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"HOW TO MODEL LABOR MARKETS IN DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MACROECONOMIC MODELS"

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Abstract

The present study presents several methods on how to model labor markets in a dynamic general stochastic equilibrium macroeconomic framework.

We begin with the neoclassical theory of demand and supply. Price (wage rate) and quantity (number of employed individuals) are jointly determined by the forces of supply and demand. Households supply labor and receive income. They maximize their utility and face a tradeoff between leisure and income generating activities. There is no involuntary unemployment, output is supply determined and interest rate brings goods market in equilibrium. By Walras' Law if all markets are in equilibrium, then the labor market is in equilibrium too- an opposite result to that of the New Keynesian theory.

Following, we discuss the real business cycle (RBC) theory. A basic RBC model is presented, as a benchmark. While the initial RBC models explain very well business cycles of most macroeconomic variables, they perform very poor when it comes to labor market time series. We present a model by Danthine and Donaldson (1995) with two types of workers (insiders-outsiders) also introducing a non Walrasian feature in the form of minimal wage, which performs well. We continue by presenting the important concept of indivisible labor. Hansen (1985), Rogerson (1988) and Rogerson and Wright (1988) (HRW) introduced this non- convexity by assuming that an individual either works a specific positive amount of time or not works at all. The model performed really well in U.S. postwar data. Finally, we discuss the work of Maffezzoli (2001) who tests the HRW model in the Italian market and concludes that it is not suitable for the European markets which have a high degree of unionization. He suggests a model with the present of monopoly unions (MU) and indeed obtains good results for the Italian economy. The MU model comes to a non Pareto optimal and is strictly dominated by that of the HRW model, as it is suboptimal due to the presence of the monopoly unions.

Next, we attempt an introduction to the search theory. We explain the central role of trade frictions in the process of finding a good match. Also, we introduce the concepts of the matching

function, the job creation, the job destruction and the Beveridge curve all presented in a simple model suggested by Pissarides (2000).

In the final chapter we discuss issues in the New Keynesian framework. We compare it with the other approaches and stress its main characteristics. We present a basic New Keynesian model suggested by Gali (2008) as well as a more complicated one by Blanchard and Gali (2006) combining the New Keynesian with the search theory. The model concludes that unemployment displays persistence beyond that inherited from productivity.

Introduction

This survey aims to study the several ways labor markets can be modeled in a dynamic general equilibrium macroeconomic framework.

We begin in the first chapter with the neoclassical framework. Neoclassical theory sees the labor market as similar to other markets in that the forces of supply and demand jointly determine price and quantity, i.e. wage rate and the number of employed individuals. Despite that, there are several differences from the rest of the markets, with the main one being the function of supply and demand in setting price and quantity

Households are suppliers of labor. They are supposed to be rational and maximize their utility function and choose between income and leisure while being constrained by the working hours available to them.

There is a trade-off between leisure activities and income generating activities. As there only 24 hours in a day, individuals must allocate their time to leisure activities and working. This decision is represented by the indifference curve in which all combinations of leisure and work that will give the individual a specific level of utility are indicated. The short-run equilibrium is the point where the highest indifference curve is just tangent to the constraint.

If we assume that consumption is measured by the value of income obtained, the slope of the budget constraint becomes the wage rate. The optimization point is where the wage rate equals the marginal rate of substitution, leisure for income, which is the slope of the indifference curve. Note that the marginal rate of substitution, leisure for income, is also the ratio of the marginal utility of leisure (MU^L) to the marginal utility of income (MU^Y).

This implies that $\frac{MU^L}{MU^Y} = \frac{dY}{dL}$

The main points of the Neoclassical model can be summarized as follows:

- Factor market equilibrium determines the real wage (w/p)* and the level of employment N* and thus is no involuntary unemployment ($N_d = N_s = N^*$).
- Output Y* is determined by the production function Y = f (N) thus from the level of employment determined in the factor markets, N*(i.e. it is supply-determined)
- Interest rate brings goods markets into equilibrium by interest rates. In equilibrium, aggregate demand equals aggregate supply, savings equal investment and demand equals the supply of loanable funds.
- We have money neutrality, i.e. changes in the supply of money only affect the absolute price level and do not affect any real variables.

In conclusion, Neoclassical macroeconomic theory, suggests that, by Walras' Law, if all markets for goods are in equilibrium, the market for labor must also be in equilibrium. It therefore comes to contradiction with the Keynesian result that negative excess demand and consequently, involuntary unemployment, may exist in the labor market, even if all markets for goods are in equilibrium. The argument of Keynesian theorists is that this neoclassical approach ignores financial markets, which may experience excess demand that permits an excess supply of labor and thus leads to temporary involuntary unemployment, even if all markets for goods are in equilibrium.

In chapter two we continue with the real business cycle models. The real business cycle (RBC) theory has its roots in the 1970's but started being studied in the framework of general equilibrium theory by Robert E. Lucas, whose work became known as the new classical revolution. He stressed the need to leave the ad hoc assumptions in favor of the standard tools of economic analysis, arguing that the former come to contradiction to the general equilibrium and the rational behavior. The notable work of Kydland and Prescott (1982) and Long and Plosser (1983) has set the main reference for the RBC theory and offered an important tool methodologically and conceptually. Despite the fact that academic researchers have had great interest in the theory, it has not been proved useful for central banks and other institutions. Lucas (1976) and Sims (1980) among others have tried to increase its practical use.

Despite the fact that the RBC approach has been an important educational tool as well as a popular theory for many researchers, it has not been used practically by central banks and other institutions. There had been efforts to change those models' practical usefulness with most notable the one of Lucas (1976) and Sims (1980) while Cooley and Hansen (1989) suggests the "classical monetary model" introducing a monetary sector in the classical RBC model while keeping the assumptions of perfect competition and fully flexible prices and wages. This model generally predicts that the power of economic policy to influence output and employment developments is in the short run a result that was empirically supported by many researchers, see for example Christiano, Eichenbaum, and Evans (1999).
We proceed in the chapter by presenting a simple RBC model with fixed labor supply by Jean-Pierre Danthine and John B Donaldson (1995) that incorporate non- Walrasian characteristics. There are two type of workers; the first type workers who have a long time relationship with the firm (and who can be seen as the insiders) and the second type workers who can lose their job in any period (and can be seen as the outsiders). The Walrasian characteristic introduced is the minimum wage. The result is that in contrast to Walrasian model, there is sufficient employment variability is generated by movements in and out of employment.

Next we present a model that incorporates the idea of indivisible labor. This concept suggests that individuals can either work some positive number of hours or not at all. This assumption is based on the observation that most people either work full time or not at all. Therefore, fluctuations in aggregate hours are the result of individuals entering and leaving employment and not continuously employed individuals adjusting the number of hours worked. Benchmark papers in this fields are those of by Hansen (1985), Rogerson (1988) and Rogerson and Wright (1988). The HRW (Hansen-Rogerson-White) model made a significant improvement to the match between calibrated and actual U.S. postwar time series regarding the labor market.

The standard RBC models are designed to fit the U.S. institutional framework. In fact Maffezzoli (2001) has calibrated the HRW model using data from Italy with poor results. He suggested a model with indivisible labor where unemployment is generated by monopolistic unions in order to explain the highly institutioned European markets and especially that of Italy.

The model leads to an equilibrium that is not Pareto optimal, due to the existence of the unions. The HRW model strictly dominates the monopoly union (MU) model, as the latter is suboptimal. The calibration of the MU model shows that it explains much better the European business cycle, while the HRW has a comparative advantage in that of the U.S. The impulse response functions of the monopoly union model exhibit a higher degree of overall existence that the HRW model. Finally the business cycle statistics of the two models are similar.

Despite the fact that the neoclassical model of supply and demand in a frictionless labor market can be methodologically useful, there many issues are still not addressed with this approach. Questions such as

- why unemployed workers may choose to remain unemployed?
- what determines the length of unemployment?
- what determines the efficient amount of turnover?
- how can we have unemployed workers and unfilled vacancies at the same time?

and others, still need to be answered.

Search theory places in its center the role of trading frictions. Workers and firms have to spend resources in order to form a job agreement; a worker needs to spend time and put effort to find a job that suits her and the same happens when a firm needs to find a suitable worker in order to fill a specific vacancy. We now abandon the assumption of the classical theory that there is a centralized market where firms and workers meet and trade at a single price.

Search theory appeared realistic and that made it popular among researchers. The new concept introduced, is that of a matching function. This function serves as a "black box"; it accounts for different characteristics and frictions of the labor markets but those need not be made explicit. This function captures the main idea of a good match; it takes time to find a good match, the length of the time depend on unpredictable parameters and the more available job vacancies exist- given the number of individuals looking for jobs- the faster a match will take place.

As both the worker and the firm are concerned to find a good match, they both need to spend time and other resources before signing a contract. Each worker has different skills and firms have different requirements to fill each of their job vacancies. This approach makes unemployment neither voluntary nor involuntary, but it is instead an outcome that might not be optimal. The model was estimated by Pissarides (1986) and Blanchard and Diamond (1989) with encouraging results.

In chapter 2 we studied some benchmark RBC models. The New Keynesian approach differs in some important matters such as the economic policy implications but shares some common features with the business cycle theory. Both approaches assume the existence of a large number of identical households who maximize their utility subject to their budget constraint and a large number of firms sharing common and maximizing their profits subject to exogenous shocks. The New Keynesian theory comes in contradiction to the classical monetary models in the following way. Firstly, it assumes monopolistic competition, thus it rejects the existence of a Walrasian auctioneer as the price setter in favor of private economic agents who maximize their objectives. Secondly, prices and wages do not fully adjust in the short term. Thirdly, due to the abovementioned stickiness, there is room for economic policy in the short run.

All those characteristics were included in the old New Keynesian literature developed in the 1970s and the 1980s. The most important features of the Modern New Keynesian models are:

- The economies respond to shocks is inefficient
- The non-neutrality of monetary policy makes room for welfare-enhancing interventions in order to minimize the existing distortions.
- Those models can be used for the comparison of alternative monetary regimes, without being subject to the Lucas critique.

After presenting a basic New Keynesian model, we proceed with the one that Blanchard and Gali (2006) proposed. This model combines the New Keynesian approach with the Diamond-Mortensen-Pissarides model of search and matching. The purpose of this combination is to accommodate all the following properties, which hold in industrialized economies:

- Variations in unemployment are an important aspect of fluctuations
- The nature of wage bargaining and labor market frictions are central to understanding movements in unemployment.
- The effects of technology and other real shocks are largely determined by the nature of nominal rigidities and monetary policy

The model combines the main features of both theories; it contains labor market frictions, real wage rigidities, and staggered price setting. Those three ingredients are vital in order to explain the movements of unemployment, the effects of productivity changes on the economy, and the role of monetary policy in shaping those effects. We derive the constrained-efficient allocation and the equilibrium in the decentralized economy in the presence of real wage rigidities and sticky prices and show that the optimal monetary policy minimizes a weighted average of unemployment and inflation fluctuations. Finally, we show that unemployment displays persistence beyond that inherited from productivity.

1. The Neoclassical theory

1.1. The microfoundation

There is a large number of identical households and a big number of identical firms. We assume that output, factor employment and investment are derived from firms' profit-maximizing decision. Also, labor supply, consumption and savings are derived from household utility-maximization and market-clearing conditions are imposed on all markets.

Following Fisher (1930), assume that the representative firm faces a "short-run" production function of the form:

$$Y=f(N,I) \qquad (1.1)$$

The output of the representative firm is some function of N, the labor inputs and I, the investment inputs. It is assumed that $Y_N > 0$ and $Y_{NN} < 0$, i.e. the marginal product of labor is positive but diminishing and $Y_I > 0$ and $Y_{II} < 0$, so that we have diminishing marginal product of investment. The firm maximizes its profits which is the difference between total revenue pY and total costs wN + (1+r) I ,where w is the nominal wage and r is the rate of interest. So, letting Π denote the profits of the firm, then its problem is:

$$\max \Pi = p\, f(N, I) - wN - (1+r)\, I \qquad (1.2)$$

This yields us a pair of first order conditions:

$$p\, f_N = w \qquad (1.3)$$

$$p\, f_I = (1+r) \qquad (1.4)$$

The term on the left, p f_N is merely the marginal value product of labor, which means that the firm will hire labor until its marginal value product is equal to the nominal wage. There are two other ways of writing this. A simple one is to divide by p, so that:

$$f_N = w/p \qquad (1.5)$$

so now the firm hires labor until the marginal product of labor is equal to the real wage, w/p. Alternatively, we can write it as:

$$p = w/ f_N \qquad (1.6)$$

where w/ f_N is the marginal cost of output, i.e. the cost of hiring labor in order to increase output by a unit. Given w/p, we can determine the amount of labor demanded by the firm (N^d) via the condition $f_N = w/p$ and by assuming that f_N (.) is invertible, we can write the labor demand function as:

$$N^d = f_N^{-1}(w/p) \qquad (1.7)$$

Our prior assumption imply that $d f_N^{-1}/d(w/p) = d N^d/d(w/p) < 0$, so that the firm's labor demand curve is downward-sloping. The aggregate labor demand function is obtained by aggregating firm-level labor demand functions which are assumed to be well behaved.

We now turn to the investment decision of the firm. The first order condition $p f_I = 1 + r$ implies that the firm will invest (or employ) until the marginal efficiency of investment is equated with the rate of interest

Thus, in a Fisherian investment-labor economy, the first order conditions imply that at the optimum:f_N

$$f_N / f_I = w/(1+r) \qquad (1.8)$$

the ratio of marginal products is equal to the ratio of real factor prices (w/(1+r)). The factor price ratio is shown in Figure 1.1 as the slope of the isocost line. Once (w/(1+r)) is given, the firm automatically knows the amount of labor and investment it will undertake (N* and I*) as the tangency of the highest isoquant and the isocost curve

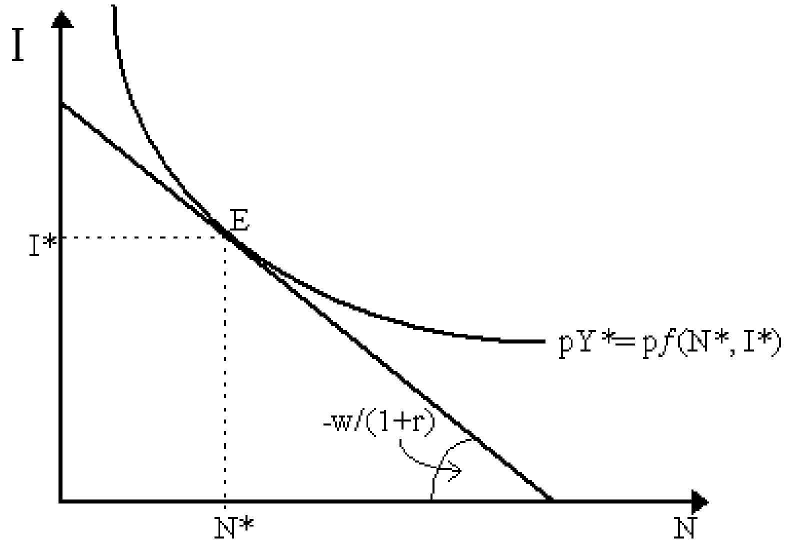

Figure 1.1 - the Production Decision

Therefore, labor demand N^d is a function of the wage, but also of the rate of interest, (1+r). Also, investment demand I is not only a function of interest but also a function of the wage rate. An increase in w/(1+r), which can be due either to a rise in w or a fall in (1+r), leads to a decline in labor demand and/or a rise in investment demand . This implies that we still obtain a downward-sloping demand function for labor and a downward sloping investment function

We now turn to the household's utility maximization problem. The choice variables are now labor supply, consumption demand and savings. The representative household wants to maximize its consumption and minimize the amount of work it has to do (or maximize its leisure time). Therefore, consumption and leisure will directly enter its utility function U=u(C,N) . Note that $U_C > 0$ and $U_N < 0$, where U_C and U_N are the marginal utilities of consumption and labor respectively, thus our assumption about signs means that consuming goods, C, is utility-increasing while supplying labor, N, is utility-decreasing while the second derivatives are $U_{CC} < 0$ and $U_{NN} > 0$.

Household consumes goods with income obtained from its labor and so it faces two parameters in its constraint: the prices of goods and factors (p, w) and the factor it is endowed with, i.e. the maximum labor supply. The budget constraint is pC = wN. In order to recognize that there is a maximum amount of labor supply (call it T - a person cannot work more than twenty-four hours a day), we shall change this constraint by adding the value of maximum labor supply (wT) to both sides so the constraint can be rewritten:

$$pC + w(T - N) = wT \qquad (1.9)$$

so the agent sells his total labor supply, T, to buy goods, C and "leisure" (T-N). The budget constraint is depicted in consumption-leisure space in Figure 1.2 by the straight line that emanates from the consumer's endowment, which is at (0, T), with slope -w/p. The agent maximizes his utility subject to this constraint:

$$\max U = U(C, N) \qquad (1.10)$$

s.t.

$$pC + w(T - N) = wT \qquad (1.11)$$

The Lagrangian is :

$$L = U(C,N) - \lambda[\, pC + w(T-N) - wT \,] \qquad (1.12)$$

so the first order conditions for a maximum will be:

$$dL/dC = U_C - \lambda p = 0 \qquad (1.13)$$

$$dL/dN = U_N + \lambda w = 0 \qquad (1.14)$$

where λ is the Langrangian multiplier.

At the utility-maximizing point E, the household chooses a level N* of labor (or a (T-N)* of leisure) and enjoys C* amount of consumption.

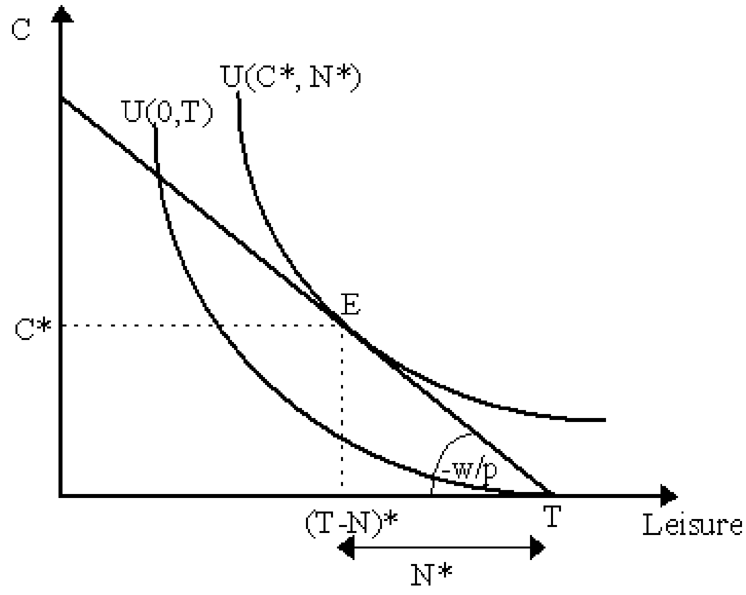

Figure 1.2 - Consumption-Leisure Choice

The relationship between real wage, w/p and labor supply N is combined by two different effects. The substitution effect argues that the higher the real wage, the more costly leisure becomes relative to consumption and thus the agent will supply more labor. The income effect argues that the higher the wage, the agent can buy the same goods with less work and thus the more appealing leisure becomes, so the agents supplies less labor.

We now turn to the consumption-savings decision, also known as the Fisher's (1930) "first approximation". A two- period model in used to illustrate the household's intertemporal maximization of consumption. The extrapolation of Fisher's two-period model to more than two periods is accomplished in the "Life Cycle Hypothesis" (LCH) of Modigliani and Brumberg (1954).

We assume that the household receives income in both periods (Y_1, Y_2) and chooses its consumption (C_1, C_2) in order to maximize its utility $U = u(C_1, C_2)$.

We also assume perfectly working financial markets, so that the household can lend some of its present income to increase future consumption (or borrow from its future income to increase present consumption). Thus constraints in each period are:

$$C_1 + S = Y_1 \qquad (1.15)$$

$C_2 = Y_2 + (1+r)\ S$ (1.16)

where S is the savings of the first period. Combining both constraints we end up with the intertemporal constraint:

$C_1 + C_2/(1+r) = Y_1 + Y_2/(1+r)$ (1.17)

which can be interpreted as saying that the present value of the stream of consumption cannot exceed the present value of the stream of income (where, by present value, we see that future consumption and income is discounted by the rate of interest). The representative household solves:

$\max\ u(C_1, C_2)$ (1.18)

s.t.

$C_1 + C_2/(1+r) = Y_1 + Y_2/(1+r)$ (1.19)

while the Lagrangian is

$L= u(C_1, C_2)- \mu\ [C_1 + C_2/(1+r) - Y_1 - Y_2/(1+r)]$ (1.20)

and the first order conditions are

$dL/dC_1 = U_1 - \mu = 0$ (1.21)

$dL/dC_1 = U_2 - \mu /(1+r) = 0$ (1.22)

where $U_{1=}$ d $u(C_1, C_2) /dC_1$ and $U_2 =$ d $u(C_1, C_2) /dC_2$ are the marginal utilities of present and future consumption respectively. The first order conditions imply that the household will allocate consumption in both periods until the ratio of marginal utilities is equal to the interest rate, i.e. until $U_1/U_2 = 1+r$.

1.2. The Macroeconomic model

The Fisherian Neoclassical model of the macroeconomy has what may be regarded as a "supply-determined" equilibrium. The main assumption are:

(1) Factor supplies and demands determine factor returns and employment.

(2) Factor employment and technological possibilities determine aggregate supply.

(3) Aggregate supply and aggregate demand determine the equilibrium rate of interest.

(4) Money demand and money supply determine the price level.

The essential features of the Neoclassical macromodel are presented in Figure 1.3, with causality running from Quadrant I (upper right) to Quadrant III (bottom left).

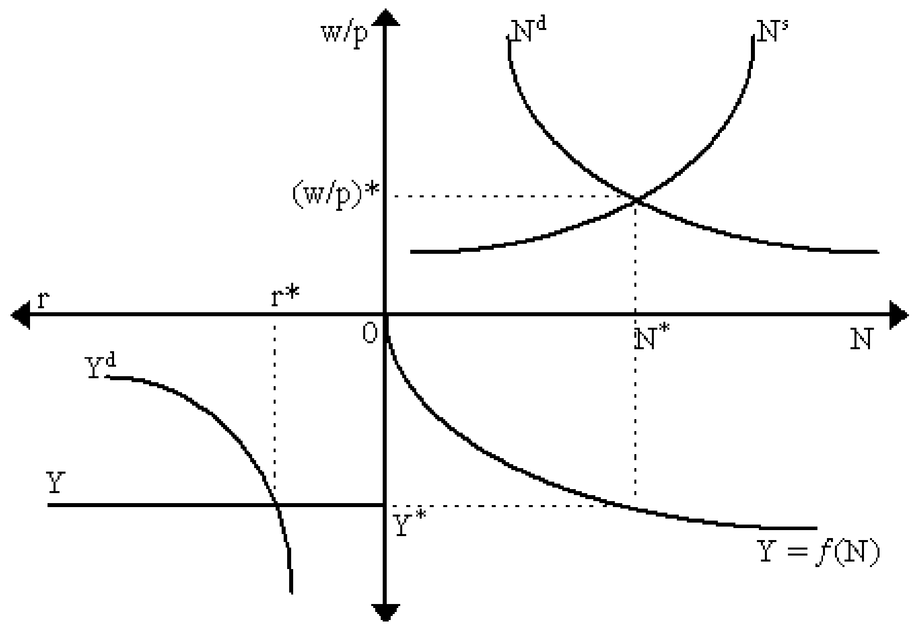

Figure 1.3 - The Neoclassical Macromodel

We can write the labor demand function as:

$N^d = N^d(w/p)$ (1.23)

where $dN^d/d(w/p) < 0$ from our previous assumptions in the optimization problem, the labor demand curve is downward sloping, as shown in Quadrant I of Figure 1.3.

For the labor supply, we have respectively

$N^s = N^s(w/p)$ (1.24)

where $dN^d/d(w/p) > 0$ so that labor supply is a positive function of the real wage. The upward-sloping labor supply function in Quadrant I of Figure 1.3 arises from substitution between work and leisure on the part of the household. The greater the real wage, the more labor is supplied.

Now, the equilibrium in the labor market and is given by:

$N^d(w/p) = N^s(w/p)$

which determines the equilibrium real wage (w/p)* and the equilibrium level of employment (N*), as shown in Quadrant I of Figure 1.3. Given these conditions, then by total differentiation we know that:

$dw/w = dp/p$ (1.25)

This relationship implies that nominal wages are fully flexible in the long run and will accompany changes in the price level by the same proportion in order to maintain (w/p)* and, by extension, N*.

Using the short-run production function we introduced before Y=f (N), then given N* from the labor-market clearing we just obtained, we can determine the aggregate supply, Y*. This is presented in Quadrant II of Figure 1.3.

In Quadrant III of Figure 1.3 we present the goods market. The aggregate supply is horizontal, and thus the output of goods in an economy is only "supply-determined". The higher the level of employment, N*, is the higher the level of output, Y*. Therefore, the supply of goods is invariant to anything happening in the goods market .

Following, the given output Y* will be factor income which can be consumed, saved or taxed away:

$$Y = C + S + T \qquad (1.26)$$

The aggregate demand, Y^d, is composed of consumption, investment and government expenditures, thus:

$$Y^d = C + I + G \qquad (1.27)$$

We equate the latter equations in order to obtain a condition for equilibrium in the goods market:

$$I = S + (T - G) \qquad (1.28)$$

It is important to note that the interest rate adjusts so that $Y^d = Y$. This differs from the Keynesian multiplier where the output itself adjusts to equate investment and savings. Therefore, interest in the Neoclassical model does not affect aggregate supply, but only affects aggregate demand.

The main relationship for the aggregate demand is:

$$Y^d = C(Y, r) + I(r) + G \qquad (1.29)$$

Note that, $Cr < 0$ and $Ir < 0$ so that the aggregate demand function is downward sloping with respect to interest. Aggregate supply is invariant with interest. Thus, the third line of the catena is fulfilled by recognizing that $Y^d = Y$ achieved by movements in the rate of interest. All this is shown in Quadrant III of Figure 1.3.

The interest rate is determined in the market financial assets. According to Fisher (1930), "loanable funds" are demanded by firms who need it for investment and are supplied by households who need a place to put their savings. In equilibrium we have that $I = B^s$ and $S = B^d$.

The relationship between the three markets is depicted in Figure 1.4. When interest clears the loanable funds market (r*) then $B^d = B^s$ and $I = S$ and therefore $Y^d = Y$. If interest is too low (r^1), so that there is excess demand for goods $Y^d > Y^s$ (or $I > S$), it implies in turn that there must be excess bond supply, $B^s > B^d$. The reverse applies if interest rates are too high (r2).

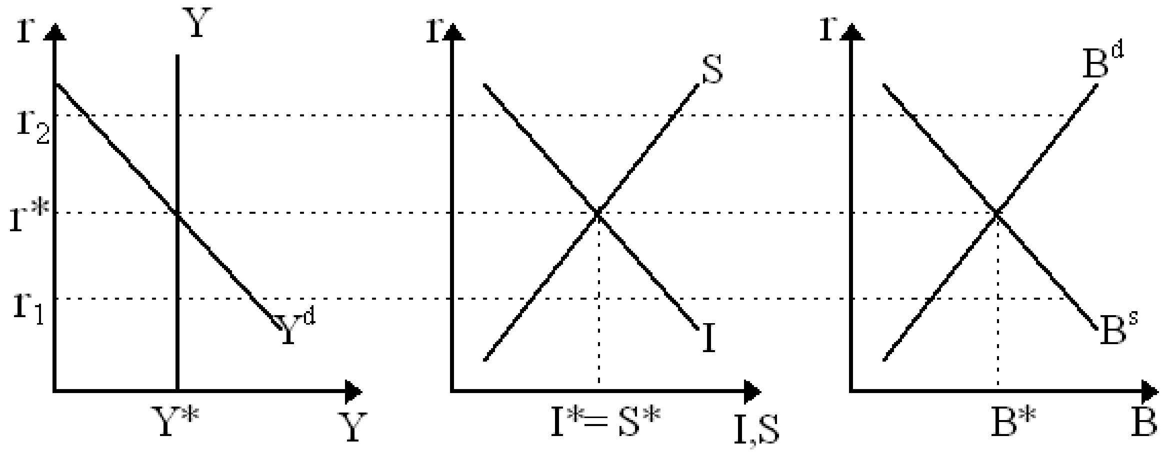

Figure 1.4 - Interest Rates

Applying standard market-clearing arguments for the loanable funds market, we see that if r is too high (r_2), the interest falls to balance the loanable funds market (B^d=B^s) and by extension the goods market (Y^d=Y^s). Step-by-step, the fall in interest leads to an increase in investment demand and a decrease in savings as an effect on intertemporal allocation of consumption.

Finally, it is important to note that we have assumed that household savings are equivalent to the demand for loanable funds without considering that they may desire to place their savings in other assets. As Keynes (1936) showed, when these other assets are taken into account, there will be some important modifications to the conclusions.

The Neoclassicals highlighted the role of money in their theory. In the Cambridge cash-balance theory developed by Marshall (1923), Pigou (1917) and Keynes (1923), the interest rate and output influence the demand for real money (M^d). i.e.

$$M^d = L(r, Y) \qquad (1.30)$$

where $L_r < 0$ and $L_Y > 0$ If real money demand, M^d, is determined by r and Y and the latter are determined in the real markets for goods and factors, then if these are unchanged, L^d will also be unchanged.

We are -exogenously- given a real supply of money:

$$M^s = M/p \qquad (1.31)$$

where M is the nominal money stock. In order to have a money market equilibrium it has to be that $M^d = M^s$, or:

$$L(r, Y) = M/p \qquad (1.32)$$

However, as money demand is exogenously determined and M is assumed to be controlled by the central bank, then the general price level ,p, will adjust this market into equilibrium. The money market is illustrated in Figure 1.5, which is drawn in nominal terms. The nominal money demand curve is given by $pM^d = pL(r, Y)$ and thus only p* is left to be determined by equilibrium condition $pL(r, Y) = M$.

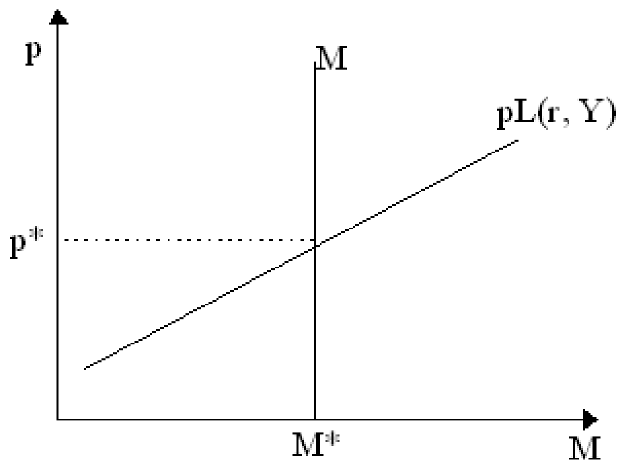

Figure 1.5 - The Money Market

Note that the determination of money market has no impact on the determination of the real variables w/p, N, Y, r and B.

1.3. Conclusions

Given that w/p is and the rest of our assumptions, there are three primary features of the Neoclassical model which are preserved. Firstly, there is a strict dichotomy between the money market and the real market, i.e. what happens in the former does not affect the latter. Secondly, and by extension, we have money neutrality, i.e. a rise in the supply of nominal money (M) will not affect any other variables other than the price level. Lastly, a change in the money supply will change the price level proportionately but not anything else.

2. The Real Business Cycle Theory

2.1. Introduction

The systematic study of real business cycles has begun in the early 1970s, but since then there have been several important methodological changes. The so called “new classical revolution”, based on the work of Robert E. Lucas, studied business cycles in the framework of competitive equilibrium theory. The main contribution of this approach was the suggestion that we need to use the standard tools of economic analysis in order to explain business cycles, rather than use ad hoc assumptions that disagree with the rational behavior and general equilibrium. This was when Kydland and Prescott (1982), Long and Plosser (1983) and Prescott (1986), provided the main reference RBC theory framework for the analysis of economic fluctuations and became to a large extent the core of macroeconomic theory, with a large methodological and conceptual dimension.

Methodologically, RBC theory suggested the use of dynamic stochastic general equilibrium models as a central tool for macroeconomic analysis, First-order conditions of intertemporal problems faced by consumers and firms, replaces behavioral equations describing aggregate variables, while the Ad hoc assumptions on the formation of expectations are replaced by rational expectations. RBC theory gives a central role to the quantitative aspects of modeling, stressing the importance of calibration, simulation, and evaluation of the models.

Conceptually, the theory was based on three basic claims. The first one is the efficiency of business cycles. Economic fluctuations in an economy with perfect competition and frictionless markets, can be viewed as an equilibrium outcome resulting from exogenous variations in real forces, such as technology. Those fluctuations do not signal an inefficient allocation of recourses. As a result, there is no room for economic policy, which in fact can be proved to be counterproductive. This comes to opposition with Keynes (1936) result who claimed that recessions are periods with an inefficiently low utilization of resources that could be brought to an end by means of stabilization policies aimed to expand aggregate demand. A second point is the source of the economic fluctuations. The traditional view of technological change as a source of economic growth, unrelated to business cycles, comes in contrast to the one RBC theory claims; exogenous technology shocks generate fluctuations in output and other macroeconomic variables. Lastly, RBC theory minimizes the role of economics policy. This theory tends to explain economic fluctuations without the existence of the monetary sector.

Despite the fact that the RBC approach has been an important educational tool as well as a popular theory for many researchers, it has not been used practically by central banks and other institutions. There had to introduce a monetary sector in the classical RBC model, while keeping the assumptions of perfect competition and fully flexible prices and wages, were not perceived as yielding a framework that was relevant for policy analysis. The resulting framework, which is referred to as the classical monetary model, generally predicts neutrality of monetary policy with respect to real variables, a result that supports the belief of central banks and many others that in the power of economic policy to influence output and employment developments is in the short run. The latter has been supported empirically by many researchers, from Friedman , to the more recent work using time series techniques, as described in Christiano, Eichenbaum, and Evans (1999).

One can claim, that the conflict between theoretical predictions and evidence, and between normative implications and policy practice, can be interpreted as a symptom that some elements that are important in actual economies may be missing in classical monetary models.

2.2. The basic RBC model

2.2.1. The Representative Household

Assume that there is a representative household maximizing its expected life-time utility:

$$E_0[\sum_{t=\infty} \beta^t U(C_t, l_t)] \qquad (2.1)$$

where $\beta \in [0,1]$, C_t is household's consumption and l_t is leisure. The utility function U(.) is increasing and concave in both arguments. Specifically, the utility function takes the form:

$$U(C_t, l_t) = \frac{1}{1-\sigma}(C_t^{\mu}(l_t)^{1-\mu})^{1-\sigma} \qquad (2.2)$$

while the household faces the usual constraint:

$$1 = l_t + h_t \qquad (2.3)$$

where h_t is the amount worked.

The household's budget constraint in each period takes the form:

$$w_t h_t + r_t K_t + \Pi_t = C_t + I_t + T_t \qquad (2.4)$$

where w_t is the real wage rate, K_t is the beginning of period physical capital, r_t is the rental rate of capital, Π_t the household's share of profits of the firm, I_t is the investment and T_t the lump-sum taxes. The law of motion of capital is:

$$K_{t+1} = (1-\delta)K_t + I_t \qquad (2.5)$$

with $\delta \in [0,1]$ denoting the depreciation rate.

Thus, in period t, representative household solves:

$$\max_{\{C_t, h_t, l_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left(C_t^{\mu}(1-h_t)^{1-\mu}\right)^{1-\sigma} \qquad (2.6)$$

subject to its budget constraint and the law of motion of capital.

The first order conditions are:

$$\mu C_t^{\mu(1-\sigma)-1}(1-h_t)^{(1-\mu)(1-\sigma)} = \Lambda_t^{\mathrm{B}} \tag{2.7}$$

$$(1-\mu)C_t^{\mu(1-\sigma)-1}(1-h_t)^{\sigma(1-\mu)-1} = \Lambda_t^{\mathrm{B}} w_t \tag{2.8}$$

$$\Lambda_t^B = \Lambda_t^K \tag{2.9}$$

$$\Lambda_t^K = \beta \mathrm{E}_t \left\{ \Lambda_{t+1}^B r_{t+1} + \Lambda_{t+1}^K (1-\delta) \right\} \tag{2.10}$$

where Λ_t^{B} and Λ_t^K are the Lagrange multipliers associated with the two constraints.

2.2.2. The Representative Firm

The representative firm produces a good Y, consumed by the households. It maximizes its profits given by:

$$\Pi_t = \mathrm{Y}_t - w_t h_t - r_t K_t \tag{2.11}$$

s.t.

$$Y_t = A_t (K_t)^{\alpha} (\Gamma_t h_t)^{1-a} \tag{2.12}$$

which is the firm's production function. Γ_t is the Harrod neutral deterministic technical progress evolving according to $\Gamma_t = \gamma \Gamma_{t+1}$ with $\gamma \geq 1$

Therefore, the firm each period solves:

$$\Pi_t = \max_{\{h_t, K_t\}} A_t (K_t)^{\alpha} \left(\Gamma_t h_t\right)^{1-\alpha} - w_t h_t - r_t K_t \tag{2.13}$$

and the first order conditions are:

$$(1-a) A_t K_t^a (\Gamma_t h_t)^{1-a} = w_t h_t \tag{2.14}$$

$$a A_t K_t^a (\Gamma_t h_t)^{1-a} = r_t K_t \tag{2.15}$$

2.2.3 The Government

The government each period has a balanced budget:

$$T_t = G_t \tag{2.16}$$

where G is the government's spending.

2.2.4. The shocks

We assume two types of shocks, one in technology and one in public spending. The former, denoted by

ε_t^{α} evolves according to

$$A_{t+1} = \overline{A} A_t^{\rho_A} \varepsilon_{t+1}^{A} \tag{2.17}$$

where ρ_A denotes the autocorrelation parameter of A. The latter, denoted by ε_t^{G} evolves according to

$$G_{t+1} = \overline{G} G_t^{\rho_G} \varepsilon_{t+1}^{G} \tag{2.18}$$

where ρ_G denotes the autocorrelation parameter of G.

2.2.4. Market Clearing

Combining the national accounts identity $aA_t K_t^a (\Gamma_t h_t)^{1-a} = C_t + I_t + G_t$ with the production function $Y_t = A_t (K_t)^{\alpha} (\Gamma_t h_t)^{1-a}$ we obtain the market clearing condition in the good's market

$$aA_t K_t^a (\Gamma_t h_t)^{1-a} = C_t + I_t + G_t \tag{2.19}$$

2.2. Labor Contracting and Wage Floor

Let us now try to incorporate non- Walrasian considerations in the real business cycle model within the general structure of the neoclassical growth paradigm. The aim is to suggest an RBC model, with a fixed labor supply, where sufficient employment variability is generated by movements in and out of employment in contrast to the Walrasian models. This model is proposed by Jean- Pierre Danthine and John B Donaldson (1995)

2.2.1 The Model

Assume that there is a large number of firms which are owned by infinitely living dynasties of shareholders. They decide upon investment and hiring plans. There is a single product produced by all firms who are assumed to face the same constant returns to scale technology, described by a production function :

$$f(k, n_p, n_s)\, z \qquad (2.20)$$

where k is the individual firm's capital stock, z is the economy wide shock to technology (and is common to all firms), n_p is the primary labor employed and n_s is the secondary labor employed.

Firm owners receive the residual income of production

$$\pi(k, K, z) \qquad (2.21)$$

that is the value of output net of the wage bill and taxes. We denote the aggregate level of capital stock by K1.

Representative shareholder solves

$$\max_{c_t, x_t} E\left(\sum_{t=0}^{\infty} \beta^t u(c_t)\right) \qquad (2.22)$$

[1] We denote aggregate level of the variable with capital letters, while the firm- specific with a lower case letter.

subject to

$$c_t + x_t \leq \pi(k_t, K_t, z_t) \tag{2.23}$$

$$k_{t+1=}(1-\delta)k_t + x_t \tag{2.24}$$

and the initial condition; k_0 given

choosing paths for her consumption c_t and her investment c_t.

2.2.2. Workers in the primary sector

Workers do not have access to the financial markets and thus their decision problem is static. Each worker supplies one unit of labor- inelastically- in each period of her life. Primary sector workers are "insiders", i.e. they benefit from a lifelong association with the firm and are considered to be "a part of the family" i.e. their utility directly enters shareholders utility function which is

$$u(c) + vn_p\bar{u}(w_p) \tag{2.25}$$

where w_p is the compensation offered to the permanent worker, $\bar{u}(w_p)$ is the one period utility which is common to both type of workers and v is an altruistic parameter. We have that

w_p (k, K, z) satisfies:

$$v = \frac{u_1[c(k,K,z)]}{n_p\bar{u}_1[w_p(k,K,z)]} \tag{2.26}$$

This is also known as the sharing rule. The magnitude of v captures the relative position of primary sector workers in relation to the shareholders. This latter relationship shows that if firm owners and primary workers can precommit to an optimal lifelong contract, that would need to satisfy that permanent workers receive a state- contingent compensation such that the ratio of their ex-post marginal utility of consumption to the marginal utility of consumption of firm's owner is constant. This is the only Pareto optimal arrangement. This is due to the fact that a constant v ensures that the intertemporal marginal role of substitution of primary workers and firm owners will be the same and this is necessary to a Pareto- optimal allocation. If v was changing over time, that would imply more gains to intertemporal exchanges between the two types of agents. But v is chosen in a way that both workers and firm owners precommit to the contract voluntarily. The contract smoothes consumption intertemporally for both parts and serves as a substitute for a complete securities market.

2.2.3. Secondary Sector Workers

In opposition to the primary sector workers, this type of workers has a short- term relationship with the firm[2]. Firms take their wage as given and hire them as long as the marginal production exceeds the real wage.

The level of employment of this type of workers is $N_s (K, z)$

and the compensation offered to the typical second type worker w_s is defined by

$$w_s (k, z)=f_3[(K, 1, N_3 (K, z)] z \qquad (2.27)$$

2.2.4. Minimum Wage

We can now introduce a non- Walrasian feature in our model; minimum wage. The secondary labor clears at the Walrasian equilibrium wage given by

$$w^* (K, z)= f_3 (K,1,1) z \qquad (2.28)$$

Assume the existence of a minimum wage so that

$$w_s= \max [w^*, w_m] \qquad (2.29)$$

where w_m is the minimum wage.

This way represents the simplest modeling of a wage floor $w_f (K, z)$

Now assuming that the wage floor and the transfer payment, t (K, z) to the unemployed are state-contingent, the government solves

[2] We assume that both types are of equal measure and normalized to 1.

$$\max_{w_t, t} \psi u[c(K,k,z)] + \overline{u}[w_p(k,z)]$$
$$+N_s(K,z)\overline{u}(w_p) + (1 - N_s(k,z)]\overline{u}(t) \quad (2.30)$$

subject to

$$w_t \geq t$$
$$1 \geq N_s(K,z)$$

where ψ is the firm owner's weight factor. The above assumes that government fails to take into account the depressing effect of its wage floor policy on the investment function of the firm owners.

2.2.5. Equilibrium

An equilibrium in this model is an investment policy X (.) and a government policy [w_f(.), T (.)] such that given X (.) and [w_f(.), T (.)] it solves

$$\max_{w_t, t} \psi u[c(K,k,z)] + \overline{u}[w_p(k,z)]$$
$$+N_s(K,z)\overline{u}(w_p) + (1 - N_s(k,z)]\overline{u}(t) \quad (2.31)$$

s.t.

$$w_t \geq t$$
$$1 \geq N_s(K,z)$$

for all (K, z), while given [w_f(.), t (.)] and X (.) is the solution to

$$\max_{c_t, x_t} E\left(\sum_{t=0}^{\infty} \beta^t u(c_t)\right) \quad (2.32)$$

s.t.

$$c_t + x_t \leq \pi(k_t, K_t, z_t)$$
$$k_{t+1=}(1-\delta)k_t + x_t$$

k_0 given

with profit defined by

$$\pi(k,K,z)=\max_{n_s}\begin{Bmatrix} f(k,n_p,n_s)z-w_p(k,K,z)n_p- \\ n_s w_s(K,z)-t(k,K,z) \end{Bmatrix} \qquad (2.33)$$

The calibration of the model using U.S. data, showed that investment is more variable than output and the latter is more variable than total consumption. The inclusion of non Walrasian characteristics is consistent with most characteristics of the business cycle and there is evidence of excessive consumption smoothing, a typical feature of the RBC models. Finally the model is able to solve the employment-productivity paradox of the U.S. data.

2.3.The Hansen- Rogerson-Wright Model

2.3.1. Introduction

Early theories of the business cycle, such as Kydland and Prescott (1982) have been accused of not being able to account for some important labor market characteristics such as the existence of unemployed workers, fluctuations in the rate of unemployment, and the observation that fluctuations in hours worked are large relative to productivity fluctuations.

Another point of critique is that those models depend too heavily on the willingness of individuals to substitute leisure across time in response to wage or interest rate changes when accounting for the last observation. This claim is based on empirical studies using panel data on hours worked by individuals that have not detected the intertemporal substitution necessary to explain the large aggregate fluctuations in hours worked ,see for example Ashenfelte (1984).
Hansen (1985) presents a model that differs from the abovementioned ones as it introduces the concept of indivisible labor; this is done by assuming that individuals can either work some positive number of hours or not at all. This assumption is based on the observation that most people either work full time or not at all. Therefore, fluctuations in aggregate hours are the result of individuals entering and leaving employment and not of continuously employed individuals adjusting the number of hours worked, a fact that is also supported by empirical studies on U.S. post war data.

In this model hours worked are found to fluctuate about the same amount as productivity. This differs from all previous models such as Prescott (1983) who found that hours worked fluctuates twice as much as productivity.

Equilibrium theories of the business cycle have typically depended heavily on intertemporal substitution of leisure to account for aggregate fluctuations in hours worked. Hansen's model exhibits results consistent with the low estimates of the elasticity of substitution between leisure in different time periods found from studying panel data see for example Altonji (1984) or MaCurdy (1981).

The main extension of the standard RBC model concerning labor market is the indivisible labor supply model, introduced by Hansen (1985), Rogerson (1988) and Rogerson and Wright (1988). The HRW (Hansen-Rogerson-White) model made a significant improvement to the match between calibrated and actual U.S. postwar time series regarding the labor market.

2.3.2 The Model

There is a big number of identical households and firms. Households are the owners of labor and capital so they can sell it to the firms. The latter produce the same product and face the same constant-returns-to-scale production function with inputs labor and capital subject to persistent but stationary productivity shocks. Both agents are price-takers on all markets. We assume that human capital accumulates through a learning-by-doing process in order to introduce endogenous growth.

2.3.3. Firms

The representative faces the following constant elasticity of substitution (CES) production function

$$Y_t = a_t[aK_t^{\eta} + (n_t H_t)^{\eta}]^{1/\eta} \qquad (2.34)$$

with η < 1 and α > 0. We denote by Y_t the aggregate level of per capita output, by K_t the aggregate per-capita level of physical capital services, by n_t the employment rate, by H_t the aggregate per-capita human capital stock, and by a_t the aggregate level of total factor productivity (TFP).

The firm maximizes the expected discounted flow of future profits

$$\max_{(n_s, K_s | a_s)_{s=1}^{\infty}} = E_t\left[\sum_{s=t}^{\infty} R_t^s (Y_s - W_s n_s - r_s K_s)\right] \tag{2.35}$$

where R_t^s is a discount factor and the wage rates, the rental rates $(W_s | a_s)_{s=t}^{\infty}$ and the human capital stocks $(r_s | a_s)_{s=t}^{\infty}$ are given.

We assume that the TFP follows an exogenous stochastic Markov process, that can be approximated by the following stationary AR(1) process

$$\ln\left(a_{t+1}\right) = \rho \ln\left(a_t\right) + u_t \tag{2.36}$$

with $u_t \sim N\left(0, \sigma^2\right)$ and the persistence parameter ρ between 0 and 1.

Also human capital evolves according to

$$H_{t+1} = H_t + \varphi n_t H_t \tag{2.37}$$

where φ>0 is a scale parameter. Thus, for as long as nt>0 for all t, the economy is characterized by endogenous growth

2.3.4 Households

Representative household has two options; either to work a fixed number of hours, here normalized to one, or not work at all. By introducing indivisible labor, we imply a non-convexity in the model. This can be introduced by including a lottery where individuals are called to choose to work a fraction $\tilde{n}$ of her days and this allocation of work is completely random. The lottery's outcomes are independent over time. Before the lottery draw the individual utility function has the form

$$\tilde{n}\left[\tilde{C}_e\upsilon(0)\right]^{1-\mu}+\left(1-\tilde{n}\right)\left[\tilde{C}_u\upsilon(1)\right]^{1-\mu} \tag{2.38}$$

where $\tilde{C}_e$ is the consumption level of an employed worker and $\tilde{C}_u$ the consumption level of an unemployed individual. We denote the ex ante probability of being unemployed by $\tilde{n}$ and the utility of leisure by υ. We have that $\upsilon(0)$ >0 and $\upsilon(1)$>0 so we can define $\upsilon(0)\equiv\upsilon_0$ and $\upsilon(1)\equiv\upsilon_1$.

Equilibrium must be characterized by the following constraint:

$$\tilde{C}=\tilde{n}\tilde{C}_e+(1-\tilde{n})\tilde{C}_u \tag{2.39}$$

By substituting in the initial utility function, we obtain the value function which enables us to present both employed and unemployed individuals in the same utility function

$$\tilde{C}^{1-\mu}\left[\tilde{n}+\left(1-\tilde{n}\right)\left(\frac{\upsilon_1}{\upsilon_0}\right)^{\frac{1-\mu}{\mu}}\right]^{\mu} \tag{2.40}$$

and aggregating, we have intertemporally

$$U_t=\left\{\sum_{t=0}^{\infty}\beta^t\,\frac{\left[\tilde{C}_t\phi(\tilde{n}_t)\right]^{1-\mu}}{1-\mu}\right\} \tag{2.41}$$

where

$$\phi(\tilde{n}_t)\equiv\left[\tilde{n}_t+\left(1-\tilde{n}_t\right)\left(\frac{\upsilon_1}{\upsilon_0}\right)^{\frac{1-\mu}{\mu}}\right]^{\frac{\mu}{1-\mu}} \tag{2.42}$$

represents the disutility of employment.

The representative household solves

$$\max_{\{\tilde{C}_t,\tilde{n}_t,\tilde{K}_{t+1}\}_{t=0}^{\infty}} U_t = \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left[\tilde{C}_t \phi(\tilde{n}_t)\right]^{1-\mu}}{1-\mu} \right\}$$

s.t.

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + r_t\tilde{K}_t + \tilde{n}_t W_t - \tilde{C}_t \tag{2.43}$$

Forming the Lagrangian

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \dfrac{\left[\tilde{C}_t \phi(\tilde{n}_t)\right]^{1-\mu}}{1-\mu} \\ -\lambda_t \left[\tilde{K}_{t+1} - (1-\delta)\tilde{K}_t - r_t\tilde{K}_t - \tilde{n}_t W_t + \tilde{C}_t \right] \end{array} \right\} \tag{2.44}$$

2.3.5. Equilibrium

Maximizing the Lagrangian we end up with six difference equations that will define the six endogenous variables $\{C_t, n_t, K_{t+1}, r_t, W_t, H_{t+1}\}_{t=0}^{\infty}$ and $\{Y_t\}_{t=0}^{\infty}$ residually, so that the following properties are satisfied:

- households maximize their utility
- firms maximize their profits
- human capital accumulation holds
- all markets clear

This equilibrium is a Pareto optimal solution. There are no rigidities or externalities thus there is no room for economic policy.

The dynamic competitive equilibrium model with indivisible labor that Hansen suggested, aimed to account for the standard deviations and correlations with output found in the U.S. post war data. Individuals either enter or exit the labor force according to the technology shocks and do not adjust their hours of work. Fluctuations in unemployment are found to be important for fluctuations in hours worked through the business cycle and most of the variability comes from the number of workers rather than the hours of work. We must also note that in this model the elasticity of substitution between leisure in different periods is independent of the elasticity of substitution implied by individuals and it is infinite in the economy. Previous models have been proved unsuccessful to catch to account the main features of the U.S. post war labor market time series. This model showed that the non-convexities suggested can be important for explaining the volatility of hours in relation to productivity even if individuals are unwilling to substitute leisure across time. They are also able to increase the size of standard deviation of all variables in relation to that of the technology shock. Finally, they allow for an equilibrium in an RBC model to exhibit fluctuations in employment.

2.4 The Monopoly Union Model

The standard RBC models are designed to fit the U.S. institutional framework. In fact Maffezzoli (2001) has calibrated the HRW model using data from Italy with poor results. He suggested a model with indivisible labor where unemployment is generated by monopolistic unions in order to explain the highly institutioned European markets and especially that of Italy.

2.4.1. The Model

There are unions that negotiate the wage on behalf of workers who are their members. Acting as monopolists, they maximize:

$$n_t W_t + \left(1 - n_t\right)\overline{W}_t \qquad (2.45)$$

where n_t is the employment rate and $\overline{W}_t$ the union's reservation wage, given the conditional labor demand of the firm and the reservation wage as given. The former implies that unions are risk-neutral maximizers of their members' wage level. Unions are small, i.e. they take the rental price of physical capital as given.

The problem of heterogeneity arises in the model; workers are divided between union members and non-members. As a solution, Maffezzoli suggests that unions pursue a redistributive goal, serving as a substitute for competitive insurance markets. This implies that in equilibrium the marginal utility of consumption will again be equalized across employed and unemployed household members.

The union solves:

$$\max_{\{W_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} R^t \left[n_t W_t + (1-n_t)\overline{W}_t \right] \right\}$$
$$s.t.$$
$$W_t = a_t \left[\alpha K_t^{\eta} + (n_t H_t)^{\eta} \right]^{\frac{1-\eta}{\eta}} n_t^{\eta-1} H_t^{\eta} \tag{2.46}$$

we form the Lagrangian

$$L = \max_{\{n_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} R^t \begin{bmatrix} a_t \left[\alpha K_t^{\eta} + (n_t H_t)^{\eta} \right]^{\frac{1-\eta}{\eta}} \\ n_t^{\eta} H_t^{\eta} + (1-n_t)\overline{W}_t \end{bmatrix} \right\} \tag{2.47}$$

the first order condition with respect to n_t:

$$\left(\frac{a_t n_t H_t}{Y_t} \right)^{\eta} \left[\eta + (1-\eta) \left(\frac{a_t n_t H_t}{Y_t} \right)^{\eta} \right] = \overline{W}_t \frac{n_t}{Y_t} \tag{2.48}$$

This is a highly non-linear equation and does not have a closed-form solution but implicitly defines the monopoly employment rate.

In this model, the firm solves a sequence of independent games. This is because:

- physical capital and labor have to be purchased in each period
- pre-commitment is ruled out
- the union takes the rental rate as given

The union is a Stackelberg leader, while the firm is the Stackelberg follower. Labor demand is the follower's reaction function, which is then substituted into the leader's reaction function. We should mention that in the union's problem once the labor demand function has been taken into account, the choice of control variable makes no difference.

2.4.2.Households

The union offers actuarially fair insurance, so the households will share the risk of being unemployed. Household's problem is the same as it was in the HRW model, with the exception that the employment rate is no longer an individual decision but it is rather determined in the game between the firm and the union.

The representative household maximizes its intertemporal utility function subject to its dynamic constraint:

$$\max_{\{\tilde{C}_t,\tilde{K}_{t+1}\}_{t=0}^{\infty}} U_t = \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left[\tilde{C}_t \phi(n_t)\right]^{1-\mu}}{1-\mu} \right\}$$

$s.t.$

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + r_t\tilde{K}_t + n_t W_t - \tilde{C}_t \qquad (2.49)$$

the first order conditions are identical to those of the HRW model but there is no Euler for the employment rate, n_t.

2.4.3. Equilibrium

We have a system of two difference equations in $\left\{\tilde{C}_t, \tilde{K}_{t+1}\right\}_{t=0}^{\infty}$, taking $\{W_t, r_t, n_t\}_{t=0}^{\infty}$ as given:

$$\frac{\tilde{C}_t^{-\mu}\left[\phi(\tilde{n}_t)\right]^{1-\mu}}{\tilde{C}_{t+1}^{-\mu}\left[\phi(\tilde{n}_{t+1})\right]^{1-\mu}} = \beta(1-\delta+r_{t+1}) \qquad (2.50)$$

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + r_t\tilde{K}_t + \tilde{n}_t W_t - \tilde{C}_t \qquad (2.51)$$

in the general equilibrium we also have

$$W_t = a_t\left[\alpha K_t^{\eta} + \left(n_t H_t\right)^{\eta}\right]^{\frac{1-\eta}{\eta}} n_t^{\eta-1} H_t^{\eta} \qquad (2.52)$$

$$r_t = a_t\left[\alpha K_t^{\eta} + \left(n_t H_t\right)^{\eta}\right]^{\frac{1-\eta}{\eta}} \alpha K_t^{\eta-1} \qquad (2.53)$$

$$H_{t+1} = H_t + \varphi n_t H_t \tag{2.54}$$

$$\left(\frac{a_t n_t H_t}{Y_t}\right)^{\eta}\left[\eta + (1-\eta)\left(\frac{a_t n_t H_t}{Y_t}\right)^{\eta}\right] = \overline{W}_t \frac{n_t}{Y_t} \tag{2.55}$$

thus we have a system of six difference equations that will define the paths of the six endogenous variables $\{C_t, n_t, K_{t+1}, r_t, W_t, H_{t+1}\}_{t=0}^{\infty}$ and residually $\{Y_t\}_{t=0}^{\infty}$ so that the following properties are satisfied:

- households maximize their utility
- firms maximize their profits
- human capital accumulation holds
- all markets clear

The above equilibrium is not Pareto optimal and this is due to the existence of unions in the MU model. The HRW model strictly dominates the MU model, since the latter is sub-optimal. Finally the main results suggest that the impulse response functions of the monopoly union model exhibit a higher degree of overall existence that the HRW model. Another point is that the business cycle statistics are similar in the two models, but the monopoly union model explains much better the European business cycle, while the HRW has a comparative advantage in that of the U.S.

3. Search Theory

3.1.Introduction

The usual model of supply and demand in a frictionless labor market can be useful for discussing some topics. Despite that, many important issues are still not addressed with this approach. Questions such as why unemployed workers may choose to remain unemployed or what determines the length of unemployment, what determines the efficient amount of turnover, how can we have unemployed workers and unfilled vacancies at the same time and others need to be answered.

Search theory suggests a new framework to work with in order to address such questions. Search theorists place in the centre of this framework the trading frictions. Both agents have to spend resources in order to form a job agreement; a worker needs to spend time and put effort to find a job that suits her and the same happens when a firm needs to fill a vacancy. Unfortunately, there is no such thing as a centralized market where firms and workers meet and trade at a single price, as assumed in classical equilibrium theory.

The idea of including search frictions in the labor market appeared in the literature in the late 1960s and became known as search theory. Some influential work was done by Stigler (1962), John McCall (1970), Mortensen (1970) and of course the papers collected by Edmund Phelps et al. (1970) also known as “the Phelps volume”. Although the idea of examining the role of frictions in the labor market has been discussed by John Hicks (1932) and William Hutt(1939), it has not been formally modeled till the late 1960s

Milton Friedman(1968) and Edmund Phelps(1967) suggested that search serves as a microfoundation for the natural rate of the unemployment an argument that was also claimed in Phelps volume. Furthermore, Axel Leijonjufvud (1968) argued that it could also serve as a microfoundation for the Keynesian effective demand, an argument that was criticized as his claims also required either wage rigidity or absence of capital markets. Peter Diamond (1971) and Michael Rothschild (1973) from a theoretical point of view and Tobin (1972) from an empirical point of view argued that the requirement of a wage distribution was not consistent with the other curve. This was the main point of critic for Phelps (1970) and Dale Mortensen (1970).

A model not involving matching problems, presented by Lucas and Prescott (1974), satisfied Rothschild's criticisms, but it involved a trivial role for workers looking for alternative jobs. Some of the first to suggest the concept of the matching function that downplay the role of reservation wages include among others Pissarides (1979). Pissarides (1979, 1984) discussed the zero-profit conditions for new jobs, leading to a closed model with endogenous demand for labor.

Diamond (1982) was the first to apply the Nash solution assuming fixed numbers of traders, though earlier work included similar sharing rules for the division of the surplus from a job match. Pissarides (1984) also applied the Nash rule to derive a wage equation.

Search theory became easily popular, as it appeared realistic. The main concept of the theory is the matching function. The background of this idea is that the job search does not simply aim to a high wage, but rather to a good job match. Also both the worker and the firm are concerned to find a good match and thus they both need to spend time and other resources before signing a contract. Each worker has different skills that make him suitable for different kind of jobs. On the other hand, job requirements vary across firms too, and thus firms are concerned on the type of the worker that will fill each position. This approach makes unemployment neither voluntary nor involuntary, but it is instead an outcome that might not be optimal. The matching function accounts for different characteristics and frictions of the labor markets but those need not be made explicit; it serves as a black box. One can recall the concept of a production function which is a black box of technology. The model was estimated by Pissarides (1986) and Blanchard and Diamond (1989) with encouraging results.

3.2. A Simple Search Model (Pissarides 2000)

We start by stating the non trivial role of trade in our economy. Trade is a decentralized activity, uncoordinated, time consuming and costly for both workers and firms who have to spend resources while participating in it. Existing jobs command rents in equilibrium and this is a property of a non Walrasian equilibrium. As abovementioned, trade is a non trivial activity and this comes us a result of heterogeneities, frictions and information imperfections. Firms and workers are not identical to each other and information acquisition is costly.

Next we assume the existence of a well behaved matching function, m(.), that gives the number of jobs in terms of the number of workers looking for jobs, the number of firms looking for workers and possibly some other variables. This function captures the implications of costly trading without the need to make the heterogeneity explicit, as happens in the case of the production function or the one of the demand for money function.

We need to clearly state the assumption on which the model is based:

- Full specialization in either trade or production
- Only vacant jobs can engage in trade
- Firms do not specialize but jobs do
- Only unemployed workers search for job
- Firms and workers act with full knowledge of job-matching and job-separation, but do not attempt to coordinate
- There are many firms and workers (not identical to each other) operating under atomistic behavior
- Lastly, we assume rational expectations

We will build the model in continuous time. There are L workers in the economy, the unemployment rate is u and the number of vacant jobs as a fraction of the labor force, i.e. the vacancy rate, is v. Only the uL unemployed workers and the vL jobs engage in matching. The numbers of jobs per unit time are given by:

mL= m (uL, vL) (3.1)

The matching function is assumed to be increased in both arguments, concave and homogeneous of degree 1[3].

Unemployed workers and job vacancies that are matched at any point are randomly selected from the sets vL and uL, thus the process changing the state of vacant jobs is Poisson with rate $\frac{m(uL, vL)}{vL}$. Introduce $q = \frac{v}{u}$, known as the labor market tightness, we can state the rate at which vacant jobs become filled:

$$q(q) = m\left(\frac{u}{v}, 1\right) \qquad (3.2)$$

[3] There are convincing empirical reasons allowing us to assume that. This leads us to a balanced growth path.

During a small time interval dt, a vacant job becomes filled with probability q(q)dt, so the mean duration of a vacant job is 1/q(q). By the properties of m(.), q´(q)=0 and -1< q(q) <0. The rate at which an unemployed worker move into employment follows a Poisson process and is equal to :

$$qq(q) = m\frac{(uL, vL)}{uL} \qquad (3.3)$$

The flow into unemployment also follows a Poisson process and is equal to I. This flow results from job specific shocks, such as change in demand or productivity shocks.

Job creation happens when a worker and a firm agree to a negotiated wage. Once a job is created, production continues until a negative idiosyncratic shock arrives. Job destruction happens when the productivity of the job moves to the low value and equals job separation. The worker moves to the unemployment and the firm can either withdraw from the market or reopen a job as a new vacancy. Job separations have the same rate I at which a worker moves from employment to unemployment. This rate follows a Poisson process which is independent and exogenous.

Labor force is assumed to be fixed. The mean number of workers entering unemployment is I(1-u)dt, while the mean number of workers leaving unemployment is mLdt or uqq(q)Ldt. The evolution of mean unemployment given the the two flows is:

$$\dot{u} = I(1-u) - qq(q)u \qquad (3.4.)$$

In the steady state $\dot{u} = 0$ so the unemployment in term of the two flows:

$$u = \frac{I}{I + qq(q)} \qquad (3.5)$$

Equation (3.5) is also known as the flow equilibrium condition. It implies that for a given I, q there is a unique equilibrium unemployment rate. I is a parameter of the model, while q is unknown.

When worker and a firm, after searching, agree to a contract there is job creation. Contract specifies a wage rule which depends on observed variables. In our model hours of work are fixed and normalized to one. We assume that firms are small, i.e. each firm had only one job that is vacant when it first enters the market. When a job is occupied, firm rents capital and produces output which is sold in competitive markets. Assume that the value of the job's output is p>0. When the job is vacant, it has a fixed cost pc>0. During hiring, workers arrive to vacant jobs at rate q(q) independently of what the firm does. The number of jobs is exogenously given by profit maximization.

Let J denote the present- discounted value of expected profit from an occupied job. Let also V denote the present- discounted value of expected profit from a vacant job. We also assume:

- Perfect capital market
- Infinite horizon
- No dynamic changes in parameters expected

then V satisfies the Bellman equation:

$$rV = -pc + q(q)\,(J-V) \qquad (3.6.)$$

Equation 3.6 gives the asset value of vacant jobs and is also known as the Beveridge curve. This is a downward sloping, convex function. A job is an asset for the firm. Here the capital cost, rV, equals the rate of return of the asset. Vacant jobs cost pc and the rate at which vacant jobs become filled, q(q), follow a Poisson process. The change of state yields a return J-V, where J and V are constants.

In equilibrium all opportunities from new jobs are exploited, therefore V=0. From 3.4.. :

$$J = \frac{pc}{q(q)} \qquad (3.7.)$$

The expected duration of a vacancy for a firm is 1/q(q). The asset value of an occupied job is:

$$rJ = p - w - IJ \qquad (3.8)$$

where p is the job output's value and w is the cost of labor, determined by bargaining. The flow capital cost of the job, rJ, equals the net return of the job, p-w, minus the product of I (the risk of an adverse shock leading to the loss of J) with J. From equations 1.6 and 1.7 we obtain the marginal condition for the demand of labor:

$$p - w - \frac{(r+I)pc}{q(q)} = 0 \qquad (3.9)$$

Equation 1.8 is also known as the job creation condition and it can be represented in the q,w space. The marginal product of labor is p while $\frac{r+I}{q(q)}pc$ is the expected capitalized value of the firm's hiring cost. If there is no hiring cost, c=0, so we have the standard marginal productivity condition for employment in the steady state, r=w. Equations 1.5 and 1.8 include four unknowns: unemployment, number of jobs, real wage rate and real interest rate. We take solutions of quantities (unemployment and number of jobs) in terms of prices (real wage rate and real interest rate).

As mentioned above, we assume that the size of labor force is fixed. Furthermore, each workers' intensity is fixed and there is common productivity in all jobs. Those assumptions imply that job-acceptance is a trivial decision and thus the only influence that workers have is through their wages.

A worker earns w when employed and some real return z when unemployed. This z includes anything worker has to give up when she gets a job, e.g. insurance benefits, real returns from home production etc. and is assumed to be constant and independent of markets returns. Let U and W be the present discounted value of the expected income stream of an unemployed and an employed worker respectively. The expected real return the unemployed enjoys is z and the probability she expects to move into unemployment is qq(q). The valuation placed on an unemployed worker by the market is:

rU= z+ qq(q) (W-U) (3.10)

The asset of an unemployed worker is her hum capital. The average expected return on the worker's human capital during search is rU and this is the minimum compensation that an unemployed worker requires to give up search, i.e. it is the unemployed worker's reservation wage while it also is her normal (permanent) income. The expected capital gain from the change of the state is qq(q) (W-U). The valuation placed on an employed worker by the market is:

rW= w+ I(U-W) (3.11)

Permanent income of employed workers, rW, is different to the constant wage, w, because of the risk of unemployment, I. Workers stay on their job for as long as W>U. If we solve equation 3.8 and 3.9 for the permanent incomes of employment and unemployment we get:

$$rU = \frac{(r+I)z + qq(q)w}{r+I+qq(q)} \qquad (3.12)$$

$$rW = \frac{Iz + [r+qq(q)]w}{r+I+qq(q)} \qquad (3.13)$$

Since w>z, employed workers have higher permanent income than unemployed workers. Without discounting they are equal in permanent- income terms as in infinite horizons all workers eventually participate equally in unemployment and unemployment.

4. The New Keynesian theory

4.1 Introduction

It is true that real business cycle models are differ from the New Keynesian ones in many aspects such as the policy implications and the absence of endogenous capital accumulation in the latter[4] but one can claim that they share some common characteristics. Those can be resumed by the following facts; they both assume a representative household who is a utility maximizer and whose utility increases with the expand of consumption and leisure and are constrained by their available budget, they also assume the existence of a big number of firms who maximize their profits subject to the common production function.
The New Keynesian approach combines the dynamic stochastic general equilibrium (DSGE) structure with assumptions that differ from those suggested by the classical monetary models.

[4] A feature that is easy to incorporate and is a common feature of medium-scale versions of Keynesian models

To begin with, they assume monopolistic competition. There is no Walrasian auctioneer to set the prices and clear all markets but they are determined by private economic agents who sick to maximize their objectives. Also, nominal rigidities are introduced. Firms cannot freely adjust the prices of goods, and workers are subject to sticky wages. Finally, there is room for monetary policy in the short run. In the presence of nominal rigidities, changes in nominal rates in the short run are not matched by one-to-one changes in expected inflation and this implies changes in the real interest rates. This cause variations in investment and consumption which in turn cause variations in employment and output. In the long term though, all prices and wages fully adjust, leading the economy to its natural equilibrium.
All those characteristics had a central role in the 1970s and 1980s literature of the New Keynesian theory. The models suggested though were not based on dynamic optimization of the private agents but rather they were static and trying to provide microfoundations. The modern New Keynesian literature works under a DSGE framework, adopting important characteristics of the RBC theory but still differing a lot from it. To begin with, the economy responds to shocks inefficiently. Also, the non-neutrality of monetary policy makes room for welfare-enhancing interventions in order to minimize the existing distortions. Finally, those models can be used for the comparison of alternative monetary regimes, without being subject to the Lucas critique.

4.2.The Basic Newkeynesian Model

4.2.1.Households

Representative household maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{4.1}$$

s.t.

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{4.2}$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-1/\varepsilon} di \right)^{\varepsilon/\varepsilon-1}$ is a consumption index with $C_t(i)$ being the consumption of good i. We also assume a continuum of goods represented by the interval [0,1]. $P_t(i)$ is the price of good i, W_t the nominal wage, B_t the one period nominally riskless discount bonds that pays one unit of money at maturity and its price is Q_t and T_t are the lump-sum additions/ substractions to period income in nominal terms.

We impose the solvency condition

$$\lim_{T\to\infty} E_t \left\{ B_T \right\} \geq 0, \forall t \tag{4.3}$$

Representative household chooses paths for the consumption, labor supply and allocation of consumption expenditures among different goods. The consumption index needs to be maximized for any given level of expenditures and thus becomes

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t \tag{4.4}$$

where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{1/1-\varepsilon}$ is an aggregate price index.

Given the above, total consumption expenditures can be written as the price index time the quantity index:

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t \tag{4.5}$$

Substituting the above to the budget constraint, we obtain:

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{4.6}$$

The first order conditions from the maximization are:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \qquad (4.7)$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \qquad (4.8)$$

4.2.2. Firms

Assume that there is a continuum of firms represented by an interval [0,1]. All firms face common technology, given by the following production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \qquad (4.9)$$

where A_t represents the common level of technology, which is exogenously given.

Each firm $i \in [0,1]$ produces a differentiated product. All firms face a common demand given by the household's optimization and take aggregate price level and aggregate consumption index as given.

4.2.3. Equilibrium

For the goods market to clear it is required that

Yt (i) = Ct (i), for $\forall i \in [0,1]$ and $\forall$ t (4.10)

We define aggregate output as

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{1-1/\varepsilon} di \right)^{\varepsilon/\varepsilon-1} \qquad (4.11)$$

so that

Yt = Ct for $\forall$ t

If we combine the above clearing condition with the consumer's Euler equation we can obtain the equilibrium condition

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \tag{4.12}$$

Finally, labor market clearing requires:

$$N_t = \left(\frac{Y_t}{A_t}\right)^{1/1-\alpha} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon/1-\alpha} di \tag{4.13}$$

where $di \equiv (1-\alpha)\log\int_0^1 \left(P_t(i)/P_t\right)^{-\varepsilon/1-\alpha}$ is a measure of price dispersion across firms.

4.3. Blanchard and Gali (2006)

4.3.1. Introduction

The following model combines two typically different streams of research; one the one hand we have the New Keynesian approach and on the other hand the Diamond-Mortensen-Pissarides model of search and matching. The purpose of this combination is to accommodate all the following properties, which hold in industrialized economies:

- Variations in unemployment are an important aspect of fluctuations
- The nature of wage bargaining and labor market frictions are central to understanding movements in unemployment.
- The effects of technology and other real shocks are largely determined by the nature of nominal rigidities and monetary policy

In this model the presence of hiring costs, increasing with the degree of labor market tightness, implies frictions in labor markets. Under the assumptions on technology and preferences, the constrained-efficient allocation implies a constant level of unemployment.

Both firms and workers need to spend resources in order to achieve a good match. This leads to a surplus with the wage determining how it is split between workers and firms. Two different wage setting structures for equilibrium are considered; Nash bargained wages reveal the property of a constant unemployment rate which characterizes the constrained efficient allocation, but in the decentralized economy the implied level of unemployment is generally inefficient. Pissarides (2000) and Shimer (2005) come to the opposite result. This happens due to the different utility specification; the reservation wage, rather than being constant, increases in proportion to consumption and thus with productivity. This proportional increase in real wages and productivity leaves all labor market flows unaffected.

Following, the scope for and the implications of real wage rigidity are studied. Real wage rigidity is found to affect hiring decisions, but not employment in existing matches[5]. The dynamic effects of technology shocks on unemployment are defined as a function of the degree of real wage rigidity and other characteristics of the labor market. Unemployment fluctuations increase as real rigidities increase, whereas the persistence of those fluctuations is higher in markets with lower average job-finding and separation rates.

Fluctuations of unemployment are always inefficient. When nominal rigidities are introduced, monetary policy can influence those fluctuations, motivating the analysis of alternative monetary policies, see for example Christoffel and Linzert (2005).

4.3.2. The Model

Assume that there is a representative household maximizing:

$$E_0 \sum \beta^t \left(\log C_t - \chi \frac{\mathrm{N}_t^{1+\varphi}}{1+\varphi} \right) \qquad (4.14)$$

where C_t is a CES function with elasticity ε and $0 \leq N_t \leq 1$ is the fraction of household members who are employed.

Assume also that there is a continuum of firms represented by the interval [0,1]. Each firm $i \in [0,1]$ produces a differentiated good, but they all share the following production function:

$$\mathrm{Y}_t(\mathrm{i}) = \mathrm{A}_t \mathrm{N}_t(\mathrm{i}) \qquad (4.15)$$

where At is the common technology for all firms which is exogenously given.

[5] See also Hall (2005)

Note that employment in firm i evolves according to

$N_t(i) = (1-\delta) N_{t-1}(i) + H_t(i)$ (4.16)

where $0 \le \delta \le 1$ is an exogenous separation rate and $H_t(i)$ the measure of workers hired.

Assume that in the labor market there is full participation, that is individuals are either employed or willing to work. The beginning of the period unemployment is denoted by U_t and is given by

$U_t = 1 - (1-\delta) N_{t-1}$ (4.17)

A fraction $H_t \equiv \int_0^1 H(i)di$ of the unemployed individuals finds a job and starts working in the same period (so $H_t \le U_t$).

Aggregate hiring process is given by:

$H_t = N_t - (1-\delta) N_{t-1}$ (4.18)

where

$N_t \equiv \int_0^1 N(i)di$ is the aggregate employment.

We define the ratio of aggregate hires to unemployment rate as the labor market tightness (or job-finding rate) $0 \le x_t \le 1$ as:

$$x_t \equiv \frac{H_t}{U_t} \tag{4.19}$$

Hiring costs for a firm are given by

$G_t H_t(i)$ (4.20)

where G_t is the cost per hire, is independent of $H_t(i)$and taken as given for each firm but is an increasing function of job finding rate.

Assume that:

$$G_t = A_t B x_t^{\alpha} \tag{4.21}$$

where α , B are a positive constants with δB<1.

The social planner solves

$$E_0 \sum \beta^t \left(\log C_t - \chi \frac{\mathrm{N}_t^{1+\varphi}}{1+\varphi} \right) \tag{4.22}$$

s.t.

$$0 \le N_t \le 1 \quad \text{and} \tag{4.23}$$

$$C_t = A_t (N_t - B x_t^{\alpha} H_t) \tag{4.24}$$

and obtains the optimality condition:

$$\begin{aligned} &\frac{\chi C_t N_t^{\varphi}}{A_t} \le 1 - (1+\alpha) B x_t^{\alpha} \\ &+\beta(1-\delta)\mathrm{E}_t \left\{ \frac{C_t}{C_{t+1}} \frac{A_{t+1}}{A_t} B(x_{t+1}^{\alpha} + a x_{t+1}^{\alpha}(1-x_{t+1})) \right\} \end{aligned} \tag{4.25}$$

The left hand side of the equation is the marginal rate of substitution between labor and consumption, while the right hand side represents the marginal rate of transformation.

4.3.3. Equilibrium under flexible prices

Each firm produces a differentiated product and its price is set optimally each period, given demand. Firm solves:

$$\max E_t \sum_k Q_{t,t+k} (P_{t+k}(i) Y_{t+k}(i) - P_{t+k} W_{t+k} N_{t+k}(i) - P_{t+k} G_{t+k} H_{t+k}(i) \qquad (4.26)$$

s.t.

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} (C_t + G_t H_t) \qquad (4.27)$$

where $Q_{t,t+k} \equiv \beta^k \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}}$ is the stochastic discount factor for nominal payoffs.

Define $M \equiv \frac{\varepsilon}{\varepsilon - 1}$ as the optimal markup. The optimal price setting rule is:

$P_t(i) = M\, P_t\, MC_t$ (4.28)

where the firm's real marginal cost is defined as:

$$\frac{W_t}{A_t} = \frac{1}{M} - Bx_t^{\alpha} + \beta(1-\delta) E_t \left\{ \frac{C_t}{C_{t+1}} \frac{A_{t+1}}{A_t} Bx_{t+1}^{\alpha} \right\} \qquad (4.29)$$

which depends on both the wage normalized by productivity and the current and expected hiring costs.

Imposing symmetry in prices it is implied that $MC_t = 1/M$

Combining all the above and rearranging we have:

$$\frac{W_t}{A_t} = \frac{1}{M} - Bx_t^{\alpha} + \beta(1-\delta) E_t \left\{ \frac{C_t}{C_{t+1}} \frac{A_{t+1}}{A_t} Bx_{t+1}^{\alpha} \right\} \qquad (4.30)$$

Note that a characterization of the equilibrium requires that a wage determination is specified. Following we take several assumption on that.

4.3.4 Equilibrium with Nash Bargaining Wages

The value of marginal employment is:

$$\frac{\chi C_t N_t^{\varphi}}{A_t} = \frac{1}{\mathrm{M}} - (1+\theta)Bx_t^{\alpha} +$$

$$\beta(1-\delta)\mathrm{E}_t\left\{\frac{C_t}{C_{t+1}}\frac{\mathrm{A}_{t+1}}{\mathrm{A}_t}\mathrm{B}({x^{\alpha}}_{t+1} + \theta {x^{\alpha}}_{t+1}(1-{x^{\alpha}}_{t+1}))\right\} \qquad (4.31)$$

where W_t^N and W_t^U are represent the value to the household of having a marginal member employed or unemployed respectively. The above implies that:

$$W_t^U = \beta\mathrm{E}_t\left\{\frac{C_t}{C_{t+1}}[x_{t+1}W_{t+1}^N + (1-x_{t+1})W_{t+1}^U]\right\} \qquad (4.32)$$

and therefore

$$W_t^N - W_t^U = W_t - \chi C_t N_t^{\varphi} +$$

$$\beta(1-\delta)\mathrm{E}_t\left\{\frac{C_t}{C_{t+1}}(1-x_{t+1})(W_{t+1}^N - W_{t+1}^U)\right\} \qquad (4.33)$$

We denote the relative weight of workers in the Nash bargain by θ.

It is required that

$$W_t^N - W_t^U = \theta G_t \qquad (4.34)$$

Imposing the above condition we obtain the Nash wage schedule:

$$\frac{W_t}{A_t} = \frac{\chi C_t N_t^{\varphi}}{A_t} + \theta B x_t^{\alpha} - \beta(1-\delta)\mathrm{E}_t\left\{\frac{C_t}{C_{t+1}}\frac{\mathrm{A}_{t+1}}{\mathrm{A}_t}(1-x_{t+1})\theta B {x^{\alpha}}_{t+1}\right\} \qquad (4.35)$$

the above implies that the equilibrium under Nash Bargaining can be described by

$$\frac{\chi C_t N_t^{\varphi}}{A_t} = \frac{1}{\mathrm{M}} - (1+\theta)Bx_t^{\alpha} +$$
$$\beta(1-\delta)\mathrm{E}_t\left\{\frac{C_t}{C_{t+1}}\frac{\mathrm{A}_{t+1}}{\mathrm{A}_t}\mathrm{B}(x^{\alpha}{}_{t+1} + \theta x^{\alpha}{}_{t+1}(1-x^{\alpha}{}_{t+1}))\right\} \tag{4.36}$$

along with

Ht = Nt - (1-δ) Nt-1 (4.37)

$$x_t \equiv \frac{H_t}{U_t} \tag{4.38}$$

$$C_t = A_t(N_t - Bx_t^{\alpha}H_t) \tag{4.39}$$

and an exogenously given technology.

It can be proved that that the equilibrium under Nash bargaining involves a constant level of employment:

$$N^{nb} = \frac{x^{nb}}{\delta + (1-\delta)x^{nb}} \equiv \mathrm{N}(x^{nb}) \tag{4.40}$$

where x^{nb} is the constant equilibrium job finding rate.

Imposing the equilibrium conditions we get the following expression for the equilibrium Nash bargained wage:

$$\frac{W_t^{nb}}{A_t} = \frac{1}{M} - (1-\beta(1-\delta))\mathrm{B}(x^{nb})^{\alpha} \tag{4.41}$$

Note that productivity shocks have no effect on employment and are reflected one-for-one in the Nash bargained wage6. Here we allow for concave preferences and an endogenous marginal rate of substitution between labor and consumption. Changes in productivity do not affect the relative bargaining of firms and workers.

[6] See Shimer (2005) for a different result.

4.3.5. Equilibrium under Real Wage Rigidities

Assume stationary technology and the following wage schedule:

$$Wt = \Theta \mathrm{A}_t^{1-\gamma} \tag{4.42}$$

where $0 \leq \gamma \leq 1$ is an index of real wage rigidities and Θ is a strictly positive constant.

Note that for $\gamma = 0$ we have the Nash bargaining wage case. When $\gamma = 1$ we get the canonical example of a rigid wage analyzed by Hall (2005).

Combining the wage schedule with the extra assumption that firms employ workers only if the will not incur any loss, which only if W_t/A_t is strictly smaller than 1/M, we get:

$$MC_t = \Theta A_t^{-\gamma} + \mathrm{B}x_t^{a} + \beta(1-\delta)\mathrm{E}_t\left\{\frac{C_t}{C_{t+1}}\frac{A_{t+1}}{A_t}Bx_{t+1}^{\alpha}\right\} \tag{4.43}$$

or rearranged:

$$Bx_t^{\alpha} = \sum_{k=0}^{\infty}(\beta(1-\delta))^k E_t\left\{\Lambda_{t,t+k}\left(\frac{1}{M} - \Theta \mathrm{A}_{t+k}^{-\gamma}\right)\right\} \tag{4.44}$$

where $\Lambda_{\mathrm{k,t+k}} \equiv \left(C_t / C_{t+k}\right)\left(A_{t+k} / A_t\right)$. Note that if wages are not fully flexible, variations in productivity affect the job finding rate. The effect is a decreasing function of the sensitivity of hiring costs to labor market condition (measured by parameter α). The above mentioned imply that real wage rigidities cause the equilibrium to be characterized by inefficient in unemployment, followed by a return to its normal level.

4.3.6. Sticky Prices

We introduce sticky prices in the model with labor market frictions, following Calvo 1983. Each period, a fraction 1-θ of firms resets prices, following the optimal rule:

$$\mathrm{E}_t\left\{\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}Y_{t,t+k|t}(P_t^* - MP_t + kMC_{t+k})\right\} = 0 \tag{4.45}$$

where P_t^* is the new price set by the firm at time t, $Y_{t,t+k|t}$ is the output level in period t+k for a price set in period t and $M \equiv \frac{\varepsilon}{\varepsilon - 1}$ is the gross desired markup. This optimal price setting equation takes the same form as in Calvo (1983). Firm chooses a price that is a weighted average of current and expected marginal costs, weighted by a function of θ.

The real marginal cost is given by:

$$MC_t = \Theta A_t^{-\gamma} + \mathrm{B} x_t^{a} + \beta(1-\delta)\mathrm{E}_t \left\{ \frac{C_t}{C_{t+1}} \frac{A_{t+1}}{A_t} B x_{t+1}^{\alpha} \right\} \qquad (4.46)$$

which is influenced by the presence of labor market frictions.

The model presented combines the main points of the New Keynesian framework and the Diamond-Mortensen-Pisarrides search model of labor market flows; it contains labor market frictions, real wage rigidities, and staggered price setting. Those three ingredients are vital in order to explain the movements of unemployment, the effects of productivity changes on the economy, and the role of monetary policy in shaping those effects. We derived the constrained-efficient allocation, and the equilibrium in the decentralized economy in the presence of real wage rigidities and sticky prices and showed that the optimal monetary policy minimizes a weighted average of unemployment and inflation fluctuations. The extent of real wage rigidities determines the amplitude of unemployment fluctuations under the optimal policy. Finally, we showed that unemployment displays intrinsic persistence, i.e. persistence beyond that inherited from productivity.

5. Conclusions

In the present study, we presented different approaches on how to model labor markets in a dynamic general equilibrium macroeconomic framework.

We started by presenting the neoclassical model of demand and supply. Labor market is assumed to be similar to the other markets in that price (wage rate) and quantity (number of people employed) are jointly determined by the forces of supply and demand. Households supply labor and receive income. They maximize their utility and choose between income and leisure while being constrained by the working hours available to them.

There is a trade-off between leisure activities and income generating activities and must allocate among those two giving the limited time they have. This decision is represented by the indifference curve in which all combinations of leisure and work that will give the individual a specific level of utility are indicated. The short-run equilibrium is the point where the highest indifference curve is just tangent to the constraint.

The important features of neoclassical theory are that (1) there is no involuntary unemployment and factor market equilibrium determines the real wage (2) output is determined by the production function and thus it is supply determined (3) interest rate brings goods market in equilibrium where aggregate demand equals aggregate supply, savings equal investment and demand equals the supply of loanable funds and money neutrality holds.

In conclusion, Neoclassical macroeconomic theory, suggests that, by Walras' Law, if all markets for goods are in equilibrium, the market for labor must also be in equilibrium, contradicting the New Keynesian theory.

We continued with the real business cycle theory and presented the basic RBC model based on the notable work of Kydland and Prescott (1982) and Long and Plosser (1983). While this model has performed very well in explaining most macroeconomic variables, it has had poor results in explaining the labor markets. A simple model with fixed labor and non Walrasian characteristics was suggested by Jean- Pierre Danthine and John B Donaldson (1995).There are two types of workers (insiders and outsiders) and the non Walrasian feature used is a minimum wage. The result is that in contrast to Walrasian model, there is sufficient employment variability generated by movements in and out of employment.

We proceeded with the very important idea of indivisible labor. Hansen (1985), Rogerson (1988) and Rogerson and Wright (1988) argued that the fluctuations in aggregate hours are the result of individuals entering and leaving employment and not of continuously employed individuals adjusting the number of hours worked. So the assumption made was that individuals either work a specific positive amount of time or not at all. The HRW (Hansen-Rogerson-White) model made a significant improvement to the match between calibrated and actual U.S. postwar time series regarding the labor market.

Despite the fact that the HRW performed really well in the U.S. data, its results on the European labor markets were very poor, as reported by Maffezzoli (2001). The latter assumed that this was due to the different institutional framework between the two datasets and suggested a monopoly union model to capture the features of the European markets –especially the Italian one- which have a high degree of unionization. The model lead to an non Pareto optimal equilibrium due to the existence of the unions. The calibration of the MU model showed that in fact it explains much better the European business cycle, while the HRW has a comparative advantage in that of the U.S.   The impulse response functions of the monopoly union model exhibit a higher degree of overall existence that the HRW model. Finally the business cycle statistics of the two models are similar.

In the third chapter we attempt an introduction to search theory. The latter tries to address issues such as why unemployed workers may choose to remain unemployed or what determines the length of unemployment and other similar topics that classical theory has failed to answer.

Trading frictions play a central role in search theory. Workers and firms spend resources in order to form a job agreement. We leave the assumption of the classical theory that there is a centralized market where firms and workers meet and trade at a single price.

An important concept is that of the matching function. Pissarides and Petrongolo (2001) described it as a “black box” capturing the different characteristics and frictions of the labor markets without making them explicit. It basically captures the main idea of a good match; it takes time to find a good match, the length of the time depend on unpredictable parameters and the more available job vacancies exist- given the number of individuals looking for jobs- the faster a match will take place. As the matching function was similar to other aggregate function in economic models, it was possible to make small equilibrium models of the labor market with features captured by tha matching function.

As both agents are concerned to find a good match, they both need to spend time and other resources before signing a contract. This approach makes unemployment neither voluntary nor involuntary, but it is instead an outcome that might not be optimal. The model was estimated by Pissarides (1986) and Blanchard and Diamond (1989) with encouraging results.

In the final chapter, we present the New Keynesian approach. This theory, like the RBC, assumes the existence of a large number of identical households who maximize their utility subject to their budget constraint and a large number of firms sharing common and maximizing their profits subject to exogenous shocks. The New Keynesian theory can be summarized as follows: (1) it assumes monopolistic competition, thus it rejects the existence of a Walrasian auctioneer as the price setter in favor of private economic agents who maximize their objectives, (2) it assumes prices and wages stickiness in the short term and (3) Due to the abovementioned stickiness, there is room for economic policy in the short run.

We describe a basic New Keynesian model and continue by presenting a model by Blanchard and Gali (2006) which incorporates the New Keynesian theory as well as the Diamond-Mortensen-Pissarides model of search and matching. The aim is to capture the following features of industrialized economies; (1)Variations in unemployment are an important aspect of fluctuations (2)The nature of wage bargaining and labor market frictions are central to understanding movements in unemployment and (3)The effects of technology and other real shocks are largely determined by the nature of nominal rigidities and monetary policy

The model combines the main features of both theories; it contains labor market frictions, real wage rigidities, and staggered price setting. Those three ingredients are vital in order to explain the movements of unemployment, the effects of productivity changes on the economy, and the role of monetary policy in shaping those effects. The model results that unemployment displays persistence beyond that inherited from productivity.

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