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Hyperbolic Discounting: Theory and Applications

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Abstract

Human behavior has been getting a lot of recognition in the economics world lately; and assumptions about economic agents are taking a different turn. The desire to accumulate encourages mankind to put weight on the future; however, impatience, desire for instant gratification, and bounded rationality push him to assign a higher weight to the present. Observed behavior of economic agents have shown that the exponential discount function adopted in standard economic models suffers from two shortcomings: it implies time consistent preferences and assumes a constant discount factor. Since discounting behavior is important in determining individual inter-temporal choice, deviating from the standard geometric discounting has many implications in growth theory and welfare analysis. This paper discusses a new approach to discounting, the hyperbolic discount function, its applications and implications, and reveals the importance of adopting new methods of optimization that are more consistent with human behavior.

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Chapter 1

Introduction

"There is no man perhaps, to whom a good enjoyed today,would not seem of very different importance, from one exactly similar to be enjoyed twelve years hence,even though the arrival of both were equally certain...Everywhere we see that to spend is easy, to spare,hard." (Rae[1834] 1905, p.54)

Since the inception of time, mankind has been put on crossroads in every aspect of his life. At every crossroad, a decision is to be made, and every decision involves weighing trade-offs, most of which are trade-offs between the present and future. In todays' world, our choices have become steered by many temptations, that we find ourselves unable to stick to diets, follow an exercise program, or save enough for tomorrow. We make decisions today ignoring the fact that "the only constant in life is change", and that tomorrow's self will asses things in a way inconsistent with that of yesterday's.

Inter-temporal choices have a significant impact on multiple aspects of our lives, from health, wealth, to overall happiness. Adam Smith pointed out that these individual decisions are determinants of the economic prosperity of nations. John Rae studied the psychological and sociological determinants of these inter-temporal choices. According to Rae, "the effective desire of accumulation" is responsible for the different levels of savings and investment among societies. Rae described inter-temporal choices as "a joint product of conflicting psychological motives" which, depending on the situation, either support or curb the effective desire for accumulation (Frederick, Loewenstein and O'Donoghue 2002). Both the bequest motive and the tendency for self restraint support the desire for accumulation. However, uncertainty of human existence is a limiting factor. Rae observed that those living in safe places where climates are suitable for human life tend to be more cautious than those living in unhealthy and unsafe occupations. Another factor limiting the effective desire for accumulation is the inclination to avoid delayed gratification and the thrill from immediate consumption.

Jevons (1888) believed individuals favor immediate utility, and delay of gratification will only happen if it produces an "anticipal" utility that compensates the forgone utility from current consumption. Senior(1836) introduced the abstinence perspective which explained variations in inter-temporal choices on the basis of differences in psychological discomfort related to self denial: *"To abstain from the enjoyment which is in our power, or to seek distant rather than immediate results, are among the most painful exertions of the human will"*(Senior 1836:60). Eugen von Böhm-Bawerk(1889), added to the psychological factors mentioned above. He recognized that individuals have limited ability and vision to asses future well being and have the tendency to underestimate future wants. Nowadays, this limitation is refered to as "bounded rationality".

Such time preferences have been discussed from many angles in an attempt to understand what steers these inclinations. Fisher(1930) argued that such tendency to favor the present reflects a lack of self control; he stressed the importance of "fashion" in determining time preference and believed it to be "of vast importance ... in its influence both on the rate of interest and on the distribution of wealth itself." This phenomena is what drives some to save to become wealthy and the rich "to live in an ostentatious manner. (Fisher 1930:87)

Hence, in the early beginnings of the 20th century, "time preference" was viewed as a combination of various intertemporal motives. The discount utility model fuses these motives into the discount rate; as we will see in the next chapters, time discounting has crucial implications on intertemporal behavior and its effects. This paper is organized as follows; in the Introduction, we discussed several psychological motives behind inter-temporal choices. Chapter 2 describes exponential discounting and its limitations, introducing hyperbolic discounting along with some examples. Chapter 3 focuses on three important applications to hyperbolic discounting: 1) applications on growth; 2) on consumption and commitment; 3)and on stochastic games. The final chapter concludes.

Chapter 2

Beyond Exponential Discounting

It is common consensus that humans are impatient, a trait reflected through intertemporal behavior; small current rewards are favored over later but higher ones, and immediate costs are always avoided. Those tendencies are referred to as "present-biased preferences". When two future moments are assessed, present-biased preferences assign a relatively higher weight to the earlier moment the closer it is in time. Economists have captured this impatience by assuming that individuals discount their utiliy exponentially. Evidence has shown that those preferences are prone to change with passage of time and what seemed optimal in the present is discarded when the future arrives.(O'Donoghue, Rabin 1999)

The process of assigning weights to future and present choices is embodied by the discount factor and the rate of time preference. Frederick, Loewenstein and O'Donoghue (2002) distinguish between the two conflicting tendencies. Time preference refers to the individual's tendency to favor current to future utility(impatience). While time discounting reflects caring less about any future utility or any factors that might influence this utility.

A first application of discounting was the integration of inter-temporal welfare in Ramsey's (1928) optimal growth model. Ramsey, from an ethical standpoint, argued against discounting of the future welfare (Andrle, Bruha 2003). He prefered to use a zero rate of time preference. In a normative context, he argues that "we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible." Barro(1999)

In 1937, Samuelson's discounted utility model combined all psychological aspects to discounting into a single parameter, the discount rate. This discount rate was modeled as a constant, exogenous parameter that did not capture the psychological motives behind inter-temporal utility maximization. However, it became entrenched as the dominant theoretical framework for modeling inter-temporal choice, which was due mainly to its simplicity and familiar resemblance to the compound interest formula, and not as a result of empirical research proving its validity.

The standard discounted utility model (in discrete time) is of the following form:

$$U(c_t) = \sum_{k=0}^{T-t} D(k)u(c_{t+k})$$

where $U(.)$ is a time separable function and an individual's lifetime utility discounted to time t, $u(c_{t+k})$ for $k \in 0, 1, 2, ...$, the instantaneous utility or the felicity function, and $D(k)$, the discount function of the following exponential form:

$$D(k) = \left(\frac{1}{1+\rho}\right)^k$$

such that ρ is the constant discount rate.

Nowadays, the exponential discount function is used as an arbitrary approach to the intertemporal valuation mainly because of its analytical convenience and the fact that it expresses time consistent preferences. It is adopted by a number of mainstream economists and is widely used in macroeconomic textbooks. Time consistent intertemporal preferences imply that, for any two consumption profiles $(c_1, c_2, ..., c_T)$ and $(c'_1, c'_2, ..., c'_T)$, $U(c_1, c_2, ..., c_T) \geq U(c'_1, c'_2, ..., c'_T)$ if and only if $U_j(c_j...c_T) \geq U_j(c'_j..., c'_T)$ for $j = 2...T$, for $c_1 \geq c'_1$.

To demonstrate time consistency, we consider the following simple maximization problem with a log utility function:

$$\begin{cases} max \sum_{k=0}^{T-t} \delta^k log c_{t+k} \\ \ subject \ \ to \\ a_{t+1} = Ra_t - c_{t+1} \\ c_t \geq 0 \\ a_t \geq 0 \\ a_0 \ given \end{cases}$$

where:

1. a_t is a state variable (such as capital)
2. R is the growth rate of the state variable
3. δ^t is the discount function
4. a_0 is the endowment at time 0

First order conditions imply:

$$c_{t+1} = R\delta.c_t$$

An optimum consumption plan has to satisfy the following condition: $\sum_{t=1}^{n} \frac{c_t}{R^t} = a_0$

Results from budget constraint and first order conditions:

$$\begin{cases} a_t = R\delta a_{t-1} = (R\delta)^t a_0 \\ c_t = R(1-\delta)a_{t-1} \quad \forall t \geq 1 \end{cases}$$

In each period, the individual will consume $R(1-\delta)$ of the endowment. He will stick to his decision taken at time $t = 0$ in any future time $t \geq 1$. If we repeat the maximization problem for $t' \geq 0$ the results are consistent.

No alternatives to the exponential discount function have been discussed explicitly; however, this approach is potentially problematic as evident in several researches in psychology and behavioral economics (Strulik 2014). Irving Fisher's economic theory of intertemporal choice states that in perfect capital markets, firms and individuals borrow or lend until their marginal rate of substitution between today and tomorrow's consumption is equal to the interest rate. Economists however have been skeptical about whether consumers act as theory predicts. Böhm-Bawerk(1889) and Strotz(1956) have both speculated that people act as if their discount rates vary with the length of time to be waited.

Research in behavioral economics suggests that time preference rates are declining over time, possibly in a hyperbolic manner, and "for a given delay, discount rates vary across different types of intertemporal choices: gains are discounted more than losses, small amounts more than large amounts, and explicit sequences of multiple outcomes are discounted differently than outcomes considered singly"(Frederick, Loewenstein and O'Donoghue 2002).

Strotz(1956) followed by many economists, proposed alternatives to exponential discount functions. Not one hyperbolic functional form was adopted; several functions in the family of generalized hyperbolas were proposed. Such functions impose declining discount rates. George Ainslie (1975) suggested the form $D(t) = 1/t$.

Richard Herrnstein (1981) and James Mazur (1987) suggested $D(t) = 1/(1 + \alpha t)$.
George Loewenstein and Drazen Prelec (1992) suggested $D(t) = 1/(1 + \alpha t)^{\beta/\alpha}$.

The hyperbolic discount function is consistent with human behavior; however, it lacks the mathematical elegance of exponential discounting. An alternative way of discounting is the Quasi-hyperbolic discount function(QHD) analyzed mostly by Laibson (1997) in the theory of consumption functions and Phelps and Pollack (1968) in the intergenerational altruism problem. QHD features qualitative properties of the hyperbolic function (i.e. a declining discount rate in time). This form captures the most basic idea of present-biased preferences with a simple two-parameter model (also referred to as the "β, δ" form) that modifies exponential discounting.

QHD is given by the following functional form :

$$D(k) = \begin{cases} 1 \quad \text{if } k = 0 \\ \beta\delta^k \quad \text{if } k \geq 1 \end{cases}$$

where $0 \leq \beta \leq 1$ and $0 \leq \delta \leq 1$ In this model, δ represents long-run, time consistent discounting. β on the other hand, represents a "bias for the present".

Strotz(1956) discussed how the individual's future behavior will be inconsistent with the optimal plan of the present. If a person were free to reconsider his plan in the future, he will deviate from the path previously perceived optimal, and this behavior will be repeated in every consecutive period. This inconsistency stems from many roots; delayed rewards seem abstract and therefore the individual may not be able to evaluate their full impact in advance. To demonstrate time inconsistency, we consider the following optimization problem:

$$
\begin{cases}
max c_1 + \beta \sum_{k=1}^{T-t} \delta^k log c_{t+k} \\
\ subject \ \ to \\
a_{t+1} = Ra_t - c_t \\
c_t \geq 0 \\
a_t \geq 0 \\
a_1 \ given
\end{cases}
$$

where:

1. a_t is a state variable (such as capital)
2. R is the growth rate of the state variable
3. δ^t is the discount function
4. a_1 is the beginning endowment
5. β represents present bias

Results:

$$
\begin{cases}
c_1 = \frac{R(1-\delta)}{\beta} a_0 \\
c_t = R(1-\delta) a_t \quad \forall t \geq 1
\end{cases}
$$

The individual in this scenario consumes a fraction $\frac{R(1-\delta)}{\beta}$ in the current period and plans to consume a fraction of $R(1-\delta)$ in all future periods. To check if his decision is consistent we resolve this problem at time $t = 1$.
We obtain $c_1 = \frac{R(1-\delta)}{\beta} a_0$ while initially at time $t = 0$ he has chosen $c_1 = R(1-\delta)a_0$. That is when time $t = 1$ arrives, he changes the previous plan and decides to consume more. From

the point of view of time 0, this change represents a deviation from the optimal consumption plan, an act that reduces the total expected utility computed at time 0.

When consumers present preference reversals, and cannot pre-commit their future selves to a plan of action, literature defines two type of agents: naive and sophisticated. Naive consumers erroneously believe that their future selves will abide by the present consumption plan, and need to revise their plans at any instant in time. While sophisticated agents, aware of their self control problem, know that their future selves are less patient than currently perceived (Cabo, Martin-Herran, Martinez-Garcia 2015).

Are people sophisticated or naive? Committing to a retirement plan, getting a yearly subscription at a gym, or joining Christmas clubs demonstrates a degree of sophistication. Only sophisticated individuals are aware that tomorrow's self is unpredictable. Despite the fact that sophistication exists, it appears that humans underestimate "the degree to which their future behavior will not match their current preferences over future behavior." (O'Donoghue, Rabin 1999).

People often complain about their lack of will power and self control, and this awareness demonstrates what some refer to as "partial naivete", or as Thaler(2015) puts it:
"I share a view...that the truth is somewhere between the two extremes: partial niavete. Most of us realize that we have self-control problems, but we underestimate their severity. We are naive about our level of sophistication"

This realization has led many individuals to sign up for commitment plans that ensure they don't deviate from their initially set plans easily. Strotz(1956) proposed two strategies an individual, aware of his self control problem can follow, either commit to a plan of action or adopt a strategy of consistent planning ignoring plans he knows his future self won't follow. In the next chapter, among the applications discussed is an example on a commitment strategy proposed by Laibson, which demonstrates how individuals can curb their need for immediate gratification by investing a portion of their wealth in illiquid assets.

Chapter 3

Applications to Hyperbolic Discounting

Hyperbolic discounting is observed in everyday life, from simple choices like quitting smoking tomorrow versus now, to complex ones like saving for retirement against enjoying consumption today. Applications to hyperbolic discounting have changed the way things were done. Previously, people were assumed rational decision makers who could foresee the future and whose later selves consistently followed the plans previously set. This myopic behavior led many economists to apply hyperbolic discounting to many areas that were handled differently; those of which assumed rational behavior and no preference reversal. The following are some of many applications to hyperbolic discounting. We discuss two applications on growth using non constant discounting. The third application is a simple version of Laibson's prominent Golden Eggs model which tackles the issue of commitment strategies to curb inconsistent behavior. The fourth and last topic is Ronald Peeters' introduction of hyperbolic discounting to Markov games and testing for optimal payoffs.

3.1 Neoclassical Growth Under Hyperbolic Discounting

There has been a recent growing interest in the effect on economic growth of moving from exponential to non-constant discounting. Many of the basic frameworks in macroeconomics including the neoclassical growth model assume that households have a constant rate of time preference. Barro(1999) is the first to deal with the question for a neoclassical growth model. He modified the neoclassical growth model to include a variable rate of time preference by integrating Laibson style preferences with Ramsey's growth model.

The model:

$$U(\tau) = \int_{\tau}^{\infty} u[c(t)].e^{-[\rho(t-\tau)+\phi(t-\tau)]}dt, \tag{3.1.1}$$

where $U(.)$ is the individual's lifetime utility discounted to time τ, $u(c_t)$ is the instantaneous utility, τ is the current date, and time discounting for period t depends only on the distance in time $t-\tau$, from the current. $\phi(t-\tau) \geq 0$ is the function featuring time preference and $e^{-\rho(t-\tau)}$ is the standard exponential function with $\rho > 0$ the constant rate of time preference.

Assumptions:

1. $u'(c) > 0$ and $u"(c) < 0$

2. $\phi(0) = 0$

3. $\phi(.)$ is continuous and twice differentiable

4. $\phi'(v) \geq 0$, $\phi"(0) \leq 0$, and $\phi'(v)$ approaches zero as v goes to infinity (Laibson 1997a)

5. No commitment ability; the household can't commit to lowering $c(\tau)$ at time τ and increasing $c(t)$ at some future dates.

The full solution for log utility, $u(c) = log(c)$:

Consumer chooses $c(t)$ at time τ as the constant flow $c(\tau)$ over the short discrete interval $[\tau, \tau + \epsilon]$ where ϵ will eventually tend to zero and thereby results will be generated in continuous time.

The full integral can be broken down into two chunks[1]:

$$U(\tau) = \int_{\tau}^{\tau+\epsilon} u[c(t)].e^{-[\rho(t-\tau)+\phi(t-\tau)]}dt + \int_{\tau+\epsilon}^{\infty} u[c(t)].e^{-[\rho(t-\tau)+\phi(t-\tau)]}dt \tag{3.1.2}$$

$$\approx \epsilon.log[c(\tau)] + \int_{\tau+\epsilon}^{\infty} u[c(t)].e^{-[\rho(t-\tau)+\phi(t-\tau)]}dt \tag{3.1.3}$$

The neoclassical production function : $y = f(k)$, where y is output and k is capital per worker with $f'(k) > 0$ and $f'(k) < 0$. Also population is assumed constant and technological progress nil. The economy is closed and the consumer picks how much to consume and save at time τ; this choice influences consumption at a later date t by affecting the assets at that later date, call it $\tau + \epsilon$. Choosing optimal $c(\tau)$ requires addressing two problems:

1. how consumption at time τ affects assets at time $\tau + \epsilon$ and
2. how assets affect consumption for later date t where $t > \tau + \epsilon$.

The household budget constraint solves the first problem:

$$\frac{dk}{dt} = r(t).k(t) + w(t) - c(t), \tag{3.1.4}$$

[1]the approximation comes from taking $e^{-[\rho(t-\tau)+\phi(t-\tau)]}$ as equal to unity over the interval $[\tau, \tau + \epsilon]$

For a given starting stock of assets $k(\tau)$, the stock of assets at time $\tau+\epsilon$ is determined by:[2]

$$k(\tau+\epsilon) \approx k(\tau).[1+\epsilon.r(\tau)] + \epsilon.w(\tau) - \epsilon.c(\tau) \tag{3.1.5}$$

this implies:

$$d[k(\tau+\epsilon)/d[c(\tau)]] \approx -\epsilon \tag{3.1.6}$$

More consumption today implies less assets later.

To solve the second problem that relates $k(\tau+\epsilon)$ and $c(t)$ for $t > \tau+\epsilon$, i.e. the propensity to consume, it is known that in the standard Ramsey model with log utility, $c(t)$ is a constant fraction ρ of wealth (present value of wages plus $k(t)$). ρ is constant simply because under log utility, income and substitution effects related to future interest rates cancel each other. Barro assumes they still cancel eachother under log utility and variable time preference with no commitment; however, the new constant of proportionality denoted by λ need not be equal to ρ.

We have the following equation for consumption:

$$c(t) = \lambda.[k(t) + \textit{present value of wages}] \tag{3.1.7}$$

where $t > \tau+\epsilon$ and some constant $\lambda > 0$.

[2] approximation comes from ignoring compounding over the interval $(\tau, \tau+\epsilon)$ and treating the variables $r(t)$ and $w(t)$ as constants

It is assumed that consumpton grows at the rate $r(t) - \lambda$ for $t > \tau + \epsilon$. This implies that consumption can be determined from the following :

$$log[c(t)] = log[c(\tau + \epsilon)] + \int_{\tau+\epsilon}^{t} r(v)dv - \lambda.(t - \tau - \epsilon) \tag{3.1.8}$$

Upon substituting, the utility can be written as :

$$U(\tau) \approx \epsilon.log[c(\tau)] + log[c(\tau + \epsilon)].\int_{\tau+\epsilon}^{\infty} e^{-[\rho.(t-\tau)+\phi(t-\tau)]}dt \tag{3.1.9}$$
$$+terms\ that\ are\ independent\ of\ c(t)\ path$$

Defining the integral

$$\Omega \equiv \int_{0}^{\infty} e^{-[\rho v+\phi(v)]}dv, \tag{3.1.10}$$

which a constant expression that corresponds to the integral in equation (3.1.9) as ϵ goes to zero.

The marginal effect of $c(\tau)$ on $U(\tau)$:

$$\frac{d[U(\tau)]}{d[c(\tau)]} \approx \frac{\epsilon}{c(\tau)} + \frac{\Omega}{c(\tau + \epsilon)}.\underbrace{\frac{d[c(\tau + \epsilon)]}{d[k(\tau + \epsilon)]}}_{\lambda}.\underbrace{\frac{d[k(\tau + \epsilon)]}{dc(\tau)}}_{-\epsilon} \tag{3.1.11}$$

setting $d[U(\tau)]/d[c(\tau)]$ to zero implies:

$$c(\tau) = [c(\tau + \epsilon)]/\Omega\lambda \tag{3.1.12}$$

If the solution is correct, then $c(\tau+\epsilon)$ approaches $c(\tau)$ as ϵ tends to zero. Otherwise, $c(t)$ exhibits jumps at all points in time and the expected answer will be wrong. This happens for a unique value of λ:

$$\lambda = \frac{1}{\Omega} = \frac{1}{\int_0^\infty e^{-[\rho v+\phi(v)]dv}} \tag{3.1.13}$$

Inspection revealed that $\lambda = \rho$ in the standard Ramsey model in which $\phi(v) = 0$ for all v. Rewriting the above equation as

$$\lambda = \frac{\int_0^\infty e^{-[\rho v+\phi(v)]}.[\rho + \phi'(v)]dv}{\int_0^\infty e^{-[\rho v+\phi(v)]}dv} \tag{3.1.14}$$

is helpful since it shows that λ is a time invariant weighted average of the instantaneous rates of time preference, $\rho + \phi'(v)$. Since $\phi'(v) \geq 0$, $\phi"(0) \leq 0$, and $\phi'(v)$ approaches zero as v goes to infinity, it follows that:

$$\rho \leq \lambda \leq \rho + \phi'(0) \tag{3.1.15}$$

λ is intermediate between long-run rate of time preference ρ and the short run instantaneous rate $\rho + \phi'(0)$.

The effective rate of time preference λ is constant, this implies that the model with log utility and no commitment is observationally equivalent to the conventional neoclassical growth model.

Concluding remarks:

Allowing for variable rates of time preference doesn't change the basic properties of the neoclassical growth model. Results show that consumption is dependent on an effective rate of time preference, which is a weighted average of future instantaneous rates. The modified model is observationally equivalent to the standard one. This variable rate of time preference has implications on savings and growth as well as welfare outcomes. Barro discusses the possibility of commitment strategies and their implications on the propensity to save. He observed that economies with a greater capacity to commit future consumption have lower effective rates of time preference on the long run and therefore higher steady state levels of saving and capital accumulation. However, the short term effects can be counter effective; saving and capital accumulation go in opposite direction from commitment technology improvements.

3.2 Endogeneous Growth Under Hyperbolic Discounting

Strulik(2014) investigates time-inconsistent savings plans in the context of endogenous growth. He has analyzed AK endogenous growth models using a controlled experiment under the assumption of an identical overall impatience. The controlled experiment concluded same growth rate under both discounting methods(strong equivalence) Strulik provides an analytical solution to the standard model of endogenous growth when consumers are present biased and naive.

In his paper, he shows that the rate of economic growth in the standard endogenous growth model is invariant to the non constant discounting given a restriction: the constant stream should provide the same present value under both hyperbolic and exponential discounting. Controlling for the level of impatience helps focus on the declining time preference and time inconsistency. Result shows that time inconsistent saving plans do not affect economic growth.

Deriving the results

Households.

The economy is populated by identical (present biased) households who supply one unit of labor, and maximize their lifetime (log) utility given a budget constraint:

$$U(\tau) = \int_{\tau}^{\infty} log[c(t)].D(t,t_0)dt, \tag{3.2.1}$$

Subject to the budget constraint

$$\dot{k}(t) = rk(t) + w(t) - c(t) \tag{3.2.2}$$

where $c(t)$ and $k(t)$ denote consumption and capital and $w(t)$ and $r(t)$ denote wage and interest rate, and the hyperbolic discount function:

$$D(t,t_0) = \frac{1}{[1+\rho_0\beta(t-t_0)]^{1/\beta}}, \tag{3.2.3}$$

where β controls the present bias and ρ_0 controls the instantaneous discount rate of the next instant in time.

Assumptions:

$\beta < 1$ for integral to be bounded. Strulik prefers to use this form of hyperbolic discount function because it contains the exponential discounting as a limiting case.

The associated Hamiltonian:

$$H(t,t_0) = log[c(t)].D(t,t_0) + \lambda(t)[rk(t) + w(t) - c(t)] \tag{3.2.4}$$

First order conditions:

$$\frac{D(t,t_0)}{c(t)} - \lambda(t) = 0 \tag{3.2.5}$$

$$\lambda(t)r = -\dot{\lambda}(t) \tag{3.2.6}$$

The transversality condition: $lim_{t\to\infty}\lambda(t)k(t) = 0$

which implies:

$$c(t) = \frac{D(t,t_0)}{\lambda(t_0)}\; e^{r(t-t_0)} \tag{3.2.7}$$

substituting for $c(t)$ in the budget constraint and solving the differential equation for capital stock at time T gives:

$$k(T) = k(t_0)e^{r(T-t_0)} + \int_{t_0}^{T} w(\tau)e^{r(T-\tau)}d\tau - \frac{1}{\lambda(t_0)}\int_{t_0}^{T} D(\tau,t_0)e^{r(T-t_0)}d\tau \tag{3.2.8}$$

Result[3]:

$$c(t) = \frac{k(t_0) + \int_{t_0}^{\infty} w(\tau)e^{-r(\tau-t_0)}d\tau}{\int_{t_0}^{\infty} D(\tau,t_0)d\tau} D(t,t_0)e^{r(\tau-t_0)} \implies c(t) = \frac{k(t) + \int_{t}^{\infty} w(\tau)e^{-r(\tau-t)}d\tau}{\int_{t}^{\infty} D(\tau,t)d\tau} \tag{3.2.9}$$

[3] Divide equation (3.2.8) by $e^{r(T-t_0)}$, take limit as $T \to \infty$ and insert transversality condition, solve obtained equation for λ_0 and then insert into equation (3.2.7)

The naive consumer believes that his future selves will stick to the initial consumption plan but in fact he only sticks to the plan only at the instant it was made. As time proceeds this plan is constantly revised. Setting $t_0 = t$ provides equation (3.2.9).

Firms.

Following Romer(1986), firm i uses capital input $k(i,t)$ and labor input $l(i,t)$ to produce output $y(i,t) = \tilde{A}(t)k(i,t)^{\alpha}l(i,t)^{1-\alpha}$. Under perfect competition, the factor prices are given by $r(t) = \alpha\tilde{A}(t)k(i,t)^{\alpha-1}l(i,t)^{1-\alpha}$ and $w(t) = (1-\alpha)\tilde{A}(t)k(i,t)^{\alpha}l(i,t)^{-\alpha}$

As in Romer(1986) there is a learning by doing such that the technology available to any firm is a positive function $y(t) = Ak(t)$, wages $w(t) = (1-\alpha)Ak(t)$ and interest rate $r(t) = \alpha A$.

Economic Growth

Inserting wages and the discount factor into the above consumption equation:

$$c(t) = \rho_0(1-\beta)\left[k(t) + (1-\alpha)A\int_t^{\infty} k(\tau)e^{-}r(\tau-t)d\tau\right] \tag{3.2.10}$$

Suppose capital grows at a constant rate $g_k < r$, then consumption simplifies to a linear function of the capital stock:

$$c(t) = a.k(t), \quad a \equiv \rho_0(1-\beta)\left[1 + \frac{(1-\alpha)A}{\alpha A - g_k}\right] \tag{3.2.11}$$

Now the equation of motion can be written as :

$$\dot{k}/k = g_k = A - \alpha \tag{3.2.12}$$

solving for $a = (1-\alpha)A + \rho_0(1-\beta)$, we obtain

$$c(t) = [(1-\alpha)A + \rho_0(1-\beta)]k(t) \tag{3.2.13}$$

$$g_k = \alpha A - \rho_0(1-\beta) \tag{3.2.14}$$

The solution confirms the initial assumption of the existence of a constant growth rate. The rate of economic growth is declining in the initial time preference rate ρ_0 and increasing in the speed of declining impatience β. The fact that growth is constant and that consumption is a fraction of capital makes the model observationally equivalent to the standard endogenous growth model with exponential discounting. This confirms for the endogenous growth case the result found by Barro in the context of neoclassical growth and quasi hyperbolic discounting.

Comparing growth under hyperbolic and exponential discounting

The condition that provided the above equivalence actually equates overall impatience under the two discounting methods, a restriction that helps pinpoint effects of inconsistent behavior. To demonstrate that inconsistency of behavior and hyperbolic discounting are not harmful for growth, the following comparison is held.

Under exponential discounting, we have that $D(t, t_0) = e^{-\bar{\rho}t}$, and first order conditions lead to the Ramsey rule $\dot{c}/c = r - \bar{\rho}$. In the context of endogenous growth it simplifies to $\dot{c}/c = \alpha A - \bar{\rho}$. If we suppose consumption grows at a constant rate of capital, $c = bk$, it implies $g_k = A - b$, then we obtain the solution $b = (1-\alpha)A + \bar{\rho}$, that is

$$c(t) = [(1-\alpha)A + \bar{\rho}]k(t), \tag{3.2.15}$$

$$g_k = \alpha A - \bar{\rho} \tag{3.2.16}$$

Equations (3.2.14) and (3.2.16) confirm structural similarity between the two discounting methods. Strulik proposes that long growth is higher (lower) under exponential discounting for $\bar{\rho} < (1-\beta)\rho_0$ (for $\bar{\rho} > (1-\beta)\rho_0$) To check for the possibility of equal growth rates under both discounting mehtod, Strulik imposes the condition that both methods generate the same present value of the infinite flow of utilities, that is,

$$\int_0^\infty e^{-\bar{\rho}t} dt \equiv \int_0^\infty (1+\rho_0\beta t)^{-1/\beta} dt \tag{3.2.17}$$

This restriction holds for $\bar{\rho} = \rho_0(1-\beta)$; under this equality, and along with equations (3.2.14) and (3.2.16) it concludes that equivalent present values lead to the same rate of economic growth for both hyperbolic and exponential discounting.

3.3 David Laibson and the Golden Eggs model

David Laibson has dedicated a significant amount of work to inter-temporal behavior, mainly dealing with consumption decisions and self control problems. Based on several psychological experiments, he considers the possibility that people may be particularly impatient when faced with intertemporal decisions. He shows that under this assumption, consumption would be more sensitive to movements in income than the permanent income hypothesis predicts. This can be seen in many empirical observations. For example, some individuals borrow excessively using credit cards with high interest rates; others have difficulty saving even when there are increases in income.

One of his prominent contributions is the "golden eggs" model, where commitment devices are introduced to control for the time-inconsistent consumer behavior. Laibson sees precommitment as a technique followed by those who suffer from lack of self control. Those individuals find it impossible to stick to diets, commit to exercise, follow savings plans, and meet deadlines. Precommitment instruments such as investing in illiquid assets have the property of the goose that laid the golden eggs. These assets promise to generate benefits on the long run; however, those benefits are almost impossible to realize immediately and early attempts to do so will result in capital loss.Laibson(1997)

A simple version of the model:

The individual invests in two kinds of assets: a liquid asset x, and an illiquid asset z. Any sale of the illiquid asset z has to be made one period before receiving the actual earnings. This model assumes the same rate of return for both assets x and z; the consumer begins with exogeneous endowments $x_0, z_0 \geq 0$ and the consumption/savings decision is in discrete time $t \in 1, 2, ..., T$.

Each time t is divided into four subperiods:

Subperiod 1: Production takes place; both assets produce a gross return of $R_t = 1 + r_t$ (from both x_{t-1} and z_{t-1})

Subperiod 2: Consumer receives labor income y_t, and has access to the liquid savings $R_t.x_{t-1}$

Subperiod 3: Consumer chooses consumption such that $c_t \leq y_t + R_t.x_{t-1}$

Subperiod 4: Consumer chooses new asset allocation x_t and z_t subject to $y_t + R_t(z_{t-1} + x_{t-1}) - c_t = z_t + x_t$ where $x_t, z_t \geq 0$

Consumers have a time-additive utility function U_t and the discount function is the same as Phelps' and Pollak's(1967) in their model of intergenerational altruism:

$$U_t = E_t \left\{ u(c_t) + \beta \sum_{\tau=1}^{T-\tau} \delta^{\tau} u(c_{t+\tau}) \right\}$$

This individual has time inconsistent preferences if $\beta \leq 1$; $\beta = 1$ results in the exponential discount rate case where the individual has consistent preferences. We consider the following maximization problem that captures the idea of the golden eggs model with log utility. The individual is aware of her inconsistent preferences and can only consume asset x, that is why she turns to a commitment strategy where she divides her current endowment in liquid and illiquid assets.

She is faced with the following optimization problem:

$$max\ c_1 + \beta \sum_{t=1}^{\infty} \delta^t log c_t$$

subject to

$$\begin{cases} x_{t+1} + z_{t+1} = R(x_t + z_t) - c_t \\ 0 \leq c_t \leq R x_t \\ x_t, z_t \geq 0 \\ x_0, z_0 \ given \end{cases}$$

First order conditions imply:

$$\begin{cases} c_2 = R\beta\delta c_1 \\ c_t = R\delta c_{t-1}, \text{ for } t \geq 3 \end{cases}$$

Using the relation $\sum_{t=1}^{\infty} \frac{c_t}{R^t} = y_0$, where $y_t \equiv x_t + z_t$ is the income, the solution is:

$$\begin{cases} c_1 = \frac{R(1-\delta)}{\beta} y_0 \\ c_1 = R\beta\delta c_0 \\ c_t = R\delta c_{t-1} \forall\ t \geq 2 \end{cases}$$

which imply:

$$\begin{cases} c_1 = \frac{R(1-\delta)}{\beta} y_0 \\ c_t = R(1-\delta) y_t,\ \forall\ t \geq 1 \end{cases}$$

Strategy

This individual consumes a fraction of the endowment equal to $\frac{R(1-\delta)}{\beta}$ in the current period and plans to consume a fraction $R(1-\delta)$ in all future periods.

At time t the agent plans to consume $c_{t+1} = R(1-\delta)y_{t+1}$ but when the next period arrives, such that $t = t+1$, she will want to consume $c_{t+1} = \frac{R(1-\delta)}{\beta} y_{t+1}$.

In order to avoid this deviation, she will commit to the following investment strategy:

At time t she invests $(1-\delta)y_{t+1}$ in the liquid asset x and $\delta yt + 1$ in the illiquid asset z. So when period $t+1$ arrives, she can only consume a fraction $Ry_{t+1} = R(1-\delta)x_{t+1}$

At time $t = \bar{t}$ he plans to consume $\bar{c}_{\bar{t}+1}$ but he knows he will switch to $\hat{c}_{\bar{t}+1} > \bar{c}_{\bar{t}+1}$

At time $t = \bar{t}$ she invests a quantity $\bar{c}_{\bar{t}+1}/R$ of her endowment in the liquid asset $x_{\bar{t}+1}$ and the remaining in the illiquid asset $z_{\bar{t}+1}$.
Therefore, in period $t = \bar{t}+1$, she wants to consume $\hat{c}_{\bar{t}+1} > \bar{c}_{\bar{t}+1}$, but she cannot because $x_{t+1} = \bar{c}_{\bar{t}+1}/R$ and $c_{\bar{t}+1} \leq Rx_{\bar{t}+1}$. The best she can do is to consume all the possible, that is $\bar{c}_{\bar{t}+1} = c_{\bar{t}+1}$.

There's more to the golden eggs model than the simple example discussed above. The model helps clear some ambiguities about consumer behavior; it predicts that consumption tracks income and explains why consumers have different propensities to consume out of labor income. The model also explains why Ricardian equivalence should not hold in economies of infinitely lived representative agents. Finally, the model suggests that financial innovation provides an overflow of liquidity which has negative implications on welfare (Laibson 1997).

3.4 Hyperbolic Discounting in Stochastic Games

Ronald Peeters (2004) introduces a hyperbolic reward system to stochastic games where optimal strategies in a markov decision problem differ according to degree of sophistication. Result shows the existence of a delayed stationary equilibrium corresponding to one type of individual and for general hyperbolic discounted stochastic games. Discounted stochastic games have been introduced by Shapley (1953) who considered two-person zero-sum finite stochastic games. Shapley proved that such games have a value and that both players possess optimal stationary strategies with respect to the discounted payoff criterion.

Peeters introduces an example of a stochastic game showing that an individual can suffer from the problem of self-control. Peeters distinguishes between 3 types of individuals, resolute, naive , and sophisticated. A resolute individual pre-commits to play a certain strategy and therefore never suffers, but pre-commitments lead to irrational behavior in later stages. A naive individual revises his strategy at any instant in time and therefore displays dynamic inconsistent behavior: in future stages he will not play the action as he planned to play at forehand. An individual who is sophisticated will realize eventual future self-control problems and therefore decides to play a dynamic consistent strategy.

Results show that resolute individuals have a delayed-stationary strategy that is optimal; that naive individuals have a delayed-stationary strategy that is optimal, but will after all play stationary; and that sophisticated individuals have an optimal dynamic consistent strategy that is stationary.

One player: Markov decision problem:
A stochastic game with only one player is a Markov decision problem and can be denoted by:

$$\Gamma = \left\langle \Omega, \{S_w\}_{w \in \Omega}, u, \delta, \beta \right\rangle$$

in which N denotes the finite set of players, Ω the finite set of states, and $\{S_w\}$ is the finite set of actions available for the player in state $\omega \in \Omega$. The instantaneous payoff is given by $u(\omega, s_\omega)$, and δ and β are both discount factors

A Markov decision problem is displayed in figure 1 where in the first state the individual can decide between the options T (top) and B (bottom). If T is chosen, the player receives an immediate reward of 1 and stays in the first state. If B is chosen, the player receives an immediate reward of 0 but moves the second state in the next period. Upon arriving to the second state, no choice is to be made; the player is simply rewarded by 3 and returns to the

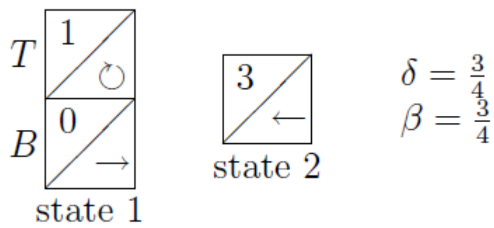

Figure 1: Markov decision problem.

first state in the subsequent period. Without any loss of generality it is assumed that the first state is the initial state.

The stream of expected payoffs is evaluated by:

$$U(\omega, \sigma) := U^0(\omega, \sigma) + \beta \sum_{k=1}^{\infty} \delta^k U^k(\omega^k, \sigma)$$

Below Peeters considers six possible strategies(believed to be sufficient for the analysis), with the corresponding payoffs. The table below shows the expected payoffs for two values of β, where $\beta = 1$ corresponds to the exponential discounting case.

Expected Payoffs		
Strategy	β=1	β=3/4
TTT.....	4	13/4 ≈ 3.25
BBB......	36/7 ≈ 5.1429	27/7 ≈ 3.8571
TBB...	34/7 ≈ 4.8571	109/28 ≈ 3.8929
BTT...	9/2 ≈ 4.5	27/8 ≈ 3.375
TBTB...	34/7 ≈ 4.8571	109/28 ≈ 3.8929
BTBT....	36/7 ≈ 5.1429	27/7 ≈ 3.8571

Results according to table:

TBB $\cdots$ and TBTB $\cdot\cdot$ are optimal strategies.

Any strategy in which action T is played in the initial state and where B is played when the first state is the active state is optimal.

It can also be shown that when the Markov decision problem is slightly perturbed such that the transition dynamics is no longer determinate, TBB $\cdots$ will be the unique optimal strategy.

In each moment in time the strategy TBB $\cdots$ is optimal and therefore the player plans to play TBB $\cdots$ every moment in time.

Optimal strategies corresponding to types of individuals

The resolute individual. The resolute individual would commit to playing the optimal strategy TBB $\cdots$ and will play action B in all periods except the initial one.

This implies that "for all hyperbolic discounted Markov decision problems with a resolute individual an optimal strategy exists in which the individual plays stationary from the first period on."[4]

The naive individual. This individual realizes in the second stage that playing T is optimal at any instance in time given that B will be played in all future periods. He will end up playing TTT.....Peeters concludes that "for all hyperbolic discounted Markov decision problems with a naive individual an optimal strategy such that the individual plays stationary exists."[5]

The sophisticated individual. A sophisticated individuals is aware of self control problems and realizes that he would deviate in any instance and therefore would play T stationary. Knowing that T will be played in all future events, the optimal action in the present stage

[4,5,6]Proof in Peeters, R. 2004."Stochastic Games with Hyperbolic Discounting". mimeo, Maastricht University.

is to play B again.

Another way to find optimal strategies in infinite horizon models is to extend the finite horizon model. Supposedly the game ends after stage K is reached, we go through backward induction(using Figure 2):

(1) In stage K, if the first state is the active state, playing T is the optimal action.

(2) In stage $K-1$, regardless of action chosen in stage K, playing B is the optimal action.

(3) Knowing that in the final two stages action B and subsequently action T will be chosen, it is optimal to play T in stage $K-2$, since $1+\frac{3}{4}.\frac{3}{4}(0+\frac{3}{4}.3)=\frac{145}{64}>\frac{135}{64}=0+\frac{3}{4}.\frac{3}{4}(3+\frac{3}{4}.1)$.

(4)Knowing that in the final three stages TBT will be played, action B is optimal in stage $K-3$.

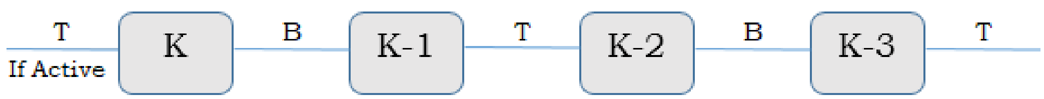

Figure 2: Otpimal Strategy by Backward Induction

The optimal dynamic consistent strategy in the finite horizon model is easily shown to be BTB...BT if K is odd(and the number of periods even) and TBT...BT if K is even.

The reasoning above shows that both TBTB $\cdot\cdot$ and BTBT $\cdot\cdot$ are optimal dynamic consistent strategies for the infinite horizon game. When the individual plans to play BTB $\cdot\cdot$ from tomorrow on, it is optimal to play action T today as $\frac{109}{28}>\frac{837}{224}=0+\frac{3}{4}.\frac{3}{4}(3+\frac{3}{4}.\frac{34}{7})$ and when individual intends to play TBT... from tomorrow on, it is optimal to play action B today as $\frac{27}{7}>\frac{209}{56}=1+\frac{3}{4}.\frac{3}{4}.\frac{34}{7}$.

Although both optimal strategies are not stationary, they are Markovian since they depend on time and not on history.

A stationary optimal dynamic consistent strategy using mixed actions

Suppose the individual plays T stationary with probability p, this implies the present value of the stream of payoffs from the second stage and further on is:

$$v(1,p) = p + \frac{3}{4}.p(v(1,p) + (1-p).v(2,p)) \text{ and } v(2,p) = 3 + \frac{3}{4}.v(1,p)$$

This implies

$$v(1,p) = \frac{36-20p}{7-3p} \text{ and } \; v(1,p) = \frac{48-24p}{7-3p}$$

The present values of payoffs when individual chosses T versus B from the first stage on:

$$U_T = 1 + \frac{3}{4}.\frac{3}{4}.v(1,p) = \frac{109-57p}{28-12p} \text{ and } \quad U_B = 0 + \frac{3}{4}.\frac{3}{4}.v(2,p) = \frac{108-54p}{28-12p}$$

The individual here is indifferent between playing either action in the first stage, and willing to play a mixed game; if we equate the two utilities, we obtain $p = \frac{1}{3}$. This implies that the stationary strategy $(\frac{1}{3}, \frac{2}{3})$ is an optimal dynamic consistent strategy. Therefore "for hyperbolic discounted Markov decision problems with a sophisticated individual there exists a stationary optimal dynamic consistent strategy."[7]

[7]Proof in Peeters, R. 2004."Stochastic Games with Hyperbolic Discounting". mimeo, Maastricht University.

Chapter 4

Conclusion

This paper discusses an alternative approach to exponential discounting, stressing how this new approach, though yielding results contradicting with those under the standard geometric discounting, reflects human behavior more realistically. Behavioral observations and empirical analyses have shown that humans are myopic and "predictably irrational". They make decisions today whose future selves don't abide by, leading to dynamic inconsistencies.

Hyperbolic discounting is introduced as the closest representation of a discount function which embodies human behavior. Some of many applications to hyperbolic discounting (and its simpler forms) are discussed here, elaborating the effect on growth and other areas of interest such as consumption/savings decisions. Results have shown the effect on economic growth of non constant discounting need to be taken into consideration. Also, commitment strategies were shown to play a much needed role to control for future inconsistent behavior. Stochastic games under hyperbolic discounting have the potential to predict cooperative behavior in teams, something highly significant in today's team driven world.

The economic agents are our families, friends, kids, etc., whose decisions combined, are the center of gravity of the economic system. Whether it's my son favoring a certain brand of cereal or my husband adopting "vaping" for his nicotine addiction, or my father committing to a retirement plan, these choices combined are factors affecting economic growth as well as society's welfare. Many areas including finance, marketing and others clearly depend on economic agents making decisions whose effects are ripples that create a propagating wave. An understanding of the importance of behavior will definitely create predictive scenarios more realistic than ever and with more satisfying results.

The need to incorporate psychology into economics is becoming apparent by the day, and economists are working hard to create better models of economic behavior. While in the process of putting together my thesis, the prominent economist Richard Thaler was awarded the Nobel prize in economics for his contribution to the world of behavioral economics. In his own words: *"In order to do good economics, you have to keep in mind that people are human."* Thaler dedicated years analysing human behavior and trying to figure out why individuals make decisions that contradict with textbook theories. His aim is to usefully employ those "behavioral" insights in order to improve social welfare and policy effectiveness.

More experiments need to be executed to better understand inter temporal decisions; this might lead to adjustments in many areas of economics. Hopefully this change will bring about strategies that better the welfare of societies now and in the future. This blend of psychology and economics will surely put forward alternative methods to cookie cutter solutions and propose policies that serve the growth of societies better. This endeavor might change the way things were implemented and revolutionize the world of economics.

References

[1] Ainslie, George. 1975. “Specious reward: A behavioral theory of impulsiveness and impulse control.” Psychological Bulletin, 82:4, pp. 463-96.

[2] Andrle, M.; Bruha, J. 2004.”Hyperbolic Discounting in Economic Modelling”. Politická Ekonomie. 2004, Vol. 52, No. 6, pp. 757–771.

[3] Barro, R., 1999.”Ramsey Meets Laibson in the Neoclassical Growth Model”. Quarterly Journal of Economics 114(4), pp. 1125-1152.

[4] Böhm-Bawerk, Eugen von. (1889), 1970.”Capital and interest”. South Holland: Libertarian Press.

[5] Caboa, F., G. Martín-Herrána and M.P. Martínez-García 2015. ”Non-constant discounting and Ak-type growth models”. Economics Letters, 131, pp. 54–8.

[6] Fisher, Irving. 1930. ”The theory of interest”. New York: MacMillan.

[7] Frederick, S., Loewenstein, G.,and O'Donoghue, T. 2002. ”Time discounting and time preference: a critical review”. Journal of Economic Literature 40, pp. 351-401.

[8] Herrnstein, Richard. 1981. “Self-control as response strength,” in Quantification of steady-state operant behavior. Christopher M. Bradshaw, Elmer Szabadi and C. F. Lowe eds: Elsevier/North-Holland.

[9] Jevons, William S. 1888. ”The theory of political economy”. London: MacMillan.

[10] Laibson, David. 1997. “Golden eggs and hyperbolic discounting.” Quarterly Journal of Economics, 112, pp. 443-77.

[11] Loewenstein, George and Drazen Prelec. 1992. “Anomalies in intertemporal choice: Evidence and an interpretation.” Quarterly Journal of Economics, May, pp. 573-97.

[12] Mazur, James E. (1987). "An Adjustment Procedure for Studying Delayed Reinforcement." In M. Commons, J. Mazur, J. Nevins, and H. Rachlin (eds.), Quantitative Analysis of Behavior: The Effect of Delay and of Intervening Events on Reinforcement Value. Hillsdale: Ballinger.

[13] O'Donoghue, Ted and Matthew Rabin. 1999b. "Doing it now or later." American Economic Review, 89:1, pp. 103-24.

[14] Peeters, R. 2004."Stochastic Games with Hyperbolic Discounting". mimeo, Maastricht University.

[15] Phelps, E.S. and Robert Pollak. 1968. "On Second-Best National Saving and Game-Equilibrium Growth." Review of Economic Studies, 35, pp. 185-199.

[16] Rae, J. 1834. "The sociological theory of capital (reprint 1834, ed.)".London: Macmillan.

[17] Samuelson, Paul. 1937. "A note on measurement of utility." Review of Economic Studies, 4, pp. 155-61.

[18] Senior, N. W. 1836. "An outline of the science of political economy". London: Clowes and Sons

[19] Smith, Adam. 1759. "The Theory of Moral Sentiments. London: Millar

[20] Strotz, R. H. 1955-1956. "Myopia and inconsistency in dynamic utility maximization." Review of Economic Studies, 23:3, pp. 165-80.

[21] Strulik, H., 2014. "Hyperbolic discounting and endogenous growth".Econ.Let.126, pp. 131-134.

[22] Thaler, Richard H. 2015. Misbehaving. UK: Penguin