steps in time is as follows:

$$\hat{Y}_{t+h} = \hat{w}_{t+h} + 2\hat{Y}_{t+h-1} - \hat{Y}_{t+h-2} = \sum_{j=0}^{h-1} (j+1) \hat{w}_{t+h-j} + Y_t + h(Y_t - Y_{t-1})$$

The above methodology is utilized for every value of the differences(d).

➤ Direct forecast of the Y_t :

It is proved that a non-stationary ARIMA (p,d,q) equals to a non-stationary ARMA (p+d,q):

$$\begin{aligned} Y_t &= \delta + \varphi_1 Y_{t-1} + \dots + \varphi_{p+d} Y_{t-p-d} + e_t - \theta_1 e_{t-1} - \dots - \theta_q e_{t-q} \\ &= \delta + \Phi(L)Y_t + \Theta(L)e_t \end{aligned}$$

The ARIMA (p,d,q) forecast for h step in the future results as follows:

$$\hat{Y}_{t+h} = \delta + \varphi_1 \hat{Y}_{t+h-1} + \dots + \varphi_{p+d} \hat{Y}_{t+h-p-d} + e_t - \theta_1 e_{t-1} - \sum_{j=1}^q \theta_j e_{t+h-j}$$

2.5 Empirical Analysis

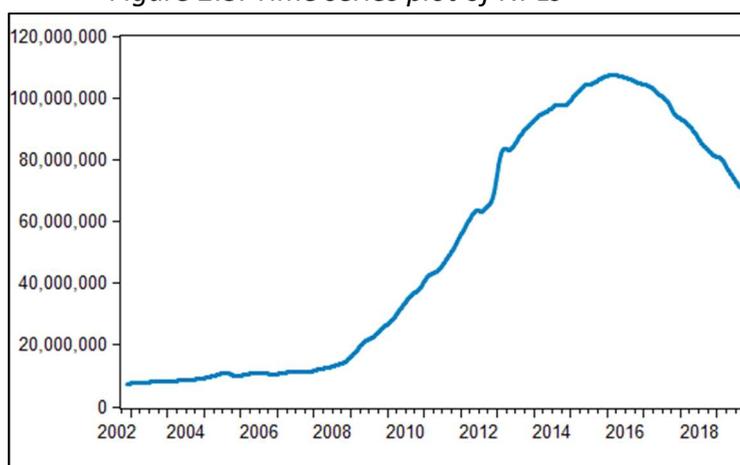
2.5.1 Data

In the following sections we analyze the non-performing loans time series using the EViews 9²⁹ package. The data used in this study are the total amount of non-performing loans and they are obtained from the Bank of Greece³⁰. The sample includes 68 observations covering the period from 12/2002 to 09/2019. According to the Bank of Greece, the data refer to on-balance sheet gross loans and advances³¹ of Greek commercial and cooperative banks. For 2002-09/2014 the data were obtained from the banking sector according to Act. 2442/1999 with the assumption that including loans have been restricted over the past 12 months. After 2014 the data were gathered from the banks according to Act. 42/2014 and European Banking Authority (EBA) policies.

Table 2.1. Descriptive analysis of NPLs

Mean	50.847.755,07
Median	43.336.054,00
Maximum	107.196.294,79
Minimum	7.145.431,16
Std. Dev.	38.742.509,86
Skewness	0,19
Kurtosis	1,34
Jarque-Bera Probability	8,26 0,02
Sum	3.457.647.344,82
Observations	68

Figure 2.3. Time series plot of NPLs



The above measures are in thousands of euros, the mean is about €50,84 billion and the maximum amount of NPLs is about €107,19 billion in 03/2016. After 03/2016 the time series is reduced due the reduction policies of the banking sector in the European Union and the general total growth of the Greek economy.

²⁹ <http://www.eviews.com/home.html>: EViews is a modern econometric, statistics, and forecasting package that offers powerful analytical tools.

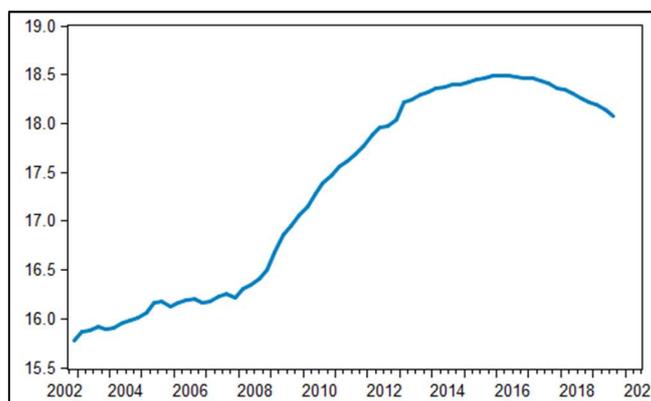
³⁰ <https://www.bankofgreece.gr/en/statistics/evolution-of-loans-and-non-performing-loans>

³¹ Advances refer to bank's credit facilities provided in an entity for a specific purpose and they must be repayable in a short term

2.5.2. Methodology and model testing

The first step of the study is to estimate the best fitted ARIMA(p,d,q) of the NPLS time series. In this study we choose to estimate the forecast model of the natural logarithm of NPLS time series in order to maximize our forecast accuracy.

Figure 2.4. $\ln(NPLS)$ time series graph



From table 2.4. it is obvious that the $\ln(NPLS)$ time series is non-stationary as it shows a sharp increase after 2008 mainly due to the effect of sovereign debt crisis that started to decline in 2016. The stationary is eliminated by differencing the series and defining the order of the differencing. There are several ways to identify this order including the Augment Dickey Fuller (ADF) unit root test, which leads to the correct order of differencing. For the unit root test and consequently for the Augment Dickey Fuller test the hypothesis testing is defined as:

H_0 : The time series has a unit root vs H_a : The time series is stationary

In the following tables the results of ADF unit root test for 1st and 2nd difference for the $\ln(NPLS)$ time series are represented.

Table 2.2: ADF test of $\Delta \ln(NPLS)_t$

Null Hypothesis: D(LN_NPLS) has a unit root Exogenous: Constant Lag Length: 3 (Automatic - based on SIC, maxlag=3)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-0.778748	0.8181
Test critical values:		
1% level	-3.538362	
5% level	-2.908420	
10% level	-2.591799	
*Mackinnon (1996) one-sided p-values. Augmented Dickey-Fuller Test Equation Dependent Variable: D(LN_NPLS,2) Method: Least Squares Sample (adjusted): 3/01/2004 9/01/2019 Included observations: 63 after adjustments		

Table 2.3. ADF test of $\Delta^2 \ln(NPLS)_t$

Null Hypothesis: D(LN_NPLS,2) has a unit root Exogenous: Constant Lag Length: 2 (Automatic - based on SIC, maxlag=3)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-8.438048	0.0000
Test critical values:		
1% level	-3.538362	
5% level	-2.908420	
10% level	-2.591799	
*Mackinnon (1996) one-sided p-values. Augmented Dickey-Fuller Test Equation Dependent Variable: D(LN_NPLS,3) Method: Least Squares Sample (adjusted): 3/01/2004 9/01/2019 Included observations: 63 after adjustments		

The ADF test results in table 2.2 show that the absolute value of the ADF Test Statistic (-0.77) is less than the absolute values of the test critical values at 1%, 5%, and 0% level, thus we conclude that the $\ln(NPLS)$ time series is non stationary at its 1st level of differencing.

Next. We test whether the series become stationary in the second differences. The ADF test results in table 2.3, show that the value of ADF Test Statistic (-8.43) is less than the values of the test critical values at 1%, 5%, and 0% level, we conclude that the ADF showed that the $\ln(NPLS)$ time series is stationary at its 2nd level. Concluding, the ARIMA parameter d equals to 2 and we proceed with the estimation of the other two parameters (p,q).

The p and q values are estimated according to the Autocorrelation function(ACF) and the Partial Autocorrelation function(PACF). The output of ACF and PACF for the 2nd difference shows that the best fitted ARIMA is the ARIMA(3,2,2) of the $\ln(NPLS)_t$

Table 2.4. ACF and PACF plots of 2nd difference of $NPLS_t$

Autocorrelation		Partial Correlation		AC	PAC	Q-Stat	Prob	
				1	-0.350	-0.350	8.4620	0.004
				2	-0.194	-0.360	11.092	0.004
				3	0.012	-0.271	11.102	0.011
				4	0.197	0.021	13.917	0.008
				5	0.019	0.133	13.944	0.016
				6	-0.213	-0.068	17.351	0.008
				7	-0.009	-0.124	17.356	0.015
				8	0.241	0.104	21.852	0.005
				9	-0.179	-0.118	24.372	0.004
				10	-0.025	-0.037	24.422	0.007
				11	0.029	-0.010	24.488	0.011
				12	0.162	0.125	26.662	0.009
				13	-0.103	0.044	27.555	0.010
				14	-0.226	-0.196	31.946	0.004
				15	0.204	-0.033	35.612	0.002
				16	0.131	0.068	37.160	0.002
				17	-0.114	0.089	38.358	0.002
				18	-0.007	0.194	38.363	0.003
				19	-0.107	-0.094	39.463	0.004
				20	0.290	0.117	47.689	0.000
				21	-0.329	-0.299	58.477	0.000
				22	0.052	-0.086	58.752	0.000
				23	0.020	-0.192	58.792	0.000
				24	0.067	-0.017	59.271	0.000
				25	-0.107	-0.039	60.529	0.000
				26	-0.028	-0.026	60.616	0.000
				27	0.101	0.016	61.781	0.000
				28	0.109	-0.006	63.174	0.000

The p value is determined by the partial autocorrelations of the appropriately differenced series. The partial autocorrelations were cut off after three lags, the third lag with a large value is the estimation of the p value and therefore equals to 3. The q value is determined by the autocorrelations of the appropriately differenced series. The autocorrelations were cut off rapidly after the second lag, the second lag with a large value is the estimation of the q value and therefore

equals to 2. In conclusion, the best fitted model of the $\ln(\text{NPLS})$ time series is the ARIMA(3,2,2) and the estimated equation of the model is as follows:

$$\Delta^2 \ln(\text{npls}) = c + a_1 \Delta^2 \ln(\text{npls})_{t-1} + a_2 \Delta^2 \ln(\text{npls})_{t-2} + a_3 \Delta^2 \ln(\text{npls})_{t-3} - \vartheta_1 u_{t-1} - \vartheta_2 u_{t-2}$$

Further in our analysis, we add a dummy variable in order to account for the effect of the Greek debt crisis in the increase of the stock of non-performing loans. The dummy variable of the Greek debt crisis describes the steep increase of NPLs from 2009 to 2018 and denoted as $DUMMY_t$, which is specified as:

$$DUMMY_t = \begin{cases} 1, & 3 / 2009 \leq \text{time} \leq 12 / 2018 \\ 0, & \text{time} < 3 / 2009 \text{ and } \text{time} > 12 / 2018 \end{cases}$$

Finally, the estimated model is described in the following equation:

$$\Delta^2 \ln(\text{NPLS}) = c + a_1 \Delta^2 \ln(\text{NPLS})_{t-1} + a_2 \Delta^2 \ln(\text{NPLS})_{t-2} + a_3 \Delta^2 \ln(\text{NPLS})_{t-3} - \vartheta_1 u_{t-1} - \vartheta_2 u_{t-2} + DUMMY_t$$

Where: - c is the constant term

- $\ln(\text{NPLS})_{t-i}$ is the lags of the $\ln(\text{NPLS})$ variable

-The indicator t denotes the quarters³² (t=1,2,...,66) and the indicator i denotes the number of lags

Having identified the appropriate forecast model, the next stage is to estimate its parameters with the maximum likelihood method (ML). The results of the estimated equation are represented in table 2.7 and the above model is formulated as:

$$\Delta^2 \ln(\text{NPLS}) = 0.0024 - 0.47 \Delta^2 \ln(\text{NPLS})_{t-1} - 0.65 \Delta^2 \ln(\text{NPLS})_{t-2} - 0.35 \Delta^2 \ln(\text{NPLS})_{t-3} + 0.17 u_{t-1} - 0.24 u_{t-2} + DUMMY_t$$

S.E. (0.003) (0.314) (0.238) (0.187) (0.331) (0.276)

³² The quarters are 68 minus 2 (total 66), since in the 2nd degree differencing of $\ln(\text{NPLS})$ we lost 2 observations.

Table 2.5. Maximum Likelihood estimation results

Dependent Variable: D(D(LN_NPLS))				
Method: ARMA Maximum Likelihood (BFGS)				
Sample: 6/01/2003 9/01/2019				
Included observations: 66				
Convergence achieved after 18 iterations				
Coefficient covariance computed using observed Hessian				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002580	0.003462	0.745336	0.4591
DUMMY	-0.006451	0.004425	-1.457948	0.1502
AR(1)	-0.474209	0.314186	-1.509327	0.1366
AR(2)	-0.645509	0.238891	-2.702106	0.0090
AR(3)	-0.353701	0.187431	-1.887105	0.0642
MA(1)	-0.169997	0.331583	-0.512683	0.6101
MA(2)	0.240364	0.276698	0.868687	0.3886
SIGMASQ	0.001466	0.000255	5.741577	0.0000
R-squared	0.350256	Mean dependent var		-0.002174
Adjusted R-squared	0.271838	S.D. dependent var		0.047863
S.E. of regression	0.040843	Akaike info criterion		-3.431436
Sum squared resid	0.096751	Schwarz criterion		-3.166023
Log likelihood	121.2374	Hannan-Quinn criter.		-3.326559
F-statistic	4.466559	Durbin-Watson stat		1.926675
Prob(F-statistic)	0.000497			
Inverted AR Roots	.03-.82i	.03+.82i		-.53
Inverted MA Roots	.08+.48i	.08-.48i		

The final stage of the estimation is to verify if the residuals are normally distributed, have constant variance, have zero mean and if they are uncorrelated. The normal distribution of the residuals is tested through the Jarque-Bera normality test. The statistic is computed as :

$$Jarque - Bera = \frac{N}{6} \left(S^2 + \frac{(K-3)^2}{4} \right)$$

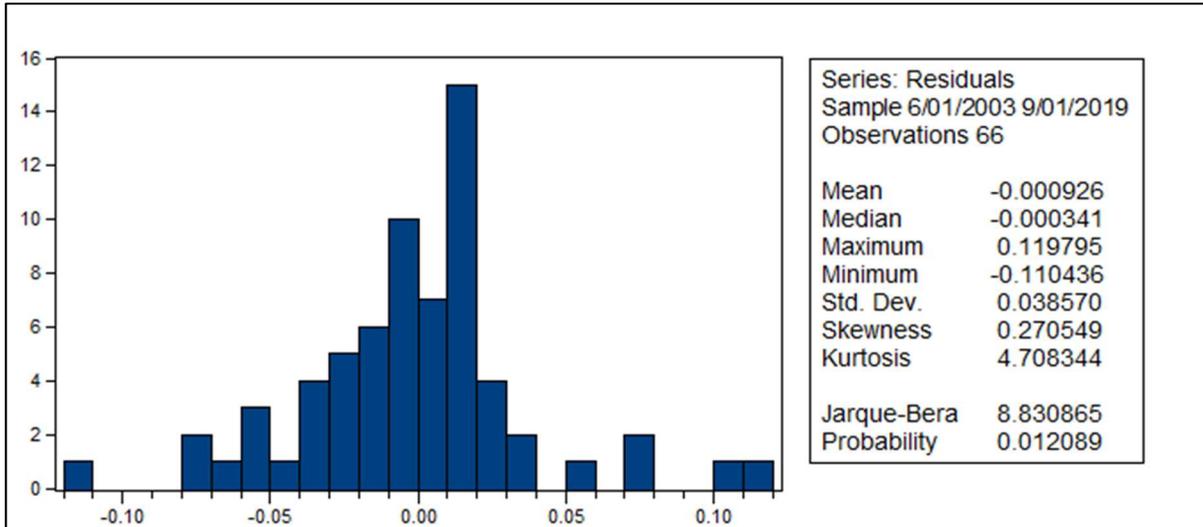
Where, - S is the skewness

- K is the kurtosis

- N is the number of observations

Under the null hypothesis of a normal distribution, the Jarque-Bera statistic is distributed as χ^2 with 2 degrees of freedom. For our model the results in the Eviews 9 are represented in the table 2.6.

Table 2.6. Histogram-Normality test of the residual



Therefore, we conclude that we reject the null hypothesis ($p\text{-value}=0.012<5\%$) and their mean equals to $-0.000926(\approx 0)$.

The second residual diagnostic is the condition that the residuals should have constant variance. There are different heteroskedasticity tests, in this study the used method is the Autoregressive Conditional Heteroskedasticity test (ARCH)³³. The null hypothesis of the ARCH test is that the residuals display no conditional heteroskedasticity against the alternative hypothesis that the residuals show heteroskedasticity. The ARCH(m) model has the following form:

$$\sigma_t^2 = a_0 + \sum_{j=1}^m a_j e_{t-j}^2 \quad \text{and} \quad e_t \sim N(0, \sigma_t^2)$$

Where there is at least one $a_j \neq 0$ ($j=0, \dots, T$) and T is the sample size. The statistic test is the statistic TR^2 , where R^2 is the coefficient of determination from fitting the ARCH(m) model for a number of lags (m) via regression. The statistic TR^2 follows a chi-squared distribution with m degrees of freedom.

³³ Engle (1982): Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation

Table 2.7. ARCH heteroskedasticity test outputs

Heteroskedasticity Test: ARCH				
F-statistic	1.582445	Prob. F(1,63)	0.2131	
Obs*R-squared	1.592677	Prob. Chi-Square(1)	0.2069	
Test Equation:				
Dependent Variable: RESID^2				
Method: Least Squares				
Sample (adjusted): 9/01/2003 9/01/2019				
Included observations: 65 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001205	0.000397	3.035424	0.0035
RESID^2(-1)	0.156226	0.124191	1.257953	0.2131
R-squared	0.024503	Mean dependent var	0.001437	
Adjusted R-squared	0.009019	S.D. dependent var	0.002846	
S.E. of regression	0.002833	Akaike info criterion	-8.864749	
Sum squared resid	0.000506	Schwarz criterion	-8.797844	
Log likelihood	290.1043	Hannan-Quinn criter.	-8.838351	
F-statistic	1.582445	Durbin-Watson stat	1.994803	
Prob(F-statistic)	0.213051			

Considering the results of the ARCH test(table 2.7), the residuals display no conditional heteroskedasticity (p-value=Pro of Chi-square=0.2069>5%) and therefore have constant variance in the sample period.

The last test of the residuals diagnostics is the autocorrelation test. The statistic for the test of this condition is the Q statistic³⁴. The hypothesis testing form is :

$$H_0: \rho_1=\rho_2=\dots=\rho_k=0 \text{ against } H_a: \text{at least one } \rho_j \neq 0 \text{ for } j=1,\dots,k$$

The statistic function for the Q statistic of Ljung and Box is the following:

$$Q^{LB} = N(N + 2) \sum_{j=1}^k \frac{\hat{\rho}_j(e)^2}{N - j}$$

Where: - N is the number of observations

- $\hat{\rho}_j(e)^2$ is sample autocorrelation at lag j

Under null hypothesis the statistic Q asymptotically follows a X^2 distribution with k-p-q (parameters of the ARIMA(p,q,d)) degrees of freedom.

³⁴ Ljung and Box(1978)



Table 2.8. Q statistic outputs

Sample: 12/01/2002 9/01/2019 Included observations: 66 Q-statistic probabilities adjusted for 5 ARMA terms and 1 dynamic regressor						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob*	
		1	0.018	0.018	0.0232	
		2	0.002	0.002	0.0235	
		3	-0.001	-0.001	0.0236	
		4	-0.093	-0.093	0.6503	
		5	0.032	0.036	0.7249	
		6	-0.082	-0.084	1.2297	0.267
		7	-0.056	-0.053	1.4666	0.480
		8	0.041	0.035	1.5969	0.660
		9	-0.121	-0.119	2.7529	0.600
		10	0.093	0.085	3.4445	0.632
		11	0.018	0.008	3.4711	0.748
		12	0.009	0.011	3.4776	0.838
		13	-0.117	-0.154	4.6330	0.796
		14	-0.159	-0.132	6.8033	0.658
		15	0.200	0.204	10.311	0.414
		16	0.142	0.145	12.127	0.354
		17	0.056	0.049	12.411	0.413
		18	0.020	-0.033	12.449	0.491
		19	-0.207	-0.203	16.541	0.281
		20	0.028	0.010	16.619	0.342
		21	-0.302	-0.314	25.713	0.058
		22	-0.020	0.031	25.753	0.079
		23	-0.051	-0.099	26.029	0.099
		24	-0.033	0.092	26.145	0.126
		25	0.003	-0.073	26.146	0.161
		26	-0.002	-0.066	26.147	0.201
		27	0.081	-0.014	26.899	0.215
		28	0.079	0.014	27.633	0.230

According to the table 2.8, the values of Q-stat leads to high values of p-value(Prob.>5%) and therefore we accept the null hypothesis that the residuals are uncorrelated.

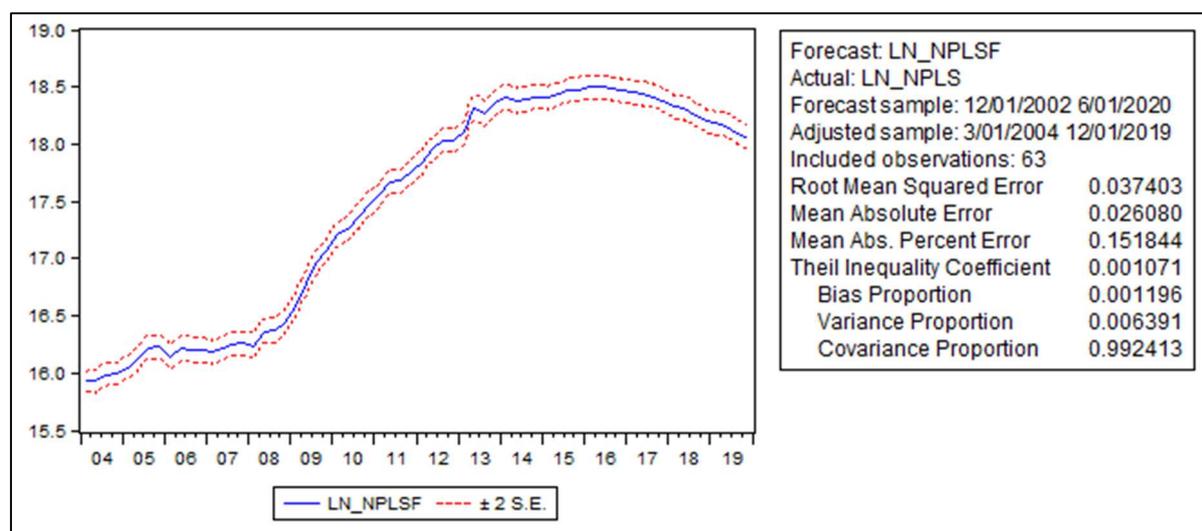
Finally, considering the results of all the residuals diagnostics we conclude that the estimated model satisfies all necessary conditions with the exception of normality which is not unusual for such models.

2.5.3 Forecasting and results

Having estimated the best fitted ARIMA according to the Box-Jenkins approach, the final step of the study is to forecast the future stock of non-performing loans for the next three quarters. The forecast results separated into ex-post and ex-ante for out-of-sample forecasting.

The errors of the model are estimated through the ex-post forecast and the results are presented in figure 2.5. The RMSE, MAE, MAPE are 3.74%, 2.60%, 0.15% respectively. The Theil Inequity Coefficient is about 0.0011 and is analyzed in Bias, Variance and Covariance proportion. The Bias proportion refers to the difference between the real and forecasted data, which indicates the level of the model's bias. Further, the Variance proportion refers to the difference between the variability of the real and forecasted data. The values of the Bias and Variance proportion are very low; therefore, the forecast model does not need further specification and the remaining systematic errors are accumulated in the Covariance proportion.

Figure 2.5. Forecast results

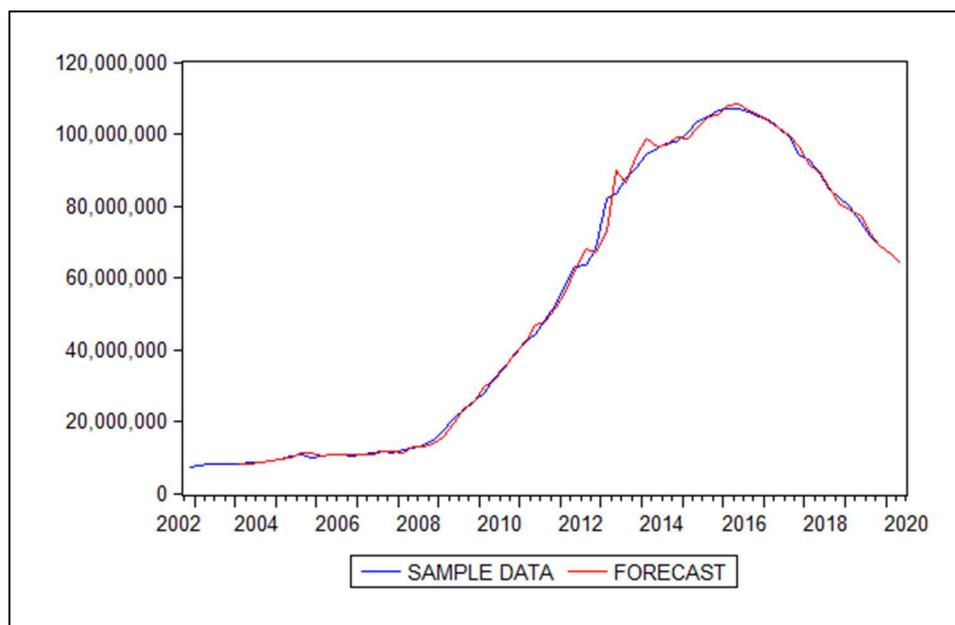


The future values for the next three quarters are represented in the following two tables:

Table 2.9. Forecast values

Quarters	ln(NPLS) ↓	Gross NPLS (in thousands of €) ↓
Dec-2019	18,05	68.690.409,53
Mar-2020	18,01	66.541.129,33
June-2020	17,97	63.855.994,96

Figure 2.6. Forecast graph



Considering the results of the future values of the forecast model it is obvious that the stock of non-performing loans continues to shrink for the next three quarters. The amount of non-performing loans started to decline after 2016 due to the European Union strategies and the banking sector's policies. Moreover, the general economic growth after the sovereign debt crisis and the effect of the different macroeconomic determinants resulted in this reduction. Some of these determinants (such as GDP growth, unemployment rate etc.) will be analyzed and tested in the next unit of the Vector Autoregressive forecast models.

2.6 Definition and analysis of VAR multivariate modelling

The multivariate forecasting uses multiple variables to forecast the future possible outcome. In the previous chapter the univariate forecast uses time series lags and has one time-dependent variable. However, in the multivariate method the forecasting model has more than one time-dependent variable. Besides the dependency on the time series lags, they are also depend on other variables to predict the future values of the time series. The most common method for the multivariate forecasting is the vector autoregression (VAR) model.

According to Sims³⁵(1980), the a priori exogeneity assumption for some of the variables is ad hoc and it is not based on a fully developed theory, therefore we should assume that all the variables are endogenous. Therefore, in the VAR analysis the values of the model are explained though the involved values and their lags. The VAR models could be of useful for the following reasons, which are analyzed in the next units:

- Forecasting and modelling economic time series
- Evaluating the consequences of alternative policy actions. The VAR analysis of monetary transmission mechanism started with Sims (1980), which analyze the effect of the asset prices and the general economic conditions on the monetary decisions.
- Granger-causality analysis, which investigate the effect among the model's variables
- Structural analysis, which consist of the impulse response analysis, the forecast error variance decomposition and the historical decomposition of time series.

Supposing a two-variable model, the general form of the VAR(p) model is the following:

$$X_t = d_1 + a_{11}X_{t-1} + a_{12}X_{t-2} + \dots + a_{1p}X_{t-p} + b_{11}Z_{t-1} + b_{12}Z_{t-2} + \dots + b_{1p}Z_{t-p} + e_{t1}$$

$$Z_t = d_2 + a_{21}X_{t-1} + a_{22}X_{t-2} + \dots + a_{2p}X_{t-p} + b_{21}Z_{t-1} + b_{22}Z_{t-2} + \dots + b_{2p}Z_{t-p} + e_{t2}$$

Or

$$\begin{bmatrix} X_t \\ Z_t \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} a_{11} & b_{11} \\ a_{21} & b_{21} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} a_{12} & b_{12} \\ a_{22} & b_{22} \end{bmatrix} \begin{bmatrix} X_{t-2} \\ Z_{t-2} \end{bmatrix} + \dots + \begin{bmatrix} a_{1p} & b_{1p} \\ a_{2p} & b_{2p} \end{bmatrix} \begin{bmatrix} X_{t-p} \\ Z_{t-p} \end{bmatrix} + \begin{bmatrix} e_{t1} \\ e_{t2} \end{bmatrix}$$

Or

$$Y_t = d + A_1Y_{t-1} + A_2Y_{t-2} + \dots + A_pY_{t-p} + e_t$$

Where, d is the vector of the constant terms, Y_t is the vector of endogenous variables, A_i is the coefficient matrices and e_t is the unobservable error term(or impulse responses).

³⁵ Christopher A. Sims, "Macroeconomics and Reality," *Econometrica*, vol. 48, 1980, pp. 1–48

The name of the Vector Autoregression (VAR) method is based on the form of the last equation, which is an Autoregression AR(p) model.

The VAR(p) model could be written as p-dimensional VAR(1):

$$Y_t = d + AY_{t-1} + E_t$$

Where:

$$\begin{array}{c}
 Y_t = \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-p+1} \end{bmatrix} \\
 (px1)
 \end{array}
 \quad
 \begin{array}{c}
 A = \begin{bmatrix} A_1 & A_2 & \dots & A_p \\ I_k & 0 & \dots & 0 \\ 0 & I_k & \dots & \vdots \\ 0 & \dots & I_k & 0 \end{bmatrix} \\
 (pxp)
 \end{array}
 \quad
 \begin{array}{c}
 E_t = \begin{bmatrix} e_t \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\
 (px1)
 \end{array}
 \quad
 \begin{array}{c}
 d = \begin{bmatrix} d \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\
 (px1)
 \end{array}$$

The advantages of the VAR analysis are the following:

- The method is simple due the assumption that all the variables are endogenous
- The estimation is simple by estimating the OLS method for each equation
- The forecasts of this method are better due the fact that the effect of other economic determinants is taken into account

On the other hand, the disadvantages of the VAR analysis are the following:

- It is a-theoretic because it is not based on an economic theory, but the outcomes are derived from the lags of the variables and their autocorrelations in time.
- Due to the large number of parameters, the VAR models are adaptable in sufficiently large samples
- All the variables should be jointly stationary and if they are not stationary sometimes the analysis leads to variables with a different degree of integration.

2.7 Model specification and testing

As in the univariate forecast analysis, the VAR model should follow some specifications in order to have reliable forecasts. The testing of the model and residuals diagnostics are briefly summarized in the following.

The main hypothesis of the VAR analysis is stationarity and stability of the forecast model. The VAR is stationary if the mean and covariance of the Y_t are stable over time. Moreover, the covariance between Y_t and Y_{t+k} should be dependent on their distance k and not on the between time t . Each variable of a VAR model should be $I(0)$ and stable with no trends. If a VAR model has a growing/reducing tendency, then it should be corrected through differencing or through other transformations such as logarithmic transformation. Consider the simple bivariate system:

$$\begin{aligned} Y_t &= d_1 + a_{11}Y_{t-1} - b_{12}Z_t + \gamma_{12}Z_{t-1} + \varepsilon_{yt} \\ Z_t &= d_2 - b_{21}Y_t + \gamma_{21}Y_{t-1} + \gamma_{22}Z_{t-1} + \varepsilon_{zt} \end{aligned}$$

Using matrix algebra, the system is written in structural form as:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

Or $Bx_t = D + \Gamma_1 x_{t-1} + \varepsilon_t$

Where

$$B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \quad D = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \quad \Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \quad x_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$$

multiply with B^{-1} we obtain the VAR model in standard form:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

Where

$$A_0 = B^{-1}D \quad A_1 = B^{-1}\Gamma_1 \quad e_t = B^{-1}\varepsilon_t$$

Using the operator L :

$$\begin{aligned} (I - A_1 L - \dots - A_p L^p)x_t &= A_0 + e_t \\ \Phi(L)x_t &= A_0 + e_t \end{aligned}$$

Suppose that our VAR model is stationary then $\mu = E(Y_t) = E(Y_{t-1}) = \dots = E(Y_{t-p})$ and as a consequence $\mu = E(Y_t) = \Phi(L)^{-1}A_0$. Hence, we have the stationarity condition, which requires that the matrix $\Phi(L)$ is reversible. Using the brute force method, the general form of the equation above is:

$$\begin{aligned} x_t &= A_0 + A_1(A_0 + A_1 x_{t-2} + e_{t-1}) + e_t = (I + A_1)A_0 + A_1^2 x_{t-2} + A_1 e_{t-1} + e_t \\ &\vdots \\ &\vdots \end{aligned}$$

$$x_t = (I + A_1 + \dots + A_1^2)A_0 + \sum_{i=0}^n A_1^i e_{t-i} + A_1^{n+1} x_{t-n-1}, \text{ where } \lim_{n \rightarrow \infty} A^n \rightarrow 0$$

The stability of our VAR model requires that the roots of $(1 - a_{11}L)(1 - a_{22}L) - (1 - a_{12}a_{21}L^2)$ lie outside the unit circle. Furthermore, if all the equations of the VAR model are non-stationary I(1), then the VAR model should be analyzed on the first differences in order to produce effective forecasts. In some cases, if the variables are I(1), then we should examine if a linear combination is I(0) in order to forecast on the level model. This issue is called cointegration analysis and it is analytically examined in the next unit.

The residuals e_t should obey the following hypothesis for the mean, variance and autocorrelations:

1. $E(e_t)=0$, for each t
2. $E(e_t, e_s) = \begin{cases} \Omega, & \text{when } t = s \\ 0, & \text{when } t \neq s \end{cases}$, where $\Omega = \begin{bmatrix} \text{var}(e_{t1}) & \text{cov}(e_{t1}, e_{t2}) \\ \text{cov}(e_{t2}, e_{t1}) & \text{var}(e_{t2}) \end{bmatrix}$
3. The stability of the $\text{Var}(e_{ti})$
4. uncorrelated with the lags of the Y_t .

In conclusion, the other specifications and model testing methods for the VAR process meet the conditions of the ARIMA analysis (chapter 2.1.2).

2.8 Estimation and Forecasting

The coefficients of the VAR model could be estimated through the least squares methodology for each equation separately. Considering the facts that the error terms e_{jt} are unrelated with the lags of Y_{t-j} , they are white noise and each equation has the same number of parameters, we conclude that the OLS estimations will be consistent. The number of the estimated parameters is $n+pn^2$, where n is the number of the vector of intercept terms, n^2 is the number of the coefficients -matrices and p is the order of the VAR(p) model.

Another issue of the VAR analysis is the estimation of the order p , which is the number of the lags of the VAR(p) model. The estimation of the order p is based on the purpose of our analysis. According to Lütkepohl, if the aim of our analysis is to forecast the next period, the order of p should be determined by minimizing the MSE. One way of the efficient order selection is to estimate models with different orders and compare them according to the information criteria:

Final Prediction Error (FPE), AIC, SIC and HQ). The form of the information criteria has the following forms:

$$\begin{aligned}
 FPE(p) &= \left[\frac{T + Kp + 1}{T - Km - 1} \right]^K \det(\Sigma_e(p)) \\
 AIC(p) &= \log \det(\Sigma_e(p)) + \frac{2}{T} pK^2 \\
 HQ(p) &= \log \det(\Sigma_e(p)) + \frac{\log T}{T} pK^2 \\
 SIC(p) &= \log \det(\Sigma_e(p)) + \frac{2 \log \log T}{T} pK^2
 \end{aligned}$$

Where $\Sigma_e(p) = T^{-1} \sum_{t=1}^T \hat{e}_t \hat{e}_t'$ is the residuals covariance matrix estimator for a model of order p, K is the dimension of the time series and $\det(\cdot)$ denotes the determinant.

A second way to select the most efficient order of the VAR model is the likelihood ratio test statistic³⁶. Suppose that the form of the VAR(p) model is as follows:

$$y_t = d + A_1 y_{t-1} + \dots + A_p y_{t-p} + A_{p+1} y_{t-(p+1)} + u_t, \text{ with } A_{p+1} = 0$$

The order of the model is p if $A_p \neq 0$ and $A_i = 0$ for each $i > p$ so p denotes the smallest possible order. Assuming that M is the upper bound for the VAR model, the null and alternative hypotheses are as follows:

$$\begin{aligned}
 H_0^1 : A_M &= 0 & \text{vs} & & H_1^1 &= A_M \neq 0 \\
 H_0^2 : A_{M-1} &= 0 & \text{vs} & & H_1^2 &= A_{M-1} \neq 0 \mid A_M = 0 \\
 & & & & & : \\
 H_0^i : A_{M-i+1} &= 0 & \text{vs} & & H_1^i &= A_{M-i+1} \neq 0 \mid A_M = \dots = A_{M-i+2} = 0 \\
 & & & & & : \\
 H_0^M : A_1 &= 0 & \text{vs} & & H_1^M &= A_1 \neq 0 \mid A_M = \dots A_2 = 0
 \end{aligned}$$

If the null hypothesis is rejected, then the process ends and the VAR order is selected. For example, if the H_0^i hypothesis is rejected, the order of the model is estimated as:

$$\hat{p} = M - i + 1$$

The likelihood ratio test for testing the i-th hypothesis is the following:

$$\lambda_{LR}(i) = T[\ln(\Sigma_e(M-i)) - \ln(\tilde{\Sigma}_e(M-i+1))]$$

³⁶ Lütkepohl: New introduction to multiple time series analysis (2005)



Where, $\tilde{\Sigma}_e$ denotes the ML estimator of the Σ_e for the sample T.

The likelihood ratio statistic follows $X^2(K^2)$ distribution, where K^2 denotes the number of parameters under the null hypothesis.

After estimating the order and the parameters of the VAR model, the next step is to provide accurate forecasts, which is one of the biggest advantages of the VAR analysis. Considering the purpose of our forecast, we should determine a specific cost/loss function. This function is linked to the forecast errors and is aimed to minimize the cost of the future-predicted values. In the economic series the most widespread loss function is the forecast mean errors (MSE), which summarizes all the errors of the economic analysis. Suppose the following VAR(1) process with zero mean:

$$\begin{aligned}
 y_t &= A_1 y_{t-1} + u_t \\
 y_{t+1} &= A_1 y_t + u_{t+1} = A_1(A_1 y_t + u_t) + u_{t+1} = A_1^2 y_t + A_1 u_t + u_{t+1} \\
 &\cdot \\
 &\cdot \\
 &\cdot \\
 y_{t+h} &= A_1^h y_t + \sum_{i=0}^{h-1} A_1^i u_{t+h-i}
 \end{aligned}$$

Thus, the predictor for h-th future steps equals to the following equation:

$$y_t(h) = E(y_{t+h} | y_t, y_{t+1}, \dots, y_{t+h-1}) = E(A_1^h y_t + \sum_{i=0}^{h-1} A_1^i u_{t+h-i}) = E(A_1^h y_t) = A_1^h y_t$$

The forecast error is:

$$\hat{e}_{t+h} = y_{T+h} - y_t(T+h) = e_{T+h} + A_1 e_{T+h-1} + \dots + A_1^{h-1} e_{T+1}$$

And the MSE matrix:

$$\Sigma_e = E[(y_{T+h} - y_t(T+h))(y_{T+h} - y_t(T+h))'] = \sum_{j=0}^{h-1} A_j \Omega A_j'$$



Now, we will estimate the forecast values for a general VAR(p) model with p order of lags.

As indicated the VAR(p) model could be written as VAR(1) model :

$$Y_t = d + AY_{t-1} + E_t$$

Using the brute force method:

$$Y_{t+h} = A^h Y_t + \sum_{i=0}^{h-1} A^i E_{t+h-i}$$

The forecast is estimated as:

$$Y_T(h) = A^h Y_t$$

and the forecast error as:

$$\begin{aligned} y_{t+h} - y_t(h) &= J[Y_{t+h} - Y_t(h)] = J\left[\sum_{i=0}^{h-1} A^i E_{t+h-i}\right] \\ &= \sum_{i=0}^{h-1} JA^i J' E_{t+h-i} = \sum_{i=0}^{h-1} \Phi_i e_{t+h-i} \end{aligned}$$

where, $J = [I_k \ 0 \ 0 \dots \ 0]$ and Φ_i is the MA coefficient matrices

The MSE matrix is:

$$\Sigma_e = E[(y_{T+h} - y_t(T+h))(y_{T+h} - y_t(T+h))'] = \sum_{j=0}^{h-1} \Phi_j \Sigma_e \Phi_j'$$

We have assumed that y_t and e_t are normally distributed and therefore the forecast is also normally distributed:

$$y_{t+h} - y_t(h) = \sum_{i=0}^{h-1} \Phi_i e_{t+h-i} \square N(0, \Sigma_e)$$

$$\frac{y_{k,t+h} - y_{k,t}(h)}{\sigma_k(h)} \square N(0,1)$$

where $y_{k,t}$ is the k element of y_t and $\sigma_k(h)$ is the square root of k-diagonal elements of Σ_e .

Finally the (1-a)% interval forecast of the k-th element of y_t , h-th steps in the future is:

$$[y_{k,t+h} - z_{a/2} \sigma_k(h), y_{k,t+h} + z_{a/2} \sigma_k(h)]$$

2.9 Cointegration and Vector Error Correction (VEC) models

In some cases, the economic data present a significant correlation, which is not linked to any causal relationship. This correlation is observed in non-stationary economic time series with stochastic trend, which are independent. For example, if we have two independent and non-stationary time series, there may be a linear combination with high r^2 and Durbin Watson (DW) percentage near to zero. This phenomenon is called nonsense or spurious regression between the time series. In order to eliminate this problem, we should convert our time series into stationary by differencing them. However, the disadvantage of differencing is that we estimate only short-term forecasts. Therefore, Enders (1995)³⁷ suggested that if the series are non-stationary (and stationary on the first-differences) but there is a linear stationary combination of them, then we should provide forecasts on the level model and avoid forecasting on the first-difference model. This linear combination is referred to as cointegration and was introduced by Engle and Granger (1987)³⁸.

The idea of cointegration is highly related with the equilibrium theory. Economic equilibrium is a condition in which economic forces are in a long-run balanced. Especially in econometric approach this causal relationship involves non-stationary variables, in which a linear combination of them is stationary. The analysis consists of a set of variables that are in long-run equilibrium when:

$$b_1x_{1t} + b_2x_{2t} + \dots + b_nx_{nt} = 0$$

Especially, if we denote b as the vector (b_1, b_2, \dots, b_n) and x_t as the vector (x_1, x_2, \dots, x_n) the system is in long-run equilibrium when $bx_t=0$. Further, the vector $e_t=bx_t$ is the equilibrium error and it should be stationary.

According to Engle and Granger, the components of the vector $x_t=(x_{1t}, x_{2t}, \dots, x_{nt})'$ are defined as cointegrated of order d , b , denoted by $x_t \sim CI(d, b)$ if:

- i. All components of x_t are integrated of order d
- ii. There exists a vector b as the linear combination of $b_1x_{1t} + b_2x_{2t} + \dots + b_nx_{nt}$ is integrated of order $(d-b)$ where $b>0$. The vector b is called the cointegrating vector.

The cointegration vector b is not unique and for any value $\lambda (\neq 0)$ the vector λb is also a cointegrating vector. If the variables are cointegrated in their logarithmic variable, then they are also cointegrated in their level form, but the opposite is not applicable. Moreover, Engle and

³⁷Applied econometric time series - Walter Enders (1995)

³⁸Robert F. Engle; C. W. J. Granger: Co-Integration and Error Correction: Representation, Estimation, and Testing (Econometrica vol. 55, 1987)

Granger's definition refers to variables that are integrated in the same order. In some cases, all the variables are not cointegrated, but only a number of them are. This linear combination is cointegrated with the remaining variables of the set and this phenomenon is called multicointegration.

Considering the above analysis, there is a correction error model(VECM) that restricts the long-run behavior of the endogenous variable to converge to their long-run equilibrium relationship. Engle and Granger presented the Granger representation theorem (1983,1987), which results that if two variables are cointegrated and $I(1)$, then there is a correction error model(ECM).The form of the VECM models is as follows:

$$\Delta y_t = b_0 + \sum_{i=1}^{p-1} \Gamma_i y_{t-i} + \Pi y_{t-1} + BX_t + u_t$$

, where Γ_i is the short-run dynamics, Πy_{t-1} is the error correction term, Π denotes the long-run relationships, B is a coefficient matrix, b_0 is the vector of intercepts, u_t the vector of innovations and the rank of the Π matrix equals to the number of cointegration.

The cointegration could be identified through the following two methods: the Engle-Granger and the Johansen methodology.

➤ **Engle-Granger methodology:**

This method is based on the testing of the non-stationarity of the cointegrated equation's errors for each variable separately. Suppose that two variables (x_t, y_t) are believed to be integrated of order 1 and we want to examine if they are cointegrated. The Engle and Granger (1987) suggested the following steps if $I(1)$ variables are cointegrated:

Step 1

The first step of this method is to test if the variables are integrated. In order to estimate the order of integration, we use the augmented Dickey-Fuller tests for each variable. If the variables do not have the same order of integration, then we conclude that they are not cointegrated unless we may want to determine whether the variables are multicointegrated. If the variables have the same unit root, then we continue with step 2.

Step 2

In this step we will estimate the cointegrating regression equation with the OLS estimation. Suppose that we have two variables X_t and Y_t with the same integrated order $I(1)$, the estimated long-run equilibrium equation has the following form:

$$Y_t = b_0 + b_1 X_t + e_t$$

In order to determine if the variables are actually cointegrated, we use the estimation of the residuals from the above cointegrating equation: $\hat{e}_t = Y_t - \hat{b}_0 - \hat{b}_1 X_t$

Step 3

If the two variables are cointegrated, Engle and Granger suggested the augmented Dickey Fuller (ADF) test using the form:

$$\Delta \hat{e}_t = \alpha_1 \hat{e}_{t-1} + \sum_{i=1}^n \alpha_{i+1} \Delta \hat{e}_{t-i} + e_t$$

$$H_0: \alpha_1 = 0 \text{ vs } H_1: \alpha_1 \neq 0$$

If we reject the null hypothesis, we conclude that the residuals is stationary and the variables are cointegrated.

Step 4

In this step we estimate the error-correction model (ECM). If the variables are cointegrated, the error correction form is as follows:

$$\begin{aligned} \Delta y_t &= a_1 + a_y [y_{t-1} - b_1 x_{t-1}] + \sum_{i=1} a_{11}(i) \Delta y_{t-i} + \sum_{i=1} a_{12}(i) \Delta x_{t-i} + e_{yt} \\ \Delta x_t &= a_2 + a_x [y_{t-1} - b_1 x_{t-1}] + \sum_{i=1} a_{11}(i) \Delta y_{t-i} + \sum_{i=1} a_{12}(i) \Delta x_{t-i} + e_{xt} \end{aligned}$$

Engle and Granger proposed to use the estimation of the residuals in order to overcome the cross-equations restrictions, due the fact that the two equations have the same term (b_1). Considering this, the error correction equations are the following:

$$\begin{aligned} \Delta y_t &= a_1 + a_y \hat{e}_{t-1} + \sum_{i=1} a_{11}(i) \Delta y_{t-i} + \sum_{i=1} a_{12}(i) \Delta x_{t-i} + e_{yt} \\ \Delta x_t &= a_2 + a_x \hat{e}_{t-1} + \sum_{i=1} a_{11}(i) \Delta y_{t-i} + \sum_{i=1} a_{12}(i) \Delta x_{t-i} + e_{xt} \end{aligned}$$

Step 5

The final step of the method includes all the diagnostics tests in order to ensure that the VECM model is efficient. One prerequisite is that the values of α_y and/or α_x should be significantly different from zero in order for the variables to be cointegrated. If both α_y and α_x equal to zero then we have a VAR model in first differences. Finally, all the diagnostics tests that are analyzed in the ARIMA and VAR processes should also be tested for the VECM models.

➤ **Johansen methodology:**

Step 1

The general form of the VECM model is:

$$\Delta y_t = \sum_{i=1}^{p-1} \Gamma_i y_{t-i} + \Pi y_{t-1} + BX_t + u_t$$

In this step the removal of the short run dynamics is proposed, therefore the final form of the VECM model is:

$$\Delta y_t - \Pi y_{t-1} = \sum_{i=1}^{p-1} \Gamma_i y_{t-i} + u_t$$

We estimate the regression of Δy_t on the second part of the above equation $\sum_{i=1}^{p-1} \Gamma_i y_{t-i} + u_t$ and we estimate the values of the residuals with the LS method (denote as R_{0t}). Then we estimate the regression of Y_t on the equation $\sum_{i=1}^{p-1} \Gamma_i y_{t-i} + u_t$ and we estimate the values of the residuals with the LS method (denote as R_{1t}).

Step 2

In the second step Johansen suggest estimating the eigenvalues of the covariance matrix (R_{1t} , R_{2t}) according to the following matrix-form:

$$S_{ij} = \frac{1}{N} \sum_{i=1}^N R_{it} R'_{jt} = \begin{bmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{bmatrix}$$

and the eigenvalues are derived as follows:

$$|\lambda S_{11} - S'_{10} S_{00}^{-1} S_{01}| = 0$$

where λ is the vector of the eigenvalues.

Step 3

In the final step of the Johansen's methodology, we estimate the parameters of the VECM model according to the rank of matrix Π .

-If $\text{rank}(\Pi)=0$, then the VECM is a VAR model in first differences.

-If $\text{rank}(\Pi)=p$, then the Y_t is stationary and the VECM has no point. A level VAR model is suitable for the analysis.

-If $\text{rank}(\Pi)<p$, then the columns of the matrix Π are not linear independent, which shows the existence of cointegration relationships among the variables of the vector Y_t .

For the cointegration rank the Johansen's test comes in two main forms Trace tests and Maximum Eigenvalue test.

Trace tests

The hypothesis testing form is:

$$H_0: h \leq h_0 \text{ vs } H_1: h_0 < h \leq N$$

The hypothesis H_0 tests the existence of h_0 cointegrated vectors versus the alternative H_1 , which tests that there is more than h_0 cointegrated vectors. The number of cointegrated vectors is indicated from the null hypothesis.

The hypotheses are tested according the statistic:

$$\lambda_{trace} = -T \sum_{i=h+1}^p \log(1 - \hat{\lambda}_i) \text{ for } h=0,1,2,\dots,p-1$$

When using the trace test to test for cointegration in a sample, we set h_0 to zero to test whether the null hypothesis will be rejected. If it is rejected, we can deduce that a cointegration relationship exists in the sample. Therefore, the null hypothesis should be rejected to confirm the existence of a cointegration relationship in the sample.

Maximum Eigenvalue test

The Maximum Eigenvalue test is similar to the Johansen's trace test, and the key difference between the two is the null hypothesis:

$$H_0: h=h_0 \text{ vs } H_1: h=h_0+1$$

The statistic of this hypothesis test is as follows:

$$\lambda_{max} = -T \log(1 - \hat{\lambda}_{h+1}) \text{ for } h=0,1,2,\dots,p-1$$

In a scenario where $h=h_0$ and the null hypothesis is rejected, it means that there is only one possible outcome of the variable to produce a stationary process.

2.10 Granger-Causality analysis

High levels of correlation between two variables do not imply that there is a causal relationship, but only some seemingly correlations. Granger (1969) defined causality as when the past and present information of X_t is helpful for improving the forecast of the other variable Y_t . This means that the variable X_t causes another variable Y_t and this relationship is known as Granger causality. Specifically, the X_t does not cause Y_t , if the MSE of the Y_t for h periods ahead considering only the historical values of Y_t is the same with the MSE of the Y_t for h periods ahead considering the historical values of Y_t and X_t :

$$MSE[E(Y_{t+h} | Y_t, Y_{t-1}, \dots)] = MSE[E(Y_{t+h} | Y_t, Y_{t-1}, \dots, X_t, X_{t-1}, \dots)]$$

The Granger-Causality analysis is applicable to the VAR models. Considering that we have a VAR(k) model for two variables (Y_t, X_t) with the following form:

$$Y_t = a_{10} + \sum_{j=1}^k a_{1j} X_{t-j} + \sum_{j=1}^k b_{1j} Y_{t-j} + e_{1t}$$

$$X_t = a_{20} + \sum_{j=1}^k a_{2j} X_{t-j} + \sum_{j=1}^k b_{2j} Y_{t-j} + e_{2t}$$

The hypothesis testing for the Granger causality is:

$$H_0: X_t \text{ does not cause } Y_t \text{ or } \alpha_{1j}=0 \text{ for } j=1,2,..k$$

Vs

$$H_a: X_t \text{ causes } Y_t \text{ or at least one } \alpha_{1j} \neq 0 \text{ for } j=1,2,..k$$

Which means that if X_t not causes Y_t and the coefficients α_{1j} are not statistically significant.

and

$$H_0: Y_t \text{ does not cause } X_t \text{ or } \alpha_{2j}=0 \text{ for } j=1,2,..k$$

Vs

$$H_a: Y_t \text{ causes } X_t \text{ or at least one } \alpha_{2j} \neq 0 \text{ for } j=1,2,..k$$

Which means that if Y_t does not cause X_t , the coefficients α_{2j} are not statistically significant.

The statistic of the Granger causality hypothesis test is:

$$F = \frac{(SSE^R - SSE^U) / k}{SSE^U / f} \sim F_{k,f}$$

where:

- SSE^R : the sum of square estimate of errors with the restrictions of the null hypothesis

- SSE^U : the sum of square estimate of errors for the unrestricted model

- k : the number of restrictions

- f : the degree of freedom of the unrestricted case, which is equal to $N-k$ (N is the number of the observations and k is the number of the parameters of the unrestricted model)

If the value $F >$ critical value of $F_{k,f}$ for a significant level $\alpha\%$, then we reject the null hypothesis and X_t causes Y_t . If X_t causes Y_t or Y_t causes X_t we conclude that we have unidirectional causality, otherwise if X_t causes Y_t and Y_t causes X_t we conclude that we have bidirectional causality.

2.11 Structural analysis

In the VAR models, changes in the variables are introduced by shocks which have a structural basis. In order to study the relations between the variables and the effect of the shocks, the main issues for this is the impulse response analysis and the variance decomposition analysis.

The impulse response analysis provides more information for the effect of a system's variable in comparison with the Granger-causality analysis. The F-test is not able to explain the sign and the duration of an effect on a VAR model. This analysis is more detailed with the impulse response analysis. Considering the following VAR model:

$$Y_t = a_{10} + \sum_{j=1}^k a_{1j} X_{t-j} + \sum_{j=1}^k b_{1j} Y_{t-j} + e_{1t}$$

$$X_t = a_{20} + \sum_{j=1}^k a_{2j} X_{t-j} + \sum_{j=1}^k b_{2j} Y_{t-j} + e_{2t}$$

In case that the e_{1t} and e_{2t} are uncorrelated, on the impulse response analysis we investigate the effect of a shock on the value of e_{1t} or e_{2t} effect the present and the future values of the two variables (Y_t, X_t). In order to estimate the impulse response function, it is necessary to convert the VAR model into VMA (vector moving average) model. According to the analysis of the 2.2.2 unit, the above VAR system could be written as:

$$\Phi(L)Z_t = \alpha + e_t$$

We multiply the equation above in order to estimate the VMA model, therefore we have:

$$Z_t = \mu + \Phi(L)^{-1}e_t = \mu + e_t + \Psi_1 e_{t-1} + \Psi_2 e_{t-2} + \dots$$

Where Ψ_j is the inverted matrix $\Phi(L)$, with $\Phi(L)^{-1} = 1 + \Psi_1 L + \Psi_2 L^2 + \dots$

The impulse response of e_t on the future values of Z_t is defined from the elements of the following impulse matrix:

$$\Psi_s = \frac{\partial Y_{t+s}}{\partial e_t}, \quad s=1,2,3,\dots$$

Applying impulse responses on the v forecast errors, the general form is as follows:

$$\frac{\partial Y_{t+s}}{\partial e_{1t}} d_1 + \frac{\partial Y_{t+s}}{\partial e_{2t}} d_2 + \dots + \frac{\partial Y_{t+s}}{\partial e_{vt}} d_v = \Psi_s d \quad \text{where } d = [d_1 \ d_2 \dots d_v]$$

While impulse response functions trace the effects of a shock to one endogenous variable on other variables in the VAR models, the variance decomposition technique measures the proportion of forecast error variance in one variable on itself and on the other variables. Furthermore, the variance decomposition determines how much of the forecast error variance of each of the variables can be explained by exogenous shocks to the other variables.

The h -step forecast error of a VAR models is:

$$y_{t+h} - y_t(h) = \sum_{i=0}^{h-1} \Phi_i e_{t+h-i} = e_{t+h} + \Phi_1 e_{t+h-1} + \dots + \Phi_{h-1} e_{t+1}$$

If $\Psi_i = \Phi_i^{-1}$ then:

$$y_{t+h} - y_t(h) = \sum_{i=0}^{h-1} \Psi_i e_{t+h-i} = e_{t+h} + \Psi_1 e_{t+h-1} + \dots + \Psi_{h-1} e_{t+1}$$

And the forecast error variance for the variables (k_1, k_2, \dots, k_n) is:

$$\sigma_k^2(h) = \sum_{i=0}^{h-1} (\psi_{k1,n}^2 + \dots + \psi_{kv,n}^2) = \sum_{j=1}^v (\psi_{kj,0}^2 + \dots + \psi_{kj,h-1}^2)$$

The term $\psi_{kj,0}^2 + \dots + \psi_{kj,h-1}^2$ represents the j^{th} shock to the h -step forecast error variance of the variable k .

2.12 Empirical Analysis

2.12.1 Data

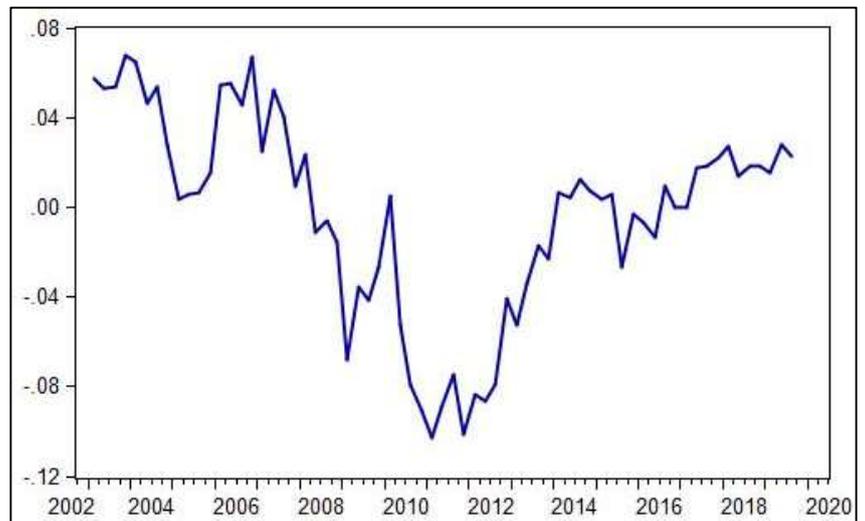
For the VAR analysis our datasets cover the period from the first quarter of 2003 until the third quarter of 2019 and they have been converted into quarterly frequency. The non-performing loans time series was provided from the Bank of Greece database³⁹ and the other time series, such as gross domestic product (GDP) growth rate and the long-term interest rate of loans rate were provided from the ECB⁴⁰. The final time series is the Harmonized unemployment rate provided from the OECD⁴¹.

The gross domestic product (GDP) measures the monetary value of final goods and services that are bought by the final user and produced in a country in a specific period (say a quarter or a year). Figure 2.7 presents the evolution of the growth rate of GDP and Table 2.10 the descriptive statistics of the GDP growth.

Table 2.10. Descriptive analysis of the GDP- growth rate in Greece

Mean	-0.002578
Median	0.005985
Maximum	0.067760
Minimum	-0.102513
Std. Dev.	0.045174
Skewness	-0.596350
Kurtosis	2.579578
Jarque-Bera Probability	4.464675 0.107277
Sum	-0.172756
Sum Sq. Dev.	0.134686
Observations	67

Figure 2.7. Time series plot of GDP growth



The next variable of the VAR model is the harmonized unemployment rate in Greece. This rate defines the unemployed as people of working age who are available to work and have taken specific steps to find a job. This indicator is measured in numbers of unemployed people as a

³⁹ See unit :2.1.4.1. Data

⁴⁰ European Central Bank, Statistical Data Warehouse: <https://sdw.ecb.europa.eu/home.do>

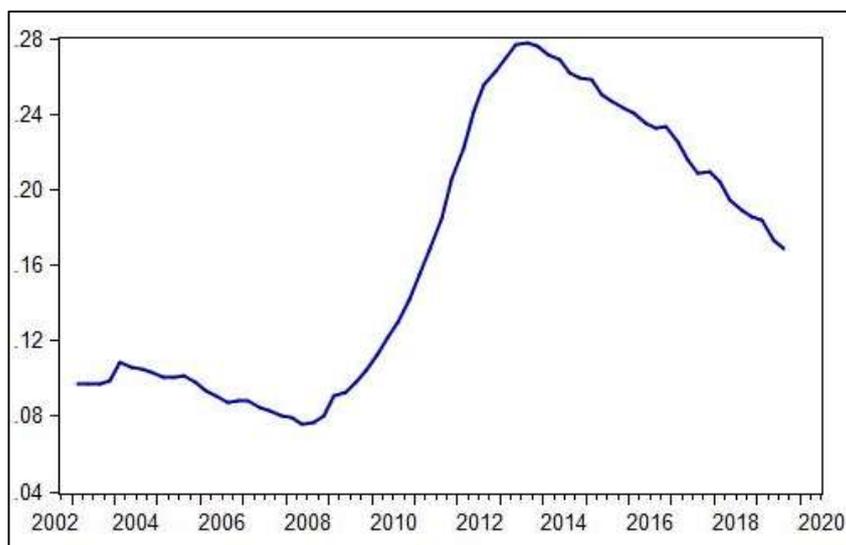
⁴¹ Organisation for Economic Co-operation and Development <https://data.oecd.org/unemp/harmonised-unemployment-rate-hur.htm>

percentage of the labour force and it is seasonally adjusted. The labour force is defined as the total number of unemployed people plus those in civilian employment and is displayed in figure 2.8.

Table 2.11. Descriptive analysis of the unemployment rate in Greece

Mean	0.165296
Median	0.168767
Maximum	0.277367
Minimum	0.075833
Std. Dev.	0.071111
Skewness	0.203217
Kurtosis	1.429984
Jarque-Bera Probability	7.342469 0.025445
Sum	11.07483
Sum Sq. Dev.	0.333750
Observations	67

Figure 2.8. Time series plot of the unemployment rate in Greece



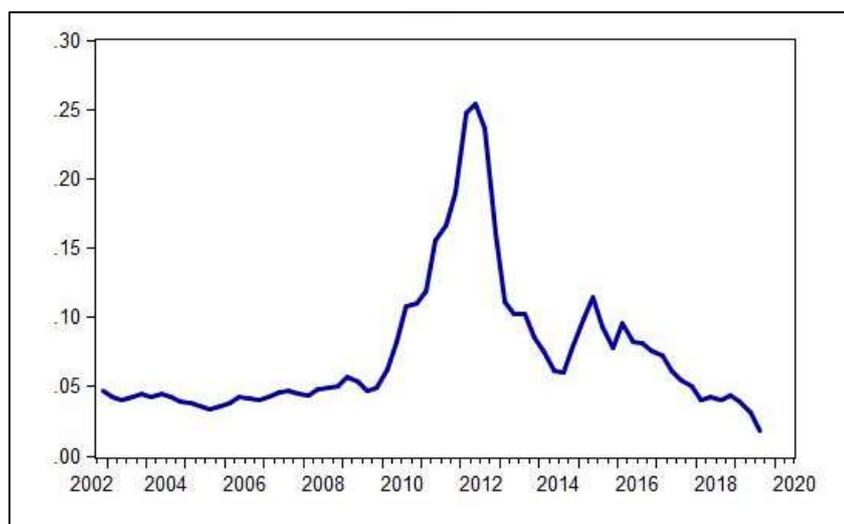
The third dataset of our analysis is the long-term interest rate of loans⁴², which was defined

The descriptive statistics and the graph of the long-term interest rate of loans are as follows:

Table 2.12. Descriptive analysis of the long-term interest rate of loans

Mean	0.074857
Median	0.050800
Maximum	0.254000
Minimum	0.018800
Std. Dev.	0.051383
Skewness	1.976882
Kurtosis	6.649951
Jarque-Bera Probability	80.83101 0.000000
Sum	5.015400
Sum Sq. Dev.	0.174257
Observations	67

Figure 2.9. Time series plot of the long-term interest rate of loans(%)



⁴² Credit and other institutions (MFI except MMFs and central banks) reporting sector - Loans including lending for house purchase. Over 1 year is calculated by weighting the volumes with a moving average (defined for cost of borrowing purposes).

2.12.2 Methodology and model testing

The initial purpose of the methodology is to test if the endogenous time series of our VAR model are stationary. The GDP growth rate (GDPR) and the interest rate (INR) are not stationary, especially they are I(1) and they have to be differenced once to become stationary. The harmonized unemployment rate and the non-performing loans time series are I(2). The aim is to estimate a VAR model in first differences, therefore we should transform the harmonized unemployment rate and the non-performing loans in order to be stationary in the first differences. First, we calculate the rate of change (ROC) for these variables, which is the speed at which a variable change over a specific period of time and it is defined as:

$$ROC = \frac{\text{current value} - \text{previous value}}{\text{previous value}}$$

and we conclude that the harmonized unemployment rate of change (HUROC) and the non-performing loans (NPLSROC) are I(1). Finally, all the tested variables are I (1) and we are able to proceed to the analysis of a VAR model in the first differences. For the previous analysis of the stationarity of the time series the Augment Dickey Fuller test (ADF) was used and the results for the first differences are briefly presented on the following table 2.20.

Table 2.13. ADF test results

Variable	ADF t-statistic	p-value
ΔINR	-4,450	0,000
ΔHUROC	-12,155	0,000
ΔGDPR	-3,438	0,000
ΔNPLSROC	-8,374	0,000

The table 2.14 presents briefly the descriptive analysis of each model's variables and the next tables (2.10 and 2.11) present the time series graphs for level and first difference presentation separately.

Table 2.14. Descriptive analysis for each variable

	INR	GDPR	NPLSROC	HUROC
Mean	0.074857	-0.002578	0.036593	0.008928
Median	0.050800	0.005985	0.026227	-0.007362
Maximum	0.254000	0.067760	0.204975	0.138075
Minimum	0.018800	-0.102513	-0.062381	-0.059783
Std. Dev.	0.051383	0.045174	0.060115	0.047361
Skewness	1.976882	-0.596350	0.746003	0.938115
Kurtosis	6.649951	2.579578	3.501038	2.849550
Jarque-Bera	80.83101	4.464675	6.915300	9.890522
Probability	0.000000	0.107277	0.031504	0.007117
Sum	5.015400	-0.172756	2.451746	0.598193
Sum Sq. Dev.	0.174257	0.134686	0.238514	0.148044
Observations	67	67	67	67

Figure 2.10. Level presentation of the variables

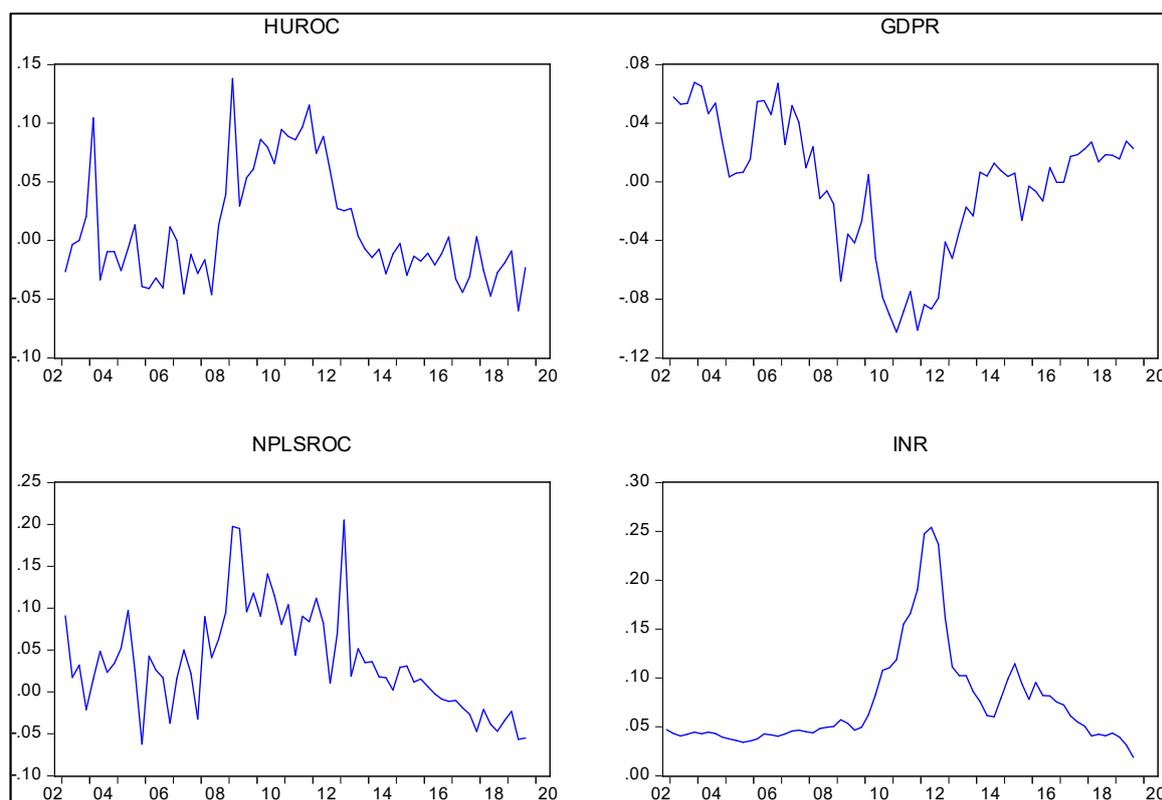
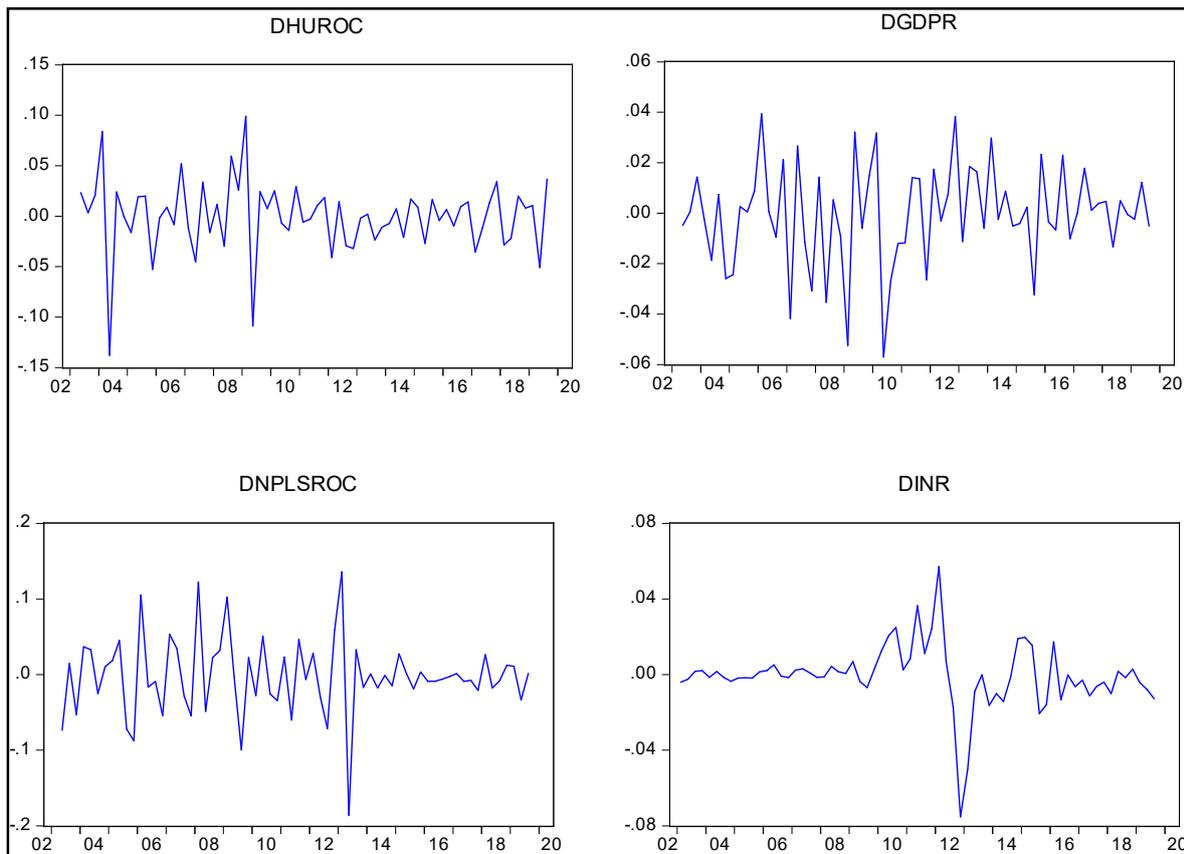


Figure 2.11. First difference presentation of the variables



Except from the endogenous variables (NPLSROC, HUROC, GDPR, INR), in our study we add an exogenous dummy variable in order to include the effect of the Greek debt crisis (as defined in the unit 2.1.4.2.) from 2009 to 2018 and denoted as $DUMMY_t$. Further, the second exogenous variable is the intercept denoted as the vector C .

The next step of the model's analysis is to determine the order of VAR lags. For this purpose, we estimate the most efficient number of lags according to Lag Length Criteria. The null hypothesis of this test is that the coefficients on lag L are jointly zero, thus we decrease the lag one at a time until we get a rejection of null hypothesis starting from the maximum lag L . The test's statistic is defined as follows:

$$LR = (T - M) [\log |\sum_{e,L-1}| - \log |\sum_{e,L}|] \sim \chi^2(k^2)$$

, where T is the number of sample's observations and M is the number of the parameters per equation. The result of the lag length criteria is presented on the next table and we conclude that the most appropriate order is the VAR(4) model. The selection based on the minimization of the FPE and AIC, both occurred on lag 4.

Table 2.15. VAR Lag Length Criteria

Endogenous variables: DGDPR DHUROC DINR DNPLSROC Exogenous variables: C DUMMY Sample: 12/01/2002 6/01/2020 Included observations: 61						
Lag	LogL	LR	FPE	AIC	SC	HQ
0	536.7991	NA	3.47e-13	-17.33768	-17.06084*	-17.22918
1	566.2566	53.11998	2.24e-13	-17.77890	-16.94840	-17.45342*
2	579.4341	22.03464	2.48e-13	-17.68637	-16.30219	-17.14389
3	595.7565	25.15248	2.51e-13	-17.69693	-15.75908	-16.93747
4	616.6921	29.51574*	2.22e-13*	-17.85876*	-15.36723	-16.88231
5	630.6407	17.83588	2.53e-13	-17.79150	-14.74630	-16.59806

* indicates lag order selected by the criterion
 LR: sequential modified LR test statistic (each test at 5% level)
 FPE: Final prediction error
 AIC: Akaike information criterion
 SC: Schwarz information criterion
 HQ: Hannan-Quinn information criterion

As we estimate the exogenous, endogenous variables and the appropriate number of lags, the next step is to estimate the stability of our VAR model. According to the unrestricted VAR estimation in E-views, the final form of the equations is provided below with the estimation outputs.

Table 2.16. Var estimated model

$$\begin{aligned} \Delta NPLSROC &= C(1,1)*\Delta NPLSROC(-1) + C(1,2)*\Delta NPLSROC(-2) + C(1,3)*\Delta NPLSROC(-3) + C(1,4)*\Delta NPLSROC(-4) + \\ &C(1,5)*\Delta GDP(-1) + C(1,6)*\Delta GDP(-2) + C(1,7)*\Delta GDP(-3) + C(1,8)*\Delta GDP(-4) + C(1,9)*\Delta INR(-1) + C(1,10)*\Delta INR(-2) + \\ &C(1,11)*\Delta INR(-3) + C(1,12)*\Delta INR(-4) + C(1,13)*\Delta HUROC(-1) + C(1,14)*\Delta HUROC(-2) + C(1,15)*\Delta HUROC(-3) + \\ &C(1,16)*\Delta HUROC(-4) + C(1,17) \\ \Delta GDP &= C(2,1)*\Delta NPLSROC(-1) + C(2,2)*\Delta NPLSROC(-2) + C(2,3)*\Delta NPLSROC(-3) + C(2,4)*\Delta NPLSROC(-4) + \\ &C(2,5)*\Delta GDP(-1) + C(2,6)*\Delta GDP(-2) + C(2,7)*\Delta GDP(-3) + C(2,8)*\Delta GDP(-4) + C(2,9)*\Delta INR(-1) + C(2,10)*\Delta INR(-2) + \\ &C(2,11)*\Delta INR(-3) + C(2,12)*\Delta INR(-4) + C(2,13)*\Delta HUROC(-1) + C(2,14)*\Delta HUROC(-2) + C(2,15)*\Delta HUROC(-3) + \\ &C(2,16)*\Delta HUROC(-4) + C(2,17) \\ \Delta INR &= C(3,1)*\Delta NPLSROC(-1) + C(3,2)*\Delta NPLSROC(-2) + C(3,3)*\Delta NPLSROC(-3) + C(3,4)*\Delta NPLSROC(-4) + \\ &C(3,5)*\Delta GDP(-1) + C(3,6)*\Delta GDP(-2) + C(3,7)*\Delta GDP(-3) + C(3,8)*\Delta GDP(-4) + C(3,9)*\Delta INR(-1) + C(3,10)*\Delta INR(-2) + \\ &C(3,11)*\Delta INR(-3) + C(3,12)*\Delta INR(-4) + C(3,13)*\Delta HUROC(-1) + C(3,14)*\Delta HUROC(-2) + C(3,15)*\Delta HUROC(-3) + \\ &C(3,16)*\Delta HUROC(-4) + C(3,17) \\ \Delta HUROC &= C(4,1)*\Delta NPLSROC(-1) + C(4,2)*\Delta NPLSROC(-2) + C(4,3)*\Delta NPLSROC(-3) + C(4,4)*\Delta NPLSROC(-4) + \\ &C(4,5)*\Delta GDP(-1) + C(4,6)*\Delta GDP(-2) + C(4,7)*\Delta GDP(-3) + C(4,8)*\Delta GDP(-4) + C(4,9)*\Delta INR(-1) + C(4,10)*\Delta INR(-2) + \\ &C(4,11)*\Delta INR(-3) + C(4,12)*\Delta INR(-4) + C(4,13)*\Delta HUROC(-1) + C(4,14)*\Delta HUROC(-2) + C(4,15)*\Delta HUROC(-3) + \\ &C(4,16)*\Delta HUROC(-4) + C(4,17) \end{aligned}$$

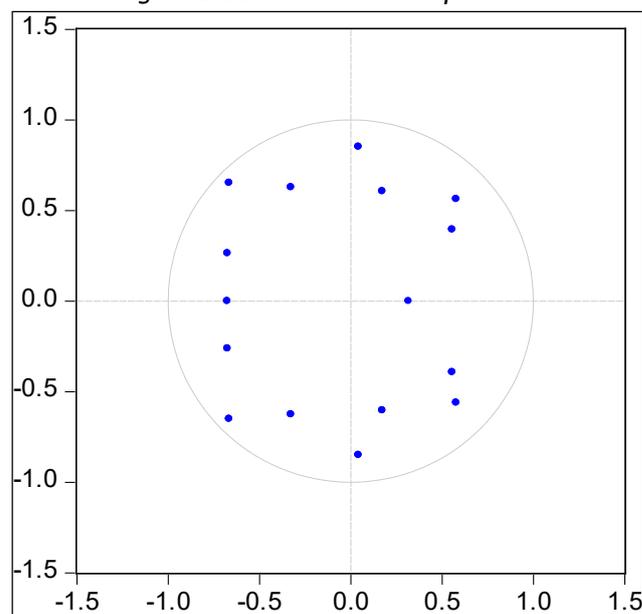
Table 2.17. Vector Autoregressive Estimates

Sample (adjusted): 6/01/2004 3/01/2020 Standard errors in () & t-statistics in []				
	$\Delta(\text{NPLSROC})$	$\Delta(\text{GAP})$	(ΔINR)	$\Delta(\text{HUROC})$
$\Delta\text{NPLSROC}(-1)$	-0.759894 (0.14574) [-5.21416]	-0.095013 (0.05744) [-1.65406]	0.026252 (0.05431) [0.48337]	0.128567 (0.10853) [1.18460]
$\Delta\text{NPLSROC}(-2)$	-0.701155 (0.17371) [-4.03636]	-0.061733 (0.06847) [-0.90164]	-0.021114 (0.06473) [-0.32617]	0.200531 (0.12936) [1.55014]
$\Delta\text{NPLSROC}(-3)$	-0.408165 (0.17356) [-2.35171]	0.004422 (0.06841) [0.06463]	-0.066438 (0.06468) [-1.02719]	0.142641 (0.12925) [1.10358]
$\Delta\text{NPLSROC}(-4)$	-0.026576 (0.14052) [-0.18912]	-0.005463 (0.05539) [-0.09863]	0.009280 (0.05237) [0.17720]	0.169956 (0.10465) [1.62406]
$\Delta\text{GDP}(-1)$	-0.571709 (0.33374) [-1.71304]	-0.204011 (0.13154) [-1.55089]	-0.065583 (0.12437) [-0.52732]	0.125825 (0.24854) [0.50626]
$\Delta\text{GDP}(-2)$	-0.310596 (0.34854) [-0.89114]	0.088879 (0.13738) [0.64697]	-0.046649 (0.12989) [-0.35915]	-0.241922 (0.25956) [-0.93205]
$\Delta\text{GDP}(-3)$	0.143910 (0.33196) [0.43352]	0.207791 (0.13084) [1.58810]	-0.058811 (0.12371) [-0.47541]	-0.165133 (0.24721) [-0.66798]
$\Delta\text{GDP}(-4)$	0.309629 (0.30722) [1.00785]	-0.510602 (0.12109) [-4.21669]	-0.086048 (0.11449) [-0.75159]	0.018227 (0.22879) [0.07967]
$\Delta\text{INR}(-1)$	-0.506646 (0.39341) [-1.28783]	-0.160030 (0.15506) [-1.03203]	0.612770 (0.14661) [4.17967]	0.396661 (0.29298) [1.35390]
$\Delta\text{INR}(-2)$	0.220569 (0.45604) [0.48366]	-0.139687 (0.17975) [-0.77713]	-0.175066 (0.16995) [-1.03013]	-0.045052 (0.33962) [-0.13266]
$\Delta\text{INR}(-3)$	-0.336337 (0.45672) [-0.73642]	0.015276 (0.18002) [0.08486]	-0.147113 (0.17020) [-0.86436]	0.095992 (0.34012) [0.28223]
$\Delta\text{INR}(-4)$	0.875677 (0.39038) [2.24315]	0.001733 (0.15387) [0.01126]	-0.013321 (0.14548) [-0.09157]	-0.091364 (0.29072) [-0.31427]
$\Delta\text{HUROC}(-1)$	0.293264 (0.17989) [1.63026]	-0.107775 (0.07090) [-1.52005]	0.088712 (0.06704) [1.32335]	-0.593632 (0.13396) [-4.43129]
$\Delta\text{HUROC}(-2)$	0.310591 (0.20935) [1.48363]	-0.034219 (0.08251) [-0.41471]	0.024647 (0.07801) [0.31593]	-0.468385 (0.15590) [-3.00437]
$\Delta\text{HUROC}(-3)$	0.173182 (0.21205) [0.81672]	0.081207 (0.08358) [0.97163]	0.022780 (0.07902) [0.28828]	-0.398650 (0.15791) [-2.52450]
$\Delta\text{HUROC}(-4)$	-0.159262 (0.18902) [-0.84257]	0.038450 (0.07450) [0.51609]	0.052571 (0.07044) [0.74633]	-0.144643 (0.14076) [-1.02755]
C	-0.002459 (0.00523) [-0.47042]	-0.001251 (0.00206) [-0.60726]	-0.000336 (0.00195) [-0.17243]	-0.002325 (0.00389) [-0.59721]

Table 2.17. Vector Autoregressive estimates(continue)

	$\Delta(\text{NPLSROC})$	$\Delta(\text{GDP})$	$\Delta(\text{INR})$	$\Delta(\text{HUROC})$
R-squared	0.506023	0.526119	0.412571	0.414028
Adj. R-squared	0.337860	0.364798	0.212596	0.214548
Sum sq. resids	0.079183	0.012301	0.010996	0.043914
S.E. equation	0.041046	0.016178	0.015296	0.030567
F-statistic	3.009127	3.261320	2.063107	2.075537
Log likelihood	123.4240	183.0093	186.5980	142.2887
Akaike AIC	-3.325751	-5.187791	-5.299938	-3.915273
Schwarz SC	-2.752298	-4.614337	-4.726485	-3.341820
Mean dependent	-0.000981	-0.000621	-0.000401	-0.002137
S.D. dependent	0.050442	0.020299	0.017238	0.034490
Determinant resid covariance (dof adj.)		6.84E-14		
Determinant resid covariance		1.99E-14		
Log likelihood		646.3112		
Akaike information criterion		-18.07222		
Schwarz criterion		-15.77841		

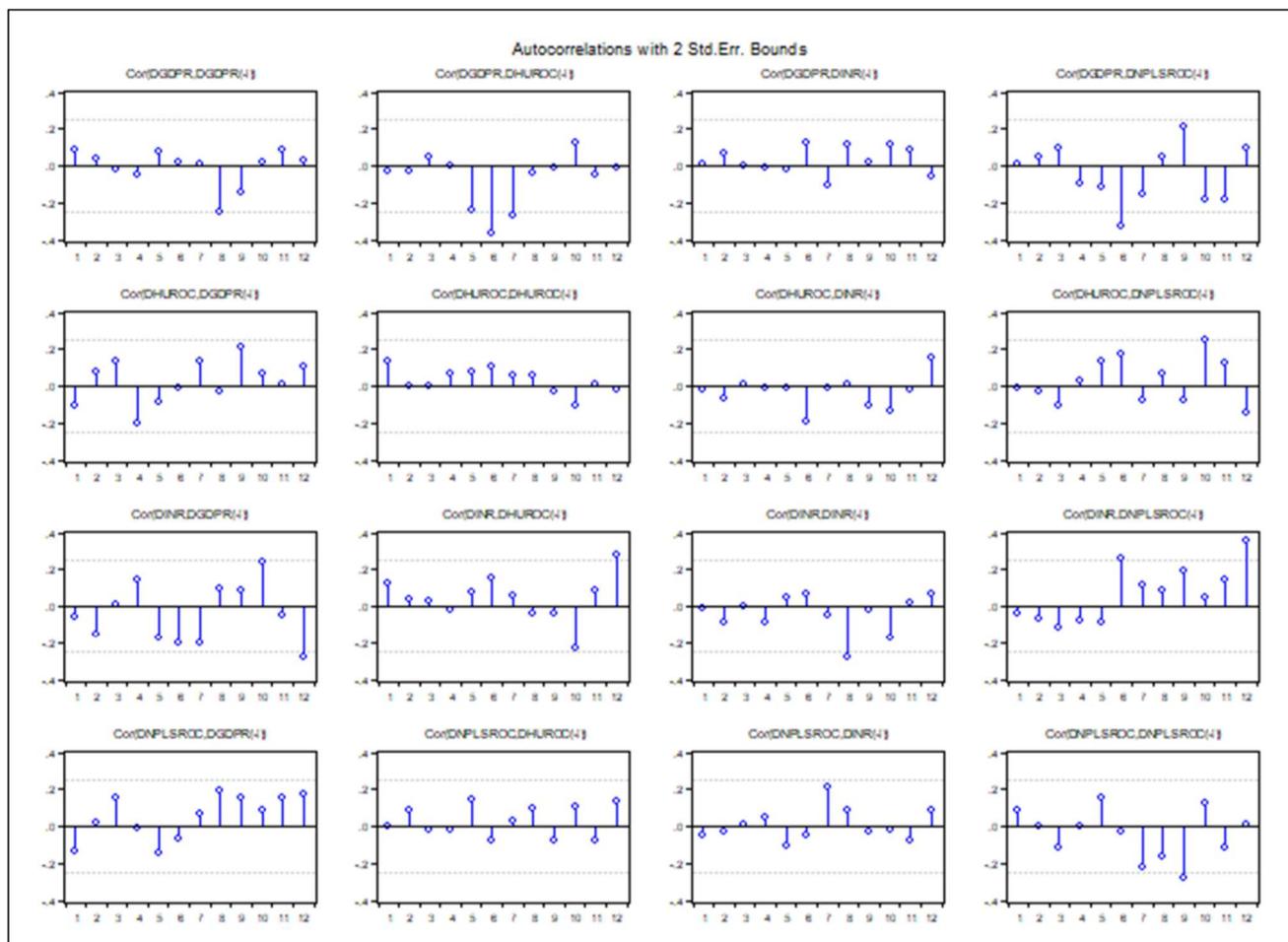
Figure 2.12. AR Roots Graph



The estimated VAR is stable (stationary), as all roots have modulus less than one and lie inside the unit circle.

The next figure is the correlogram graph for lags from 1 to 12. The dotted lines in the graphs represent plus or minus two times the approximate asymptotic standard errors of the lagged correlations. Residuals should not be autocorrelated, therefore the autocorrelations should be inside the dotted lines.

Figure 2.13. Autocorrelations graph



2.12.3 Forecasting and results

After the estimation of the most appropriate VAR for our analysis, the final step is to estimate the future values of non-performing loans and study the impulse responses.

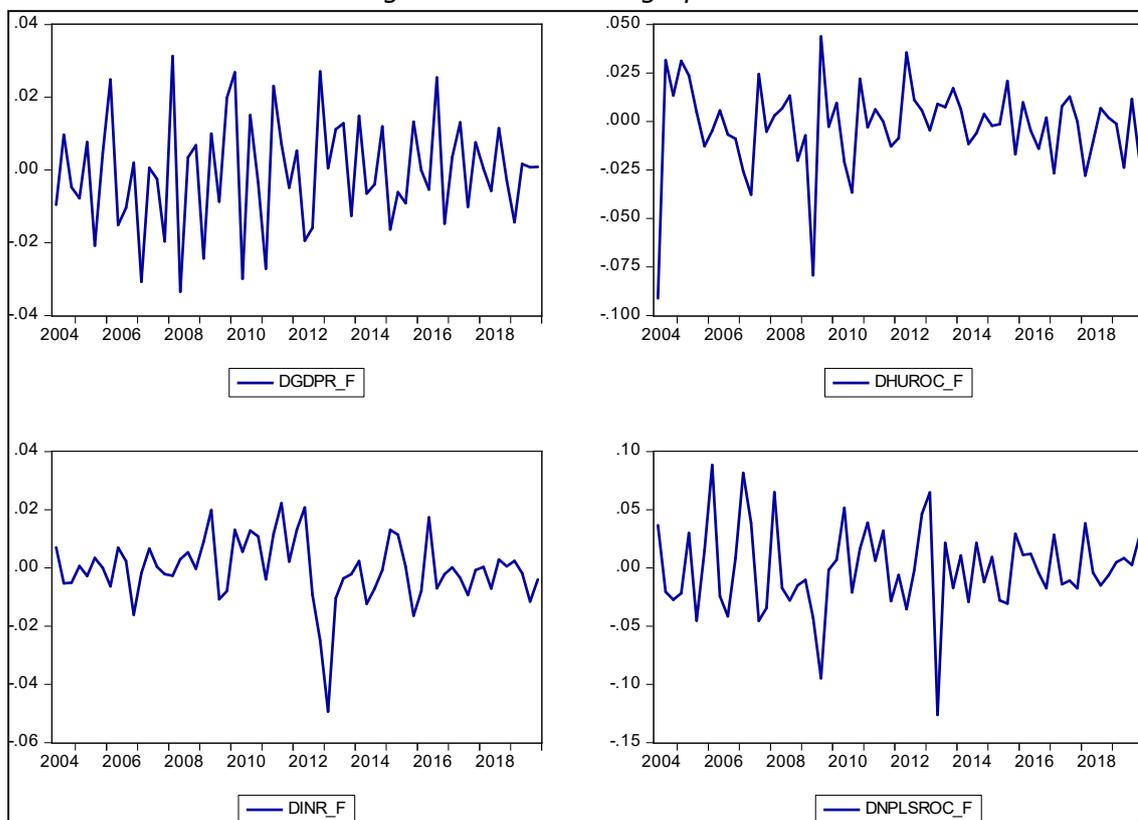
The results of forecast errors are represented in the following graph for each estimated variable. The DINR has the minimum forecast error, whereas the DNPLSROC appears the biggest forecast error.

Table 2.18. Forecast errors results

Forecast Evaluation					
Date: 05/05/20 Time: 16:32					
Sample: 6/01/2004 9/01/2019					
Included observations: 62					
Variable	Inc. obs.	RMSE	MAE	MAPE	Theil
DNPLSROC	62	0.051142	0.035569	781.4143	0.779180
DGDP	62	0.030736	0.023171	322.0006	0.717878
DINR	62	0.013318	0.008766	4161.614	0.466785
DHUROC	62	0.054590	0.043500	1435.658	0.770503

RMSE: Root Mean Square Error
 MAE: Mean Absolute Error
 MAPE: Mean Absolute Percentage Error
 Theil: Theil inequality coefficient

Figure 2.14. Forecast graph



The forecast values for the next three quarters are represented in the following table. The E-views provides the future values on the first-difference series (DNPLSROC). The forecast of the values of the NPLS (level series) is based on calculations according to the following equation:

$$\Delta(NPLSROC)_t = NPLSROC_t - NPLSROC_{t-1} \Rightarrow NPLSROC_t = \Delta(NPLSROC)_t + NPLSROC_{t-1}$$

$$NPLSROC_t = \frac{NPLS_t - NPLS_{t-1}}{NPLS_{t-1}} \Rightarrow NPLS_t = NPLSROC_t * NPLS_{t-1} + NPLS_{t-1}$$

Following the above equations, the future values of the NPLS time series for the next three quarters are as in the Table 2.19.

Table 2.19. Forecast results of NPLS for the next three quarters

Quarters	$\Delta(NPLSROC)$ ↓	NPLROC ↓	Gross NPLS (in thousands of €) ↓
Dec-2019	1,74%	-3,75%	68566581,34
May-2020	-0,97%	-4,72%	65332208,78
June-2020	0,25%	-4,46%	62416409,49

Regarding the future values of the stock of NPLS, it is obvious that a constant reduction of the stock follow the previous reduction of within-sample values.

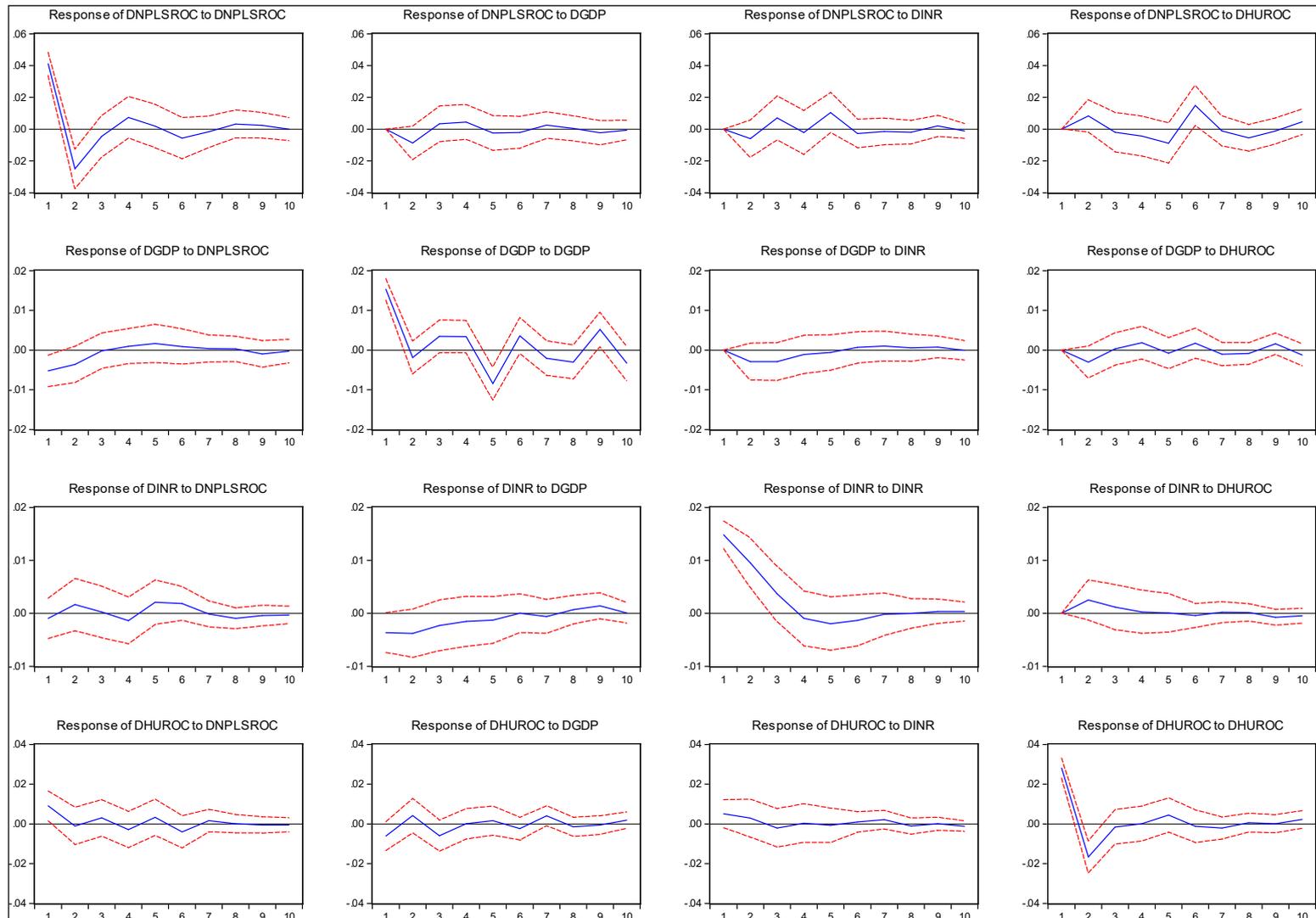
According to the findings of the estimation(Table 2.16), the estimated coefficients for the first differenced NPLSROC are significant at 5% for almost all lags and support the view that macro fundamentals affect the rate of change of non-performing loans. Further, the following table describes the signs of the DNPLSROC equation separately for each lag and variable. An increase on 3rd and 4th lags of DGDP, 2nd and 4th lags of DINR, 1st, 2nd and 3rd of DHUROC affect positively the DNPLSROC.

Table 2.20. Signs of variables

$\Delta NPLSROC(-1)$	$\Delta NPLSROC(-2)$	$\Delta NPLSROC(-3)$	$\Delta NPLSROC(-4)$
-	-	-	-
$\Delta GDP(-1)$	$\Delta GDP(-2)$	$\Delta GDP(-3)$	$\Delta GDP(-4)$
-	-	+	+
$\Delta INR(-1)$	$\Delta INR(-2)$	$\Delta INR(-3)$	$\Delta INR(-4)$
-	+	-	+
$\Delta HUROC(-1)$	$\Delta HUROC(-2)$	$\Delta HUROC(-3)$	$\Delta HUROC(-4)$
+	+	+	-

The next step of the analysis is to investigate the impulse Responses. The impulse responses provide information to analyze the dynamic behavior of a variable due to a random shock or innovation in other variables. Specifically, the Impulse Response Functions are testing the effects on current and future values of the endogenous variables of one standard deviation shock to a variable.

Figure 2.15. Impulse response graph (± 2 s.e)



The response of DNPLSROC to a shock in DGDP creates a small fluctuation and finally dies off, whereas a shock in DHUROC creates a higher fluctuation of DNPLSROC. A unit shock of DHUROC results in a significant response of DNPLSROC, this implies how the DHUROC affects the values of DNPLSROC. To conclude, DNPLSROC is affected more by DHUROC and DINDR than by DGDP.

2.13 Comparison with the ARIMA forecasting model

Our empirical analysis for non-performance loans in Greece includes two approaches. The first two approaches analyze the short-run effects of the NPLs time series based on the Bank of Greece dataset and the second analyze the determinants of NPLs from a multivariate aspect.

Considering the short-run evolution of the gross of NPLs in Greece, we develop two models: ARIMA and VAR. The ARIMA is based on the lags of NPLS time series and in the VAR model we include the macroeconomic effects (GDP, Unemployment rate and interest rate). Below we represent the evidence of the analysis both for the ARIMA and the VAR model. The gross of non-performing loans is expressed in thousands of €.

Table 2.21: Forecast of the ARIMA and VAR models of the gross of NPLs in Greece (€)

Quarters	ARIMA	VAR
Dec-2019	68.690.409,53	68.566.581,34
Mar-2020	66.541.129,33	65.332.208,78
June-2020	63.855.994,96	62.416.409,49

Table 2.22: Forecast errors of the ARIMA and VAR models

	ARIMA	VAR	Differences
RMSE	3,74%	5,11%	1,37%
MAE	2,61%	3,56%	0,95%

According to the Bank of Greece quarterly publishing of the gross of non-performing loans, the actual gross for December of 2019 was 68.510.004,13€. Considering this, the VAR forecast values was more accurate as can be seen in table 2.23.

Table 2.23: Comparison between forecasting value and actual data

Quarter	ARIMA	VAR	Actual Gross of NPLS	Deviation from the ARIMA model (%)	Deviation from the VAR model (%)
Dec-2019	68.690.409,53	68.566.581,34	68.510.004,13 €	0,2626%	0,0825%

Hence, we conclude that the multivariate analysis of the VAR model which accounts for macroeconomic determinants improves the model's forecasting ability.

CHAPTER 3: Credit risk analysis

3.1 Introduction to the main banking risks

Generally, risk implies future uncertainty about deflection from expected earnings/outcome. Further, it measures the uncertainty that an investor is willing to take a gain from an investment. The banking industry is exposed to many different types of risk. The risk may cause bank failure and the right management may affect the investor's decision and the profit of the bank. The main banking risks are analyzed in the following sections below.

Market risk

Market risk is associated with adverse movement in the market prices and defined as the risk of losses of the trading portfolio. It is also related to the commodity prices, bank's trading book, interest rate, equity prices and the foreign exchange risk position.

Liquidity risk

Liquidity risk is defined as the ability of the bank to finance its operations. These operations include funding obligations such as the inability to provide cash to customers in a time manner.

One of the issues is that the bank deposits are short term and the lending is long term, this gap between maturities results in liquidity risk. The bad management of the bank's liquidity may affect the reputation of the bank and consequently the bond pricing and ratings in the money market.

Operational risk

According to the Basel II committee, the operational risk is defined as "The risk of loss resulting from inadequate or failed internal processes, people and systems or from external events". The main causes of this risk are the human mistakes, failure of the systems (IT, internal software), failure of internal processes to transmit data & information accurately.

Credit risk

According to the Basel Committee on Banking Supervision, the credit risk defined as the possibility that a bank borrower, or counterparty, will fail to meet its payment obligations regarding the terms

agreed with the bank. It includes both uncertainties involved in repayment of the bank's dues and repayment of dues on time. It is one of the most important risks of the banking sector and is related to the possibility of the loans to not be performing in the future. Further details of the credit risk will be provided on the following units.

3.2 Types of credit risk

The quantification of credit risk is linked to many factors which concern debtors' specific criteria, macroeconomics (GDP, interest rate) and market considerations. The main factors that concerns credit risk measurements are briefly defined below.

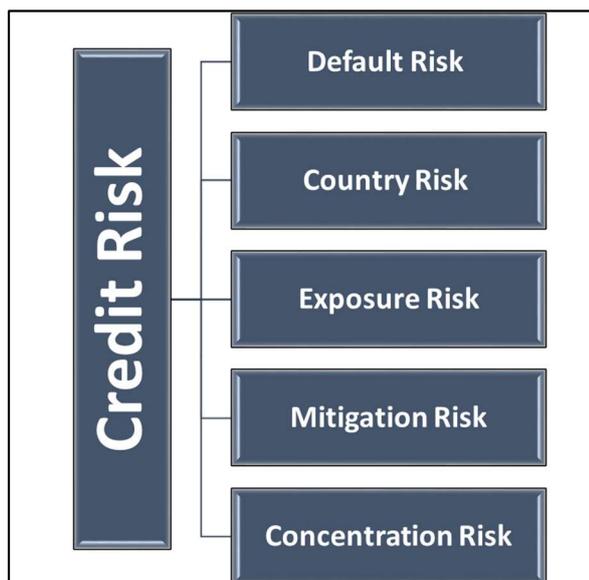
- **Probability of Default (PD)** is the likelihood of a debtor to default over some specified time horizon (usually a year). If a borrower is considered to have a high probability of default, then lenders will probably charge a higher interest rate. The banks need to estimate the PD and rate their clients. Estimating is based on the analysis of the historical data and the estimation could be carried out with the internal bank's methods or by external rating agencies (Standard & Poor's (S&P), Moody's, Fitch Group etc.).
- **Credit exposure** is the maximum potential loss (sum of the outstanding principal amount) to a bank if the counterparty defaults on payment.
- **Recovery rate** is the percentage of the recovered amount through bankruptcy proceedings when a loan defaults and the borrower is unable to fulfill the outstanding amount. A recovery rate can help banks set rates and terms for future credit transactions.
- **Exposure at Default (EAD)**: It can be defined as the gross exposure under a facility upon default of an obligor.
- **Loss Given Default (LGD)**: The amount of money that a bank losses in case of borrower's default. The LGD is expressed as a percentage loss per unit of exposure at the time of default (EAD).
- **Expected loss (EL)**: The sum of the values of all possible losses and described as:

$$EL = EAD * PD * LGD$$

Credit risk consists of five categories⁴³ as displayed in Figure 3.1:

- **Default Risk** is the risk that a counterparty defaults on its obligations or experiences material credit quality deterioration increasing the likelihood of a default.
- **Country Risk** is the risk that otherwise solvent and willing counterparties are unable to meet their obligations due to direct sovereign intervention or policies.
- **Exposure Risk** is the risk that arises from any future losses due to the potential counterparty future default. The exposure at default (EAD) is the relevant measurement of the exposure risk.
- **Mitigation Risk** is the risk of higher losses due to the unanticipated failures of risk mitigation measures.
- **Concentration Risk** is the risk of a default in a specific single counterparty, country, industry or product and is a consequence of the lack of portfolio's diversification. There are two categories of concentration risk: name concentration risk and sectoral concentration risk. The first is related to the uneven distribution of loans to its borrowers (for example a high level of credit on specific counterparties). The sectoral concentration risk is related to the uneven distribution of exposures to specific sectors such as countries, regions, industries etc.

Figure 3.1. Types of credit risk



⁴³ Deutsche Bank Annual Report 2017

3.3 Credit measurement techniques

The most known credit measurements techniques are classified in three categories:

- A. Credit Scoring Models, which concerns consumers and SME's
- B. Credit Rating Models, which concerns companies, countries, etc
- C. Newer credit measurements

An analysis of each category follows below.

A. The Credit Scoring Models are based on historic data (financial status, measures of risk) and estimate a default risk score for each debtor. These models perform a statistical analysis between lender and debtor in order to estimate the borrower's creditworthiness. There is a significant amount of studies and techniques for credit scoring. The most widely spread techniques for scoring models are briefly represented below.

- **Linear Discriminant Analysis (LDA)**

LDA is a linear function and the chosen variables are based on the dependency with the probability of default. In 1968 Altman⁴⁴ proposed the Linear Discriminant Analysis (LDA) to develop a five-factor model to predict bankruptcy in manufacturing companies. In particular, Altman used five accounting variables in order to test the contribution to credit score. The proposed bankruptcy model form is :

$$Z=0.012+0.014 X_2+0.0333 X_3+0.006 X_4+0.999 X_5$$

, where X_1 = Working Capital/Total Assets , X_2 = Retained Earnings/Total Assets , X_3 = Earnings before Interest and Taxes/ Total Assets, X_4 = Market value of Equity/Book Value of Total Liabilities and X_5 = Sales/Total Assets.

The scoring level models are classified as follows:

$Z > 2.99$ Safe Zone

$1.81 < Z < 2.99$ Grey Zone

$Z < 1.81$ Distress Zone

⁴⁴ E.Altman, "Financial ratios, discriminant analysis and the prediction of corporate bankruptcy", J.Finance 23 (1968)

- **Logistic Regression or Logit model**

Logit model has a binary dependent variable ($Y= 0$ or 1) and independent variables(X_i) as inputs. In 1980 Wiginton⁴⁵ first used this model to estimate consumer's creditworthiness. The logit regression model was defined as:

$$\log\left(\frac{p_i}{1-p_i}\right)=b_0 + b_1 X_1+ b_2 X_2+\dots+ b_n X_n$$

, where p is the probability of the event Y .

For the estimation the maximum likelihood and the least squared method are mainly used.

- **Probit Regression**

The Probit model has the same form with the Logit regression, however the main difference is that in the Probit model the function Y is assumed to be normally distributed. Roszbach⁴⁶ in 1998 used the Probit model to estimate the survival of granted loans. The form of the equation is:

$$Prob(Y = 1) = b_0 + b_1 X_1 + b_2 X_2 + \dots + b_n X_n$$

- **Classification Trees**

The scope of this method is to classify the debtors as cooperative or not, considering their specific characteristics. Makowski⁴⁷ in 1985 first used this model for credit scoring. The goal is to create a model that predicts the value of a target variable based on several input variables.

- **K-Nearest Neighbor approach**

This is a non-parametric method for classification problems and was first suggested by Fix and Hodges(1952) and applied for credit scoring in 1970 by Chatterjee and Barcum. The first step of the method is to determine the number of k , which constitutes the set of nearest neighbors. The next two parameters are the metric in order to define the distance with the neighbors and the proportion of goods/bads in order for an applicant to be classified as good/bad.

- **Artificial Intelligence-based/Machine Learning Methods**

These models are commonly used when the dependent and independent variables appear non-linear relationship and are difficult to be modeled. A self-learning AI analyzes data, learns from it,

⁴⁵ J. Wiginton "A Note on the Comparison of Logit and Discriminant Models of Consumer Credit Behavior", The Journal of Financial and quantitative analysis (1980)

⁴⁶ K Roszbach "Bank lending policy, credit scoring and the survival of loans", (1998)

⁴⁷ Credit scoring branches out, P Makowski - Credit World, 198

improves itself and provides forecast at a scale and depth of detail , a fact that is impossible for a traditional credit scoring model.

B. The Credit Rating models concern mainly companies, and countries. These models were developed from rating agencies, providing indicators for the creditworthiness as well as the investment quality. The most widely spread rating agencies are the Standard & Poor's (S&P), Moody's, and Fitch as shown in table 3.1. They use letter indicators such as A, B, C with higher grades that are intended to represent a lower probability of default.

Table 3.1. Credit ratings⁴⁸

MOODY'S	STANDARD & POOR'S	FitchRatings	Description
Aaa	AAA	AAA	Prime
Aa1	AA+	AA+	High grade
Aa2	AA	AA	
Aa3	AA-	AA-	
A1	A+	A+	Upper medium grade
A2	A	A	
A3	A-	A-	
Baa1	BBB+	BBB+	Lower medium grade
Baa2	BBB	BBB	
Baa3	BBB-	BBB-	
Ba1	BB+	BB+	Non-investment grade
Ba2	BB	BB	
Ba3	BB-	BB-	
B1	B+	B+	Highly speculative
B2	B	B	
B3	B-	B-	
Caa1	CCC+	CCC+	Substantial risks
Caa2	CCC	CCC	
Caa3	CCC-	CCC-	
Ca	CC	CC	Extremely speculative Default imminent
	C	C	
C	RD	DDD	In default
	SD	DD	
	D	D	

⁴⁸ Source: https://en.wikipedia.org/wiki/Credit_rating

C. During the recent years financial institutions have developed several models in order to provide efficient indicators for credit risk. Some of these remained proprietary and other are publicly available. Some of these models are JP Morgan's CreditMetrics, CreditRisk+, Moody's KMV, MCKinsey & Company's Credit Portfolio View etc⁴⁹.

- **Moody's KMV**

In 2002 Moody's corporation acquired KMV from Kealhofer, McQuown, and Vasicek. This model is based on Metron's model(1974)⁵⁰. It is based on the estimation of default probabilities.

- **CreditRisk +**

This model was developed by Credit Suisse in 1997 and it is based on default risk and the possible states are default or non-default.

- **Credit Metrics**

It was introduced by J.P Morgan in 1997 and adopts a similar approach to KMV . It is applicable to the valuation and risk of non-stable assets such as loans by using historic rating's data.

- **MCKinsey & Company's Credit Portfolio View**

This model was developed by Wilson in 1997 and it is an econometric model with multiple factors. One of the discriminations among other models is that it take into account the macroeconomic environment.

⁴⁹ A comparative Anatomy of Credit Risk Models Michael B. Gordy (1998).

⁵⁰ In 1974, economist Robert C. Merton proposed this model for assessing the structural credit risk of a company by modeling the company's equity as a call option on its assets. This model was later extended by Fischer Black and Myron Scholes to develop the Nobel-prize winning Black-Scholes pricing model for options.

3.4 Regulatory framework

Banks in order to address their risk should maintain enough capital to counterbalance the potential losses without militate systematic problems. For this purpose, the Basel Committee on Banking Supervision (BCBS)⁵¹, which is an international committee have developed standards for banking regulation and minimum capital requirements. . The BCBS consists of 45 members including central banks and bank supervisors from 28 jurisdictions. The Basel Accords consist of Basel I (1988), Basel II(2004) and Basel III(2010) framework and are briefly analyzed below.

The Basel I, known also as *Basel Capital Accord*, mainly focused on credit risk minimization and Risk Weighted Assets (RWA). Banks with international operation should maintain a minimum amount (8%) of capital (Tier 1 and Tier 2) based on a percent of risk-weighted assets. Bank’s assets are classified into five risk categories according to the debtor’s category.

- Tier 1 capital is a bank's core capital and includes disclosed reserves that appears on the bank's balance sheets, common stocks, non-redeemable and non-cumulative preferred stock, which can be included in certain calculation methods and equity capital. This core capital is a measure of a bank’s financial strength.
- Tier 2 capital is a bank's secondary capital. Tier 2 contains undisclosed reserves⁵², revaluations reserves⁵³, subordinated term debts⁵⁴, hybrid financial products⁵⁵ and general loss provisions .

Table 3.2. Asset categories

0%	20%	50%	100%
<ul style="list-style-type: none"> - Cash - Government debt - Bullion 	<ul style="list-style-type: none"> - Claims on OECD banks - Claims OECD public sector entities 	<ul style="list-style-type: none"> - Residential mortgage loans - Claim on municipal bonds 	All other claims such as: <ul style="list-style-type: none"> - Less developed countries debt - Non-OECD banks - Most Corporate debt

⁵¹ <https://www.bis.org/bcbs/>

⁵² Undisclosed reserves are not acceptable for some countries therefore they are excluded from the core capital

⁵³ Reserves which includes revaluated assets in order to provide a better display of the current asset. Basel II also requires that the difference between the historic cost and the actual value be discounted by 55% when using these reserves to calculate Tier 2 capital.

⁵⁴ Subordinated debt is debt that ranks lower than ordinary depositors of the bank

⁵⁵ A hybrid financial product or instrument is one that that has characteristics of multiple types of instruments, often convertible from one to another. The term covers a wide range of financial products, from convertible bonds to futures on indexes.

Regarding the above asset classification, the capital ratio is estimate as below:

$$\textit{Tier 1 capital ratio} = \frac{\textit{Tier 1}}{\textit{RWA}}$$

$$\textit{Total capital ratio} = \frac{\textit{Tier 1} + \textit{Tier 2}}{\textit{RWA}}$$

Advantages and positive effects of implementation of Basel I standards:

- Substantial increases in capital adequacy ratios of international banks.
- Worldwide adoption.
- Increased competitive equality among international banks.

Besides the advantages of Basel I, the main weaknesses of Basel I standards:

- Capital adequacy depends on credit risk and excludes other type of risk (for example operational, market etc.).
- Emphasis is on book values and not on market values.
- Inadequate assessment of risks and effects of the use of new financial instruments
- Within each RWA category the potential differences in the creditworthiness of each individual borrower are not distinguished.

The Basel II known formally as “*A Revised Framework on International Convergence of Capital Measurement and Capital Standards*” focuses on expanding the framework of Basel I, especially in term of risk diversification, detail and effectiveness. The Basel II consists of three fundamental Pillars.

Pillar 1: Minimum capital

In comparison with the Basel I, the new framework includes new parameters in estimating the capital ratio. These parameters enable three types of risk(credit, market and operational), while the total regulatory capital and the border of 8% remain unchanged.

$$\textit{Total capital ratio} = \frac{\textit{total capital}}{\textit{credit} + \textit{market} + \textit{operational risks}}$$

Additionally, the risk weighted assets are estimated by multiplying the capital requirements for market - operational risks by 12.5 and the Tier 2 capital could not overcome the 100% amount of Tier 1.

Pillar I determines the risk measurements approaches individually for each type of risk according to the following table.

Table 3.3. Risk measurement approaches⁵⁶

Credit Risk	Market Risk	Operational Risk
Standardized Approach	Standardized Approach	Basic Indicator Approach
Foundation IRB Approach	Internal Models Approach	Standardized Approach Advanced
Advanced IRB Approach		Measurement Approach

The internal rating based (IRB) approach is one of the most significant addition, in the foundation and advanced versions the IRB based on quantitative inputs provided by banks or supervisory authorities as well as they are aligned with risk weight functions described by the BCBS. Specifically, the IRB approach includes four indicators:

Probability of default (PD): It is the core input of IRB approach and measures the likelihood of a debtor's default over a given time horizon.

Loss Given Default (LGD): Measures the rate of loss to total exposure, when a default occurs.

Exposure At Default (EAD): Defined as the gross exposure under a facility upon default of a debtor.

Maturity (M): The remaining maturity of the exposure.

For the Operational Risk, Basel II developed three measurement approaches, the basic indicator approach (the capital requirement of a bank to weight the operational risk should be equal to 15% of its annual average gross income over the previous three years), the standardized approach (a bank's gross income is separated in eight business lines⁵⁷ and is multiplied by BCBS's supervisory

⁵⁶ BCBS

⁵⁷ Corporate Finance 18% Sales & Trading 18% Retail Banking 12% Commercial Banking 15% Settlement 18% Agency Services 15% Asset Management 12% Retail Brokerage 12%

factors) and the advanced measurement approach (banks are able to define a comprehensive and systematic model to estimate the minimum required capital).

For the Market risk BCBS introduced two approaches. According to the first (the internal models' approach), banks are encouraged to develop their own internal model. In the second approach (Standardized Approach) the capital requirement is the simple sum of three components: the risk charges under the sensitivities-based method⁵⁸, the default risk charge⁵⁹, and the residual risk add-on⁶⁰.

Pillar 2: Supervisor review

The Pillar 2 refers to the basic principles of an effective risk management as well as the principles of the supervisory assessment. According to the BCBS, banks are required to make comprehensive plans to improve cooperation and information exchange in order to ensure capital's adequacy. Further, the supervisors identify the importance of the internal bank's proceedings to support capital rating and the overall risk level.

The BCBS acknowledges four important principles for supervisory implementations:

- Principle 1: Banks should establish their own processes for the internal rating of capital adequacy as well as develop their strategies for maintaining their capital levels.
- Principle 2: Supervisors should review the abovementioned processes and strategies. Additionally, they should monitor and ensure that a bank's individual policies are compliant with the regulatory framework, otherwise actions are needed.
- Principle 3: Banks should operate above the level of the minimum regulatory capital, otherwise supervisors should force banks to hold capital in excess of the minimum.
- Principle 4: Supervisors should have the ability of an early intervention in case of potential capital adequacy deterioration.

⁵⁸ Capital charges for delta, vega and curvature risk factor within a prescribed set of risk classes (general interest rate risk, credit spread risk, foreign exchange risk, equity risk and commodity risk)

⁵⁹ Default risk charge for prescribed risk classes: default risk of non-securitisation, securitisation, and securitisation correlation trading portfolio.

⁶⁰ Residual risk add-on: risk weights notional amounts of instruments with non-linear payoffs.

Pillar 3: Market Discipline

The scope of this Pillar is to support the Pillar 1 and 2 frameworks. Market discipline through better disclosure of information by banks, therefore banks should publicize more differentiated data. These data concern statistics such as the aggregate amounts of capital (both Tier 1 and Tier 2), risk-weighted capital adequacy ratios, reserve requirements for credit, market, and operational risk, and a full description (with assumptions) of the risk mitigation approaches of a bank are recommended for quarterly release to the general public under Basel II's standards. The publicized information should not contradict the international accounting standards and is upon the bank's responsibility the way in which it can be achieved. The discipline could be described as the fact that an indicator of bad banking management, e.g. too many risky policies, leads the bank's clients and investors to react and put pressure on the bank to correct the situation.

The third Accord was the Basel III (or the Third Basel Accord or Basel Standards) and it was a response to the financial crisis of 2007-2009. There are three key principles for Basel III.

1. Minimum capital requirements

The BCBS agreed on detailed capital measures in order to improve the capital quality and quantity. These measures are developed at the different components of the capital structure, as well as at the regulatory adjustments. Concerning the minimum capital requirements, the main differences with the Basel II are described in the following table.

Table 3.4. Changes to the definition of the regulatory capital⁶¹

requirements	Basel II (8%)	Basel III (8%)
Core Tier 1	2%	2%
Additional Tier 1	4.5%	1.5%
Tier 2	4%	4.5%

An innovation of Basel III was the additional capital conservation buffer, equivalent to 2.5% of RWA. The Capital Conservation Buffer is a macroprudential capital adequacy requirement for all banks to build up an additional loss-absorbing capital cushion to improve their resilience to stresses. Another

⁶¹ Source: Financial stability overview, European Central Bank, December 2010



innovation is the countercyclical capital buffer, allowing regulators to require up to an additional 2.5% of capital during periods of high credit growth. According to the Basel Committee, countercyclical capital buffers require banks to hold capital when a significant growth of credit appears, so that the buffer can be reduced if the financial cycle decreases or the economic and financial environment becomes much worse. Banks can use the capital buffers they have built up during the growth phase of the financial cycle to cover losses that may arise during periods of stress and to continue supplying credit to the real economy.

2. Leverage Ratio

The Leverage ratio is expressed as the minimum loss absorbing capital and is calculated as :

$$\text{Leverage ratio} = \frac{\text{Tier 1 capital}}{\text{Total exposure}}$$

,and the minimum leverage level is 3%.

3. Liquidity framework

The Basel III introduces two liquidity ratios in order to express the minimum liquidity requirements: the Liquidity coverage ratio (LCR) and the Net stable funding ration (NSFR).

- Liquidity coverage ratio (LCR):

It is a short-term liquidity measure and describes the amount that banks hold as unencumbered, high-quality liquid assets⁶² to meet net cash outflows under a well-defined stress scenario during a period of 30 days. It is mathematically expressed as:

$$LCR = \frac{\text{High - quality liquid asset}}{\text{Total net liquidity outflows over 30 days}}$$

, and the minimum level of the LCR is 100%.

- Net stable funding ratio (NSFR)

It is a long-term liquidity measure and target to manage cases of maturity mismatch⁶³. The ratio defines a minimum amount of stable funding required by the liquidity characteristics of various assets or activities (which also comprise, for example, off-balance-sheet contingent exposures and exposures from securitisation pipelines) held by institutions over a one-year horizon.

⁶² The high-quality liquid assets include only those with a high potential to be converted easily and quickly into cash

⁶³ The mismatch happens when there are more liabilities than assets

Table 3.5. Comparison of the Basel I,II and III⁶⁴

Basel I	Basel II	Basel III
Covers only Market and Credit risk	Covers Credit, Market and Operational risk	An improvement and a more detailed aspect of the Basel III
The minimum capital requirements are based on a single RWA	The minimum capital requirements taking into account all the above types of risk and assets	Capital innovations: capital conservation buffer, countercyclical capital buffer
	3 Pillars : Minimum capital requirements, supervisory reviews and market discipline	Leverage ratio and Liquidity requirements.

⁶⁴ From Basel I to Basel III: Sequencing Implementation in Developing Economies, IMF working paper by Caio Ferreira, Nigel Jenkinson, and Christopher Wilson

CHAPTER 4: A panel approach of NPLs determinants

4.1 Introduction

This chapter presents an empirical analysis of the non-performing loans using a cross sectional time series (panel data) methodology. First, an analysis of the theoretical aspect of the Panel data modelling is provided. Moreover, the analysis concerns two approaches, fixed and random. In the final section an empirical study of the z-score metrics and the government debt ratio along with bank specific determinants is represented in order to identify their impact to the rate of non-performing loans. The datasets concern 16 European countries covering the period from 2008 to 2017.

4.2 Introduction to Fixed and Random effects model

The panel approach is a cross sectional time series analysis concerning groups of countries, individuals, states etc. over time. There is a vast literature based on the Panel analysis and it is a widely used as an econometric tool, with a significant number of economic studies. Specifically, Louzis, Voulidis (2012), Anastasiou, Louri, Tsionas (2016), Louzis, Vouldis and Metaxas (2012) are some examples of NPLs studies that focus on the determinants of non-performing loans ratio.

The main advantage of the panel approach is the increased number of observations, in comparison with univariate/multivariate analysis. Expanding the sample to more countries enables an econometric analysis with more observations and therefore more information. Further, panel data models are more effective in investigating economic phenomena across countries (behavior analysis). The main disadvantage of these models is the fact that for example, some countries do not appear data completeness in some periods of time. Furthermore, in some cases we observe an apparent relation among the variables over the time or problems with the asymptotic assumptions.

Considering the completeness of time series we can distinguish panel data between balanced and unbalanced. An unbalanced panel is one where individuals are observed for a different number of times, e.g. because of missing values.

The simplest form of the panel models is the common constant panel model and described as follows:

$$Y_{it} = a_0 + bX_{it} + e_{it}$$



, where the indicator $i=1,\dots,N$ describes the number of units, $t=1,\dots,T$ describes the time and the a_0 is the constant for each cross-sectional unit term. The main assumptions of these models are:

- i. $(Y_i, X_i) \sim iid$, which means that have produced by random sampling
- ii. $E(e_{it} | X_{i1}, X_{i2}, \dots, X_{iT}) = 0$, for each $t=1,2,\dots,T$
- iii. $Var(e_{it} | X_{i1}, X_{i2}, \dots, X_{iT}) = \sigma_\epsilon^2$, for each value of i and t
- iv. $Cov(e_{it} | X_{i1}, X_{i2}, \dots, X_{iT}) = 0$, for each $t \neq s$
- v. $e_{it} | X_{i1}, X_{i2}, \dots, X_{iT} \sim iid(0, \sigma_\epsilon^2)$

For research issues the most common approach is to assume that the constant term is different for each cross-sectional unit, which enables heterogeneity meaning. Considering this, the main and widely used approaches of Panel analysis are the fixed effects models and the random effects model, which are briefly analyzed below.

➤ **Fixed effects model:**

The constant term of each entity i is an individual effect for a given group (countries, etc.) and time invariant. The form of the fixed effect model is as follows:

$$Y_{it} = a_i + bX_{it} + u_{it}$$

, where a_i is an intercept term for each cross-sectional unit with $i=1,\dots,N$.

When using the fixed effects model, we assume that something within the individual may impact the behavior of each variable, the error term and the intercepts among individuals are being uncorrelated. In the fixed term model the unobserved variables (a_i, b) may have some associations whatsoever with the observed variables.

In order for OLS to produce unbiased estimation, it is normally distributed in large samples. For the fixed effects models the main assumptions are presented below:

- i. The error term u_i has conditional mean zero, that is $E(u_i | X_{i1}, X_{i2}, \dots, X_{iT}) = 0$, for each $t=1,2,\dots,T$
- ii. $(X_{i1}, X_{i2}, \dots, X_{iT}, u_{i1}, \dots, u_{iT})$ are i.i.d. for $i=1,\dots,n$ and $i=1,\dots,n$

Moreover, in order to have unbiased and consistent estimations, the a_i should be uncorrelated with the X_{it} . This assumption is not applied and considering this the a_i may be correlated with one or more observed variables X_i . As a result, it is necessary to expunge the intercepts term a_i and then we have OLS model's estimation for the term b . Considering this, the following methods that are analyzed below are the within-groups estimation, the first-difference estimation and the Least squares with dummy variable estimation.

A. The within-groups method produces estimation with time averages deviations for each cross-sectional individual, which eliminates the α_i term.

Step 1: We estimate the equation's time average

$$\bar{Y}_i = a_i + b\bar{X}_i + \bar{u}_i \quad \text{where } \bar{Y}_i = \sum_{t=1}^T \frac{Y_{it}}{T}, \bar{X}_i = \sum_{t=1}^T \frac{X_{it}}{T}, \bar{u}_i = \sum_{t=1}^T \frac{u_{it}}{T} \text{ and } \bar{a}_i = a_i$$

Step 2: We take the difference between the equation above and the equation $Y_{it} = a_i + bX_{it} + u_{it}$. Finally, we conclude to the following model, which does not include the constant term and it is able to produce OLS estimations :

$$Y_{it} - \bar{Y}_i = b(X_{it} - \bar{X}_i) + (u_{it} - \bar{u}_i) \text{ for } t=1,2,\dots,T$$

B. The second method to eliminate the α_i term is the first difference approach. For individual I in time period t the model may be written as:

$$\Delta Y_{it} = b\Delta X_{it} + \Delta u_{it}$$

In comparison with the first method of the within-groups estimator, both methods coincide only in the case of the two time periods ($T=2$).

C. The third method is the Least squares dummy variable (LSDV) approach. If we define a dummy for the intercept term, then we have:

$$Y_{it} = \sum_{j=1}^N a_j D_{ij} + bX_{it} + u_{it}$$

, where D_{ij} equals to 1 for all $i=j$, which means that an observation is related to individual I and 0 otherwise. It is remarkable to mention that for a large number of individuals we result to a large number of dummies, which is not proposed.

➤ **Random effects model:**

An alternative to fixed model is the random model or error components model. In this specific model the intercept is a random unobserved effect constant over time and the model's form is:

$$Y_{it} = \alpha_0 + bX_{it} + e_{it}, \quad e_{it} = \alpha_i + u_{it}$$

, where the α_0 term concerns the common intercept for all cross-sectional units and e_{it} is a composite error. The composite error consists of the random variable α_i that is different for each unit and u_i is the error term. The composite error term measures the random deviation of each individual's intercept from the common intercept α_0 . In the random effects model, the unobserved

variables are assumed to be uncorrelated with (or, more strongly, statistically independent of) all the observed variables. We assume the following specifications for the random effects model:

- i. $(Y_i, X_i) \sim iid$
- ii. $E(u_{it} | X_i)$ for each $t=1, \dots, T$
- iii. $E(\alpha_i | X_i) = \alpha_0$
- iv. $Var(e_{it} | X_i) = \sigma_e^2$, for each value of i and t
- v. $Var(u_{it} | X_i) = \sigma_u^2$, for each value of i and t
- vi. $Cov(u_{it} | X_{i1}, X_{i2}, \dots, X_{iT}) = 0$, for each $t \neq s$
- vii. $u_{it} \sim iid$ and $\alpha_i \sim iid$
- viii. $Cov(u_{it}, \alpha_i | X_i) = 0$, for each t

Considering the assumptions mentioned above, the generalized least squares (GLS) method produce unbiased and consistent estimation for the random effects model. This is due to the specificity of the component error's autocorrelation form. The OLS method would be useful only if $\sigma_\alpha^2 = 0$.

4.3 Model testing

One of the main tests of the panel analysis is to determine which type of model is more appropriate for our analysis. Other testing procedures that will be analyzed in this section are the unit root tests and the cointegration test.

A. Durbin-Wu-Hausman

The random effect model is preferred when the entities in the sample are being randomly selected, in comparison with the entities of the fixed models that are more likely to represent the entire population. Otherwise, another prerequisite is whether the unobserved effect is distributed independently of the X_i variables or not. The Durbin-Wu-Hausman test is very useful to help us choose between fixed or random effects model. The hypothesis test is:

H_0 : Both random and fixed effect estimates are consistent

versus

H_1 : The random effect estimates are consistent

The test statistics of the hypothesis testing is:

$$H = (\hat{b}_{FE} - \hat{b}_{RE})' [\text{Var}(\hat{b}_{FE}) - \text{Var}(\hat{b}_{RE})]^{-1} (\hat{b}_{FE} - \hat{b}_{RE})$$

, and follows X^2 distribution with degrees of freedom determined by the number of the variables b . The rejection of the null hypothesis indicates that the random effect estimates of b are not consistent or that the model is wrongly specified (misspecification error). It is concluded that the fixed effect model is consistent under both null and alternative.

B. Unit root test

As with the non stationarity issues of the time series in the previous chapters, the panel time series also face problems with non stationarity. We could discriminate the unit root test into two categories: first-generation and second-generation tests. This discrimination result from the dependency among the cross-sectional individuals. The first-generation tests assume the independency of the cross-section individuals (homogeneity) and the second-generation tests presuppose the dependency among the cross-sectional individuals (heterogeneity).

- First-generation unit root tests:
 - i. Levin, Lin and Chu (2002)
 - ii. IM, Pesaran and Shin (2003)
 - iii. Harris and Tzavalis (1999)

Levin, Lin, Chu and Harris ,Tzavalis are based on the assumption of the homogeneity across the individuals, whether IM, Pesaran and Shin are based on the heterogeneity across the individuals.

- Second-generation unit root tests:
 - i. Chang (2002,2004)
 - ii. Bain and NG (2004)
 - iii. Phillips and Sul (2003)
 - iv. Moon and Perron (2004)
 - v. Choi(2002)
 - vi. Pesaran (2003)

C. Cointegration test

Considering the cointegration analysis for VAR models, the same issue appears in the Panel time series. The cointegration testing in Panel analysis is more complicated as we should take into account the possibility of cointegration across cross-sectional variables as well as within groups.

Pedroni (1999,2004) following Engle-Granger methods uses the residuals u_{it} of the regression.

$$Y_{it} = \alpha_i + \gamma_{it} + b_{mi}X_{mit} + u_{it}$$

, where $m=1,\dots,M$ are the explanatory variables in the potentially cointegrating regression, $t=1,\dots,T$ and $i=1,\dots,N$.

In Pedroni test, we assume that the coefficients b and the intercepts α_i can vary across each group. The general approach is to test whether the residuals of the regression are $I(1)$ or not. According to Pedroni, the residuals regression structure is:

$$u_{it} = \rho_i u_{i,t-1} + \sum_{j=1}^{\rho_i} \psi_{i,j} \Delta u_{i,t-j} + u_{it}$$

The hypothesis testing consist of two alternative hypotheses and the form is :

H_0 : no cointegration, the residuals will be $I(1)$, $\rho_i = 1$

Versus

H_1 : $\rho_i = \rho < 1$ for all i

Or

H_1 : $\rho_i < 1$ for all i

The first alternative referred as homogenous alternative or within-dimension test or panel statistic and the second alternative as heterogenous alternative or between-dimension or groups statistic.

Kao (1999) has developed a restricted version of Pedroni's testing and assumed that the coefficients b are fixed across the groups and only the intercepts can vary. Following this, Kao proposed two cointegration tests based on the Dickey-Fuller and on the Augment Dickey-Fuller.

Maddala and Wu(1999) using the Johansen(1998) and Fisher (1932) proposed the Johansen-Fischer test for the cointegration testing. This test combines the statistics for each cross-sectional individual into one statistic for all panel models. The statistic of the hypothesis testing is:

$$-2 \sum_{i=1}^N \log(p_i)$$

, where p_i is the p-value of each cross-sectional group and N is the number of them.

4.4 Empirical analysis

4.4.1 Data

The scope of the following panel analysis is to study whether non-performing loans are affected by the bank's specific determinants and especially those which are related to the bank's credit strength (z-score & Tier 1). More specifically, the explanatory variables of our analysis are the Return on Equity (ROE), the capital adequacy ratio Tier 1, the size of the banks, the general government debt to GDP rate (DEBT) and the non-performing loans ratio (NPLS) for 16 European countries. The source of these data is the European Central Bank database. The time series of z-score rate was obtained from the database of the World Bank, and similarly for the general government debt. Considering the availability and completeness of the time series, we estimate the panel model for 16 European countries covering the period from 2008 until 2017 (annual frequency). The European countries are Austria, Belgium, Cyprus, Estonia, Finland, Germany, Greece, Italy, Latvia, Lithuania, Malta, Netherlands, Portugal, Slovakia and Spain.

According to the World Bank the z-score ratio estimates the probability of default of a country's banking system. Z-score compares the buffer of a country's banking system (capitalization and returns) with the volatility of those returns. It is estimated as

$$z - score = \frac{ROA + \left(\frac{equity}{assets}\right)}{S.D.(ROA)}$$

, where ROA is the Return on Assets and S.D.(ROA) is the standard deviation of ROA.

ROA, equity, and assets are country-level aggregate figures calculated from underlying bank-by-bank unconsolidated data from the Bankscope⁶⁵. The z-score is a metric to test the stability of the banking sector, therefore the higher this rate is, the more stable the banking sector. According to Čihák⁶⁶ (2007), the main advantage of this rate is the easy computation for a financial institution or corporation. Further, the main disadvantage of this method is represented by the fact that it does not cover the correlation between financial institutions (contagion relation).

⁶⁵ The BankScope database is a unique collection of micro-level banking information for different countries: banks.bvdinfo.com

⁶⁶ Systemic Loss: A Measure of Financial Stability*
 Martin ČIHÁK – International Monetary Fund, Washington, D.C

The next variable of the study is the Return on Equity, which is an indicator of bank’s profitability. It is estimated by dividing the net income⁶⁷ by book value of equity.

The variable Tier 1 refers to the bank’s capital adequacy ratio that includes equity capital and disclosed reserves. From the regulatory view, the Tier 1 measures the financial strength and it is estimated as the sum of the bank’s core equity capital to total bank’s asset. According to Basel III, banks and financial institutions should maintain a minimum Tier 1 capital ratio (total core capital to its total risk-weighted assets). To ensure against unexpected losses, the minimum tier 1 capital ratio is 6%. The next variable is the average bank size for each country in Euros. Finally, we use the non-performing loans ratio for 16 European countries as our dependent variable in order to test how and whether it is affected by the level of z-score ratio and how the bank’s specific indicators influence the stock of non-performing loans. The panel data are unbalanced and is indicated from the observations number for each variable.

The General government debt (also known as public debt, national debt or sovereign debt) as a percentage of GDP describes the debt owed by a central government.

Finally, table 4.21 describes briefly the above time series and the expected sign according to the analyzed determinants of non-performing loans(see unit 1.3).

Table 4.1. Definitions and hypothesis

Variable	Definition	Hypothesis	Expected sign
ROE	$\frac{\text{Net profit}}{\text{Total equity}}$	Bad management II	-
SIZE	Sum of total assets	Diversification	-
DEBT	General gross government debt as of % GDP	Sovereign debt crisis	+
TIER1	$\frac{\text{Core capital}}{\text{Total bank asset}}$	Financial strength	-
ZSCORE	$\frac{ROA + \left(\frac{\text{equity}}{\text{assets}}\right)}{S.D.(ROA)}$	Bank’s credit riskiness	-

⁶⁷ Net income is the difference between operating income (net interest income, net fee and commission income and other operating income) and operating expenses (staff costs, administrative costs and depreciation) after accounting for impairments, provisions, contributions to the resolution fund and deposit guarantee scheme and taxes.

Table 4.2. Common descriptive analysis of the variables

	DEBT	TIER1	SIZE	ROE	ZSCORE	NPLS
Mean	93.40022	0.131487	34522460	0.011132	0.130446	0.085587
Median	94.75403	0.129128	4472012.	0.055925	0.113192	0.050478
Maximum	101.8692	0.218896	4.01E+08	0.229181	0.302763	0.460923
Minimum	74.19278	0.000000	935648.0	-1.008349	0.000167	0.007777
Std. Dev.	8.082181	0.036764	84837736	0.165830	0.069806	0.097895
Skewness	-1.105391	-0.036310	3.393896	-3.843104	0.462500	2.315078
Kurtosis	3.611215	3.262220	13.27343	20.15076	2.288279	7.901735
Jarque-Bera Probability	35.07428 0.000000	0.493555 0.781314	1010.783 0.000000	2310.689 0.000000	9.081144 0.010667	303.1024 0.000000
Sum	14944.04	21.03792	5.52E+09	1.747759	20.87138	13.69388
Sum Sq. Dev.	10386.14	0.214900	1.14E+18	4.289947	0.774780	1.523769
Observations	160	160	160	157	160	160

The data of our model are unbalanced as a consistency of three missing values of ROE time series.

4.4.2 Methodology and model testing

In this study we will compare two estimated models at level (model 1) and at first-difference (model 2) transformation. For the variables Tier1 and Size we will choose the natural logarithm in order to maximize the estimation accuracy. The form of the models is as follows:

Model1:

$$NPLS_{it} = a_i + b_1ZSCORE_{it} + b_2DEBT_{it} + b_3ROE_{it} + b_4TIER1_{it} + b_5LN(SIZE) + b_5DUMMY + u_{it}$$

Model 2:

$$\Delta NPLS_{it} = a_i + b_1\Delta ZSCORE_{it} + b_2\Delta DEBT_{it} + b_3\Delta ROE_{it} + b_4\Delta TIER1_{it} + b_5\Delta LN(SIZE) + b_5DUMMY + u_{it}$$

The first step of the analysis is to test which type of effects (fixed or random) is more appropriate for our variables according to Hausman test. The rejection of the null hypothesis indicates that the fixed effect model is more appropriate.

Table 4.3. Hausman test results for the model 1

Correlated Random Effects - Hausman Test				
Test cross-section random effects				
Test Summary	Chi-Sq. Statistic	Chi-Sq. d.f.	Prob.	
Cross-section random	28.791848	6	0.0001	
Cross-section random effects test comparisons:				
Variable	Fixed	Random	Var(Diff.)	Prob.
LNSIZE	-0.083558	-0.017415	0.000643	0.0091
ZSCORE	0.228389	-0.222196	0.020286	0.0016
DEBT	0.000775	0.001093	0.000000	0.2210
TIER1	-1.360150	-1.015879	0.015949	0.0064
ROE	-0.170330	-0.171375	0.000045	0.8760
DUMMY	0.002911	0.004976	0.000004	0.3053

Table 4.4. Hausman test results for the model 2

Correlated Random Effects - Hausman Test				
Test cross-section random effects				
Test Summary	Chi-Sq. Statistic	Chi-Sq. d.f.	Prob.	
Cross-section random	5.072179	6	0.5346	
Cross-section random effects test comparisons:				
Variable	Fixed	Random	Var(Diff.)	Prob.
D(LNSIZE)	-0.042731	-0.039433	0.000035	0.5745
D(ZSCORE)	-0.183655	-0.163595	0.000791	0.4758
D(TIER1)	-0.483621	-0.467204	0.001807	0.6993
D(DEBT)	0.001048	0.001052	0.000000	0.9013
D(ROE)	-0.108150	-0.108767	0.000002	0.6314
DUMMY	-0.004184	-0.003959	0.000000	0.5765

According to the above Hausman test results, the random effects approach is not appropriate for model 1. The fixed effects models could be used for both models, therefore in order of comparison the fixed effects are indicated for our analysis.

Taking into account the previous empirical analysis in Greece, we add the same dummy variable of eurozone’s debt crisis described as follows:

$$DUMMY_t = \begin{cases} 1, & 2008 \leq t \leq 2012 \\ 0, & t > 2012 \end{cases}$$

The unit root tests for both models (level and first difference) are presented in the table 4.5. The Levin, Lin & Chu assumes common unit root process and the Im, Oesaran & Shin W-stat, ADF and PP assume individual unit root process. The null hypothesis for each assumption is the non-stationarity of the times series against the alternative of stationarity.

Table 4.5. Unit root tests

Model 1	NPLS	LNSIZE	ZSCORE	TIER1	DEBT	ROE
Levin, Lin & Chu t*	0,125	0,127	0,000	0,802	0,000	0,062
Im, Pesaran and Shin W-stat	0,634	0,611	0,140	0,983	0,002	0,064
ADF - Fisher Chi-square	0,588	0,411	0,058	0,951	0,002	0,010
PP - Fisher Chi-square	0,654	0,033	0,200	0,396	0,000	0,004
Model 2	ΔNPLS	ΔLNSIZE	ΔZSCORE	ΔTIER1	ΔDEBT	ΔROE
Levin, Lin & Chu t*	0,000	0,000	1,000	0,000	1,000	0,007
Im, Pesaran and Shin W-stat	0,027	0,032	0,742	0,0006	0,742	0,001
ADF - Fisher Chi-square	0,011	0,006	0,976	0,0002	0,976	0,000
PP - Fisher Chi-square	0,000	0,000	0,000	0,0000	0,000	0,000

In accordance with the p-values of the corresponding t-statistic, we conclude that the hypothesis of the presence of the unit root test cannot be rejected for the majority of level model variables, whereas most of the first difference variables do not present unit roots.

4.4.3 Econometric results

Using the least squares estimation method , the results for NPLs time series are presented in the following table (table 4.5). In model 1 only the Tier 1 and the ROE are statistically significant, whereas in model 2 only the $\Delta(\text{DEBT})$ and the $\Delta(\text{ROE})$ are statistically significant at a level of 5%.

Table 4.6. Estimation outputs for level and difference model

Model 1					Model 2				
Dependent variable: NPLS					Dependent variable: $\Delta(\text{NPLS})$				
Variable	Coefficient	Std, Error	t-Statistic	Prob,	Variable	Coefficient	Std, Error	t-Statistic	Prob,
LNSIZE	-0.083	0.070	-1.180504	0.239	$\Delta(\text{LNSIZE})$	-0.043	0.037	-1.160	0.248
ZSCORE	0.228	0.162	1.407220	0.162	$\Delta(\text{ZSCORE})$	-0.184	0.150	-1.222	0.224
DEBT	0.001	0.002	0.491350	0.624	$\Delta(\text{DEBT})$	0.001	0.0003	2.924	0.004
TIER1	-1.360	0.441	-3.087155	0.003	$\Delta(\text{TIER1})$	-0.484	0.317	-1.524	0.130
ROE	-0.170	0.059	-2.880845	0.005	$\Delta(\text{ROE})$	-0.108	0.037	-2.935	0.004
C	1.478	1.210	1.221963	0.223	C	0.001	0.003	0.214	0.831
DUMMY	0.003	0.012	0.238119	0.812	DUMMY	-0.004	0.005	-0.841	0.402
R-squared	0.712	Mean dependent var		0.0839	R-squared	0.441	Mean dependent var		-0.001
Adjusted R-squared	0.668	S,D, dependent var		0.097	Adjusted R-squared	0.341	S,D, dependent var		0.038
S,E, of regression	0.056	Akaike info criterion		-2.786	S,E, of regression	0.030	Akaike info criterion		-3.991
Sum squared resid	0.428	Schwarz criterion		-2.358	Sum squared resid	0.111	Schwarz criterion		-3.529
Log likelihood	240.755	Hannan-Quinn criter,		-2.612	Log likelihood	301.397	Hannan-Quinn criter,		-3.804
F-statistic	15.968	Durbin-Watson stat		0.493	F-statistic	4.427	Durbin-Watson stat		1.870
Prob(F-statistic)	0.000				Prob(F-statistic)	0,00			

Considering the estimated coefficients for both models we conclude that they are negatively associated with the NPLs ratio except of the level and first-difference of DEBT time series and the level time series of z-score for model 1.

The z-score variable is negatively associated with the NPLs ratio for the model 2 only, supporting the view that a riskier credit rate leads to a low rate of z-score and therefore to an increase of the non-performing loans. For model 1 z-score sign is positively associated with the NPLs

The second bank-specific variable is the TIER1 rate. The sum of bank's core capital ratio describes the size of a bank's core capital and subsequently the bank's strength, according to Basel policies. The TIER1 (-1.360 for Model 1) variable is negatively related with the NPLs ratio as well as the $\Delta(\text{TIER1})$ (-0,484 for Model 2). This implies that banks with higher amount of core capital tend to have an decline in the NPLs ratio.

Concerning the estimated coefficients of ROE we concluded that there is a negative association with the NPLs ratio, specifically the coefficients are -0.170, -0.108 for Model 1 and Model 2. This evidence is consistent with the empirical results of Louiz et al (2012) and confirm the bad management hypothesis II, meaning that an efficient bank performance reflects the quality of management.

The diversification opportunities are described through the sum of total assets. Proxied by the natural logarithm of total assets, the LNSIZE is negatively linked to the non-performing loans ratio (Model 1: -0.083 and Model 2: -0,043). The specific result provides support to the findings of Salas and Saurina (2002), Hu et al. (2004) and Rajan and Dhal (2003).

Finally, the government Debt has a positive impact on the ratio of non-performing loans with estimated coefficients for Model 1 and Model 2. Therefore, we could conclude that the debt affects the increase of NPLS ratio and support the view of many research studies such as Reinhart and Rogoff (2010), Perotti (1996) etc.

The next table describes the evidences of our research and compare them with former empirical studies as described in unit 1.3. Moreover, the coefficient signs follow the expected sign in most banking hypothesis.

Table 4.7. Hypothesis testing

Variable	Coefficient sign		Tested hypothesis	Results	
	Model 1	Model 2		Model 1	Model 2
LNSIZE	-	-	Diversification	Yes	Yes
ZSCORE	+	-	Credit riskiness	No	Yes
TIER1	-	-	Financial strength	Yes	Yes
DEBT	+	+	Sovereign debt crisis	Yes	Yes
ROE	-	-	Bad management II	Yes	Yes

The final step of the empirical analysis is to test for possible Granger causality among the non-performing loans determinants. The Granger causality approach is to treat panel data as one large

stacked set of data, and then perform the Granger Causality test in the standard way, with the exception of not letting data from one cross-section enter the lagged values of data from the next cross-section. This method assumes that all coefficients are the same across all cross-sections. The Granger causality output is presented below among the NPLS ratio and the independent variables of Model 1 and 2 for each null-hypothesis separately.

Table 4.8. Granger Causality testing for Model 1

Pairwise Granger Causality Tests			
Lags: 2			
Null Hypothesis:	Obs	F-Statistic	Prob.
NPLS does not Granger Cause DEBT	128	1.47893	0.2319
DEBT does not Granger Cause NPLS		1.41940	0.2458
NPLS does not Granger Cause TIER1	128	4.98589	0.0083
TIER1 does not Granger Cause NPLS		1.68532	0.1896
NPLS does not Granger Cause LNSIZE	128	6.86514	0.0015
LNSIZE does not Granger Cause NPLS		0.62141	0.5389
NPLS does not Granger Cause ZSCORE	128	3.13806	0.0469
ZSCORE does not Granger Cause NPLS		0.20699	0.8133
ROE does not Granger Cause NPLS	123	44.5372	4.E-15
NPLS does not Granger Cause ROE		11.6685	2.E-05

According to the outputs, unidirectional Granger causality is found between NPLS -ZSCORE, NPLS-TIER1 and NPLS – LNSIZE. This evidence leads to the fact that with the previous rates of NPLS we could estimate the future values of the ZSCORE, TIER1 and LNSIZE. Further, bidirectional causality is observed between NPLS and ROE time series.

Table 4.9. Granger Causality testing for Model 2

Pairwise Granger Causality Tests			
Lags: 2			
Null Hypothesis:	Obs	F-Statistic	Prob.
DNPLS does not Granger Cause DDEBT	112	0.94550	0.3917
DDEBT does not Granger Cause DNPLS		2.38243	0.0972
DNPLS does not Granger Cause DROE	106	6.96400	0.0015
DROE does not Granger Cause DNPLS		20.6728	3.E-08
DNPLS does not Granger Cause DTIER1	112	6.64620	0.0019
DTIER1 does not Granger Cause DNPLS		1.14549	0.3219
DNPLS does not Granger Cause DZSCORE	112	0.68499	0.5063
DZSCORE does not Granger Cause DNPLS		1.49235	0.2295
DLNSIZE does not Granger Cause DNPLS	112	5.63179	0.0047
DNPLS does not Granger Cause DLNSIZE		0.83764	0.4355

Considering the p-values of causality testing for Model 2, we conclude in the presence of bidirectional causality relationship between $\Delta NPLS$ and ΔROE . Finally, unidirectional relationship appeared between $\Delta LNSIZE$ - $\Delta NPLS$ and $\Delta NPLS$ - $DTIER1$.

To conclude, the empirical results support the previous findings in the non-performing loans and especially the first differenced model 2 underlines all the aforementioned hypotheses as displayed in table 4.7.

CHAPTER 5: Concluding Remarks

5.1. Summary

This Thesis is an attempt to study the evolution of non-performing loans in the Greek banking system and investigate the impact of NPLs on the bank's creditworthiness. Additionally, the regulatory framework adopted by the European authorities the regulatory framework is analyzed, considering the resolution steps as well as the loans management initiatives.

The first part of the study focuses on the theoretical analysis of the non-performing loans and their determinants as well. Based on the growing literature, both macroeconomic and bank-specific determinants affect the NPLs rate. Additionally, the European Union in order to deal with the instability of the banking sector tried to develop a comprehensive framework. In particular, the European Central Bank (ECB) and the National Competent Authorities (NCA) demonstrated the "Asset Quality Review" (AQR) which contains the methodology for valuating bank's assets from a prudential perspective. Further, the European Systemic Risk Board (ESRB) provides a guidance to the European banking sector for resolving the high stock of these NPLs.

Taking the above approach into account, the next step of the analysis was to forecast the stock of non-performing loans in Greece with two models. The first model is based on the NPLs time series using the ARIMA methodology. The second model includes the macroeconomic effect according to the VAR forecast perspective. The evidence of the empirical models is that the short-run forecasting through the VAR model leads to more reliable results when comparing the values with the out of sample rate of the Bank of Greece for the next period.

The final parts of the study consist of the credit risk theory, the Basel's regulatory framework and finally the empirical evidences concerning the z-score credit measure. Contrary to the previous models, a panel approach is used in order to analyze the impacts of z-score. Besides the z-score metrics, bank specific determinants were considered such as Return on equity, bank's size and Tier 1 ratio were employed as well as the determinant of government debt. The results support the literature of non-performing loans. More specifically, the aforementioned variables are negatively associated with the NPLs ratio except of government debt which is positively linked to the rise of NPLs. Finally, the z-score analysis underlines the importance to maintain a low level of NPLS in order for banks to be credit risk sustainable.

5.2. Proposals for further research

This study presents several limitations related to datasets, time period and econometric techniques.

Regarding data limitations there is not an easy way to deal with. An effort was made using the panel approach. We could include more countries to increase the number of observations.

Regarding the techniques, the following could be proposed. The first approach of the research could be to include bank specific determinants to forecast the evolution of non-performing loans. In our research we took only macroeconomic determinants into account for the VAR model from the period of 2002 to 2019, due to the lack of data for the Greek banking industry. Additionally, we could propose two more aspects. The first includes the analysis of the long-run equilibrium relationships suggested by the economic theory through the Vector Error Correction forecasting models. Moreover, we could compare the long-run equilibrium relationships with the short run effect of our VAR analysis. A second proposal could be the addition of explanatory variables lags of non-performing loans in the panel analysis and the use of Generalized Method of Moments (GMM), which is a methodology that accounts for serious econometric problems such as the endogeneity issues but requires additional restrictions.

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