ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

DEPARTMENT OF STATISTICS

POSTGRADUATE PROGRAM

CONTROL CHARTS FOR AUTOCORRELATED PROCESSES

By

Georgia Ekaterini A. Papaleonida

A THESIS
Submitted to the Department of Statistics
of the Athens University of Economics and Business
in partial fulfilment of the requirements for
the degree of Master of Science in Statistics

Athens, Greece
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ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ΤΜΗΜΑ ΣΤΑΤΙΣΤΙΚΗΣ

ΔΙΑΓΡΑΜΜΑΤΑ ΕΛΕΓΧΟΥ ΓΙΑ ΑΥΤΟΣΥΣΧΕΤΙΣΜΕΝΕΣ ΔΙΕΡΓΑΣΙΕΣ

Γεωργία Αικατερίνη Α. Παπαλεωνίδα

ΔΙΑΤΡΙΒΗ

Που υποβλήθηκε στο Τμήμα Στατιστικής

του Οικονομικού Πανεπιστημίου Αθηνών

ως μέρος των απαιτήσεων για την απόκτηση

Μεταπτυχιακού Διπλώματος Ειδίκευσης στη Στατιστική

Αθήνα

Δεκέμβριος 2001

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A Thesis submitted in partial fulfilment of
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CONTROL CHARTS FOR AUTOCORRELATED PROCESSES

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August 2002

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I thank God for giving me clear mind, strength and inspiration. I also thank my supervisor Lecturer S. Psarakis, whose guidance made this effort look easier and more pleasant. Finally, I am grateful to my parents, for financial and emotional support.

VITA

I was born in Kalamata in 1971. I graduated from Paralias high school in 1989. I studied Mathematics at the University of Athens. I completed my studies in 1996. I started my postgraduate studies in Statistics at the Athens University of Economics and Business in 1999.

ABSTRACT

Georgia Ekaterini Papaleonida

CONTROL CHARTS FOR AUTOCORRELATED PROCESSES

December 2001

Statistical Process Control primarily involves the implementation of control charts. Control charts are used to monitor the quality and the stability of processes.

Quality control methodology makes assumptions about the processes. One of those is the assumption of independence. In real industrial environments though, process data is often correlated or exhibits some serial dependence. More sophisticated *SPC* techniques are then needed to overcome the correlative structure of the data.

The aim of this dissertation is to present, to apply and to evaluate control charts that are designed to account for autocorrelation.

ΠΕΡΙΛΗΨΗ

Γεωργία Αικατερίνη Παπαλεωνίδα

ΔΙΑΓΡΑΜΜΑΤΑ ΕΛΕΓΧΟΥ ΓΙΑ ΑΥΤΟΣΥΣΧΕΤΙΣΜΕΝΕΣ ΔΙΕΡΓΑΣΙΕΣ

Δεκέμβριος 2001

Ο Στατιστικός Έλεγχος Διεργασιών ασχολείται κυρίως με την εφαρμογή διαγραμμάτων ελέγχου. Τα διαγράμματα ελέγχου χρησιμοποιούνται για την παρακολούθηση της ποιότητας και της σταθερότητας των διεργασιών.

Η μεθοδολογία του ελέγχου ποιότητας κάνει υποθέσεις για τη δομή των δεδομένων. Μία από αυτές είναι η υπόθεση της ανεξαρτησίας. Στην πραγματικότητα όμως πολύ συχνά τα δεδομένα παρουσιάζουν γραμμική εξάρτηση ή αυτοσυσχέτιση. Στην περίπτωση αυτή χρειάζονται πιο εξελιγμένες τεχνικές στατιστικού ποιοτικού ελέγχου .

Σκοπός αυτής της διατριβής είναι να παρουσιάσει, να εφαρμόσει και να αξιολογήσει διαγράμματα ελέγχου που έχουν ειδικά σχεδιαστεί λαμβάνοντας υπόψη τη συσχετισμένη δομή των δεδομένων.

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CHAPTER 1

Introduction

In recent years, **quality** has become one of the most important consumer decision factors in the selection among competing products and services. One can define quality, as successfully meeting requirements set by sellers or buyers.

Quality improvement processes aim to the reduction of variation in products, and are widely used in manufacturing and service industries, increasing their competitiveness, by improving product quality and decreasing production cost. Quality control in manufacturing has moved from detecting nonconforming products through inspection, to detecting quality abnormalities in the process by the use of statistical methods.

Statistical Quality Control *(SQC)* includes four major areas: **acceptance sampling** where inspection of a product leads to its acceptance or rejection based on adherence to a standard, **capability analysis** where the capability of a process to meet specification limits on key quality characteristics is assessed, **Statistical Process Control *(SPC)*** where some statistical and problem-solving techniques are employed to monitor production processes over time, to detect changes to process performance, and consequently to help achieve process stability and improve capability through the reduction of variability and **design of experiments** where important factors affecting process and product quality are identified along with their specific levels that ensure optimum performance.

A production process, regardless of how well designed or carefully maintained it is, will always hold a certain amount of inherent or natural variability. A process is said to be **in statistical control** when these patterns of variation are random, or as Dr. Walter A. Shewhart stated when a process is operating with only **chance causes** of variation.

Unfortunately, once a state of statistical control is attained, departures from statistical control in key quality characteristics may occur. After detecting these departures, explanations for them in terms of **assignable** or **special causes** are seeked. A process which is operating in the presence of special causes is said to be **out of control**.

Statistical process control, primarily involves the implementation of **control charts**. Control charts are used to monitor the quality and stability of processes in the sense of detecting the occurrence of special causes. A special cause may produce changes in a process such as a shift of its mean and/or variance affecting the quality of the output and leading to an out of control state.

Traditionally, quality control methodology makes assumptions about the process which are frequently violated in practice. New technology gives managers the option of using more sophisticated *SPC* models which more accurately reflect the process being monitored, by relaxing some of the assumptions.

One of the assumptions of control charts that is often not valid in practice is that the process whose some quality characteristic is monitored is independent. This is evident in real industrial environments where process data is often correlated or exhibits some serial dependence affecting the efficiency of *SPC* methodologies.

The aim of this thesis is to present, to apply and to evaluate control charts that are designed to account for autocorrelation. Methods for measuring the performance of control charts are also presented. Specifically, Chapter *2* illustrates the basic procedures of Control Charts while Chapter *3* some elementary topics from Time Series theory. In Chapter *4* the effects of autocorrelation to traditional control charts along with the mechanism that generates correlated processes is discussed. Control charts that are designed for monitoring the mean of correlated data are illustrated in Chapter *5*, while in Chapter *6* control charts for monitoring the variance. In Chapter *7* methods for measuring the performance of control charts are presented. In Chapter *8* and in Chapter *9* the performance of the proposed control charts is evaluated.

Finally, in Chapter *10* issues related to autocorrelated processes are stated and in Chapter *11* some general conclusions along with some topics for further research are presented.

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CHAPTER 2

Control Charts

2.1 Introduction

The basic fundamentals of control charting where proposed by **Dr. Walter Shewhart** in the 1920's and 1930's. This framework under which control charts are still constructed is presented in section *2.2* of this chapter.

Control charts may be classified into two categories, depending on whether the monitored quality characteristic can be measured on some continuous (or even quantitative) scale or not. If this is the case, the **variables control charts** are used to monitor the mean or /and the variance of the process. These are presented in section *2.3*. When the quality characteristic in question can not be measured continuously, and each unit of the product can only be judged as either conforming or non-conforming, the so called **attributes control charts** are used which are presented in section *2.4*.

In section *2.5* the **Exponentially Weighted Moving Average** *(EWMA)* and the **Cumulative Sum** *(CUSUM)* charts are briefly discussed, while at section *2.6* some ideas in the design area of control charting.

2.2 Basic fundamentals of Control Charts

Control Charts are the basic tool of *SPC* techniques in the effort to achieve and to maintain stability in industrial or other processes.

The aim of control charts is to establish practical ways of detecting the lack of statistical control of the processes that are monitored.

Checking for a state of statistical control can be regarded (although this is a disputable fact see e.g. *Woodall(2000)*) as an hypothesis testing assuming that the quality characteristic under investigation, is normally distributed with mean μ and variance σ^2 . When the process is in-control with target values for μ and σ^2 as μ_0 and σ_0^2 respectively, monitoring a process in terms of its statistical control while on target, is equivalent to testing the hypotheses

$$H_0: \mu=\mu_0 \qquad\qquad H_0: \sigma=\sigma_0$$
$$H_1: \mu\neq\mu_0 \qquad\qquad H_1: \sigma\neq\sigma_0$$

for monitoring the mean and the variance respectively.

Beyond normality, another basic assumption in calculating the statistical properties of control charts is that the observations from the process are also independent. Control charts are constructed under the assumption that the observations are independent and identically distributed normal around a central mean

$$x_t=\mu+\varepsilon_t$$

where:

x_t= the observation at time t

μ=the fixed process mean

$\{\varepsilon_t\}$ = a sequence of normal independently distributed errors with mean *zero* and variance σ^2_{ε}.

Suppose that m random samples taken at regular intervals are available from a process, each containing n observations $X_1,X_2,\ldots,X_n$. When the process is in-control, the values for μ and σ^2 are the target values μ_0 and σ_0^2 correspondingly, and the general structure of a Shewhart type control chart is

$$\mu_0 \pm c\sigma_0$$

where c is some constant.

A control chart is the graphical representation of the above scheme. The above random samples are used to obtain estimates of a "process average" at each point, which are then plotted on the control chart.

A control chart consists also of a **Center Line** representing the expected value of the process parameter under investigation corresponding to its in-control state and two lines the **Upper Control Limit** *(UCL)* and the **Lower Control Limit** *(LCL)* usually set at three standard deviations from the center line. The chart may also contain upper and lower warning limits set at two standard deviations from the center line. A typical control chart is represented in *Figure 2.1*.

Typically, when all the plotted points are between the limits the process is considered to be in-control. On the other hand, once a point falls outside the control limits the process is declared to be out-of-control.

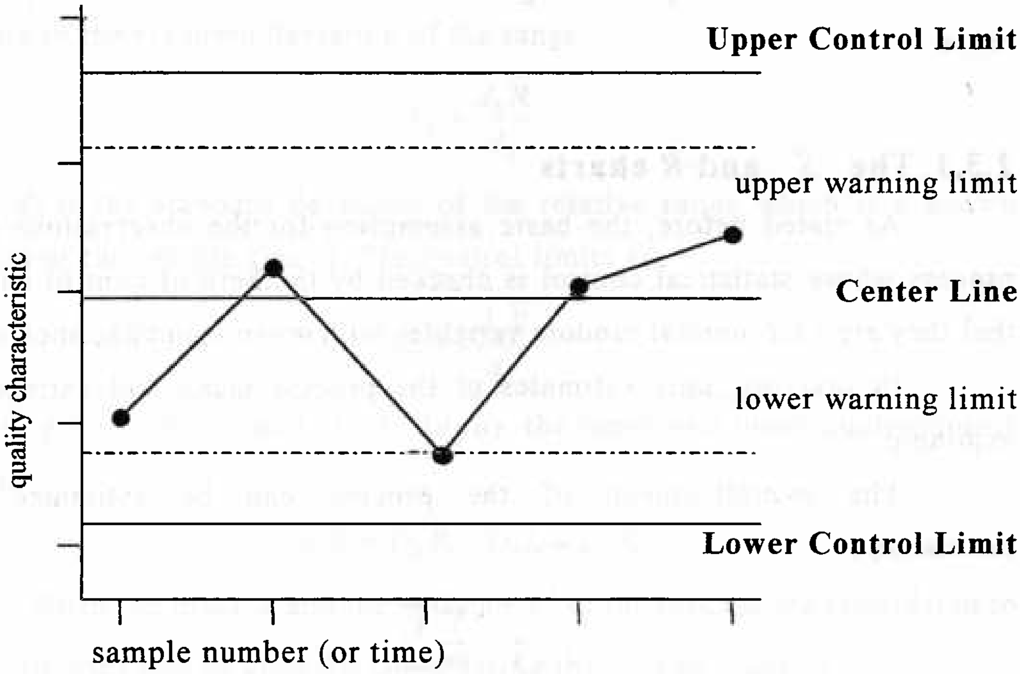

Figure 2.1: **A Typical Structure of a Control Chart.**

Sometimes though, a control chart may indicate an out-of-control state even when all the points are plotted between limits. This is possible when the plotted points exhibit some non random pattern of behavior. A sequence of observations that are of the same type is called a **run**. Many run rules are used in practice to increase the sensitivity of control charts especially to small process shifts. For example a run of eight consecutive points plotted on one side of the center line is an indication of an out-of-control situation. *Montgomery (2001)* as well as *Nelson (1985)* and *Champ and Woodall (1987)* give a detailed discussion on this matter.

2.3 Control Charts for Variables

Many quality characteristics are continuous random variables and can be expressed in terms of a numerical measurement When monitoring such processes usually is necessary to check the statistical control of both the mean and the variance. The mean is usually monitored by the $\overline{X}$-chart and the variance sometimes by the range or R-chart, and other times by the S-chart.

2.3.1 The $\overline{X}$ and R charts

As stated before, the basic assumption for the observations of the process whose statistical control is checked by the help of control charts is that they are *i.i.d.* normal random variables with mean μ and variance σ^2.

In practice, only estimates of the process mean and variance are available.

The overall mean of the process can be estimated by the statistic

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{m} \overline{X_i}}{m}$$

where $\overline{X_i} = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$ is an estimate of the mean within the ith sample.

To estimate the variance of the process in the case where the sample size n is relative small, the statistic $\overline{R}/d_2$ is used, where $\overline{R}$ is the average range. The average range is defined by

$$\overline{R} = \frac{R_1 + R_2 + \ldots + R_m}{m}$$

where $R_i = x_{max} - x_{min}$ is the range within sample i of size n.

The $\overline{X}$-chart or averages Shewhart chart, is used for monitoring the mean of the process and the control limits of the control chart when the variance is estimated by the range are

$$\overline{\overline{X}} \pm \frac{\overline{R}}{d_2}\frac{3}{\sqrt{n}}$$

By defining $A_2 = \dfrac{3}{d_2\sqrt{n}}$ the control limits become

$$\overline{\overline{X}} \pm A_2\overline{R}$$

In the meantime, when constructing control limits to monitor the variability, the distribution of the relative range $W=R/\sigma$ is used to derive an estimate of the standard deviation of the range

$$\hat{\sigma}_R = \frac{d_3\overline{R}}{d_2}$$

where d_3 is the standard deviation of the relative range which is a known function of the sample size n. The control limits are

$$\overline{R} \pm 3\frac{d_3\overline{R}}{d_2}$$

By letting $D_3=1-3d_3/d_2$ and $D_4=1+3d_3/d_2$ the upper and lower control limits become

$$UCL = D_4\overline{R}, \quad LCL = D_3\overline{R}$$

When the mean μ and the variance σ^2 of the process are considered to be known, they can be used for constructing the $\overline{X}$ and R charts.

The control limits for the $\overline{X}$-chart are

$$\mu \pm 3\frac{\sigma}{\sqrt{n}}$$

or by letting the quantity $3\sqrt{n}$ be represented by the constant A the control limits are $\mu \pm A\,\sigma$.

The control limits for the R-chart are

$$d_2\sigma \pm 3d_3\sigma$$

where d_2 is the mean of the distribution of the relative range and d_3 is the standard deviation of the distribution of the relative range.

The above results are also presented at the summarizing tables appearing in *Figure 2.1* and *Figure 2.2* at the end of this section.

2.3.2 The $\overline{X}$ and S charts

The $\overline{X}$ and R charts are widely used, but when the sample size n is relatively large, it is preferable to estimate the unknown variance of the process σ^2 by the sample standard deviation

$$S = \sqrt{\frac{\sum_{j=1}^{n}(x_j - \overline{X})^2}{n-1}}$$

If the underlying distribution is normal, then the mean of S is $c_4\sigma$ where c_4 is a constant depending on the sample size n and the standard deviation of S is $\sigma\sqrt{1-c_4^2}$.

When the unknown standard deviation is estimated by the average sample deviation $\overline{S} = \dfrac{S_1 + S_2 + ... + S_m}{m}$ the control limits for monitoring the mean of the process are

$$\overline{\overline{X}} \pm \frac{3\overline{S}}{c_4\sqrt{n}}$$

By letting $A_3 = 3/\left(c_4\sqrt{n}\right)$ the control limits become $\overline{\overline{X}} \pm A_3\overline{S}$.

The control limits for monitoring the variance are

$$\overline{S} \pm 3\frac{\overline{S}}{c_4}\sqrt{1-c_4^2}$$

By letting $B_3 = 1 - \dfrac{3}{c_4}\sqrt{1-c_4^2}$ and $B_4 = 1 + \dfrac{3}{c_4}\sqrt{1-c_4^2}$, the control limits become $UCL = B_4\overline{S}, LCL = B_3\overline{S}$.

When a standard value for σ is given (or comes from process data) the sample standard deviation can be used to construct the *3-sigma* control limits for S. The limits are

$$c_4\sigma \pm 3\sigma\sqrt{1-c_4^2}$$

By noting $B_5 = c_4 - 3\sqrt{1-c_4^2}$ and $B_6 = c_4 + 3\sqrt{1-c_4^2}$ the control limits for the *S*-chart are $UCL=B_6\sigma$, $LCL=B_5\sigma$.

The above results along with the results from paragraph *2.3.1* are summarized at the table appearing in *Figure 2.2* when the parameters are considered known and at the table in *Figure 2.3* when only their estimates are available.

Variables Control charts μ,σ known			
Chart		**Center Line**	**Control Limits**
$\overline{X}$	***(μ,σ given)***	μ	$\mu \pm A\sigma$
R	***(σ, given)***	$d_2\sigma$	$UCL=D_2\sigma$, $LCL=D_1\sigma$
S	***(σ, given)***	$c_4\sigma$	$UCL=B_6\sigma$, $LCL=B_5\sigma$

Table 2.1 : Formulas for control charts for Variables, Standards Given.

All the quality control factors A, A_2, A_3, B_3, B_4, B_5, B_6, D_1, D_2, D_3, D_4 are tabulated in the literature *i.e., Montgomery (2001)* and $\chi^2_{\alpha/2,n-1}$, $\chi^2_{1-\alpha/2,n-1}$ denote the upper and lower $\alpha/2$ percentage points of the chi-square distribution with $n-1$ degrees of freedom.

Variables Control charts μ,σ unknown			
Chart		**Center Line**	**Control Limits**
$\overline{X}$	***(using R)***	$\overline{\overline{X}}$	$\overline{\overline{X}} \pm A_2\overline{R}$
$\overline{X}$	***(using S^2)***	$\overline{\overline{X}}$	$\overline{\overline{X}} \pm A_3\overline{S}$
R		$\overline{R}$	$UCL = D_4\overline{R}, LCL = D_3\overline{R}$
S		$\overline{S}$	$UCL = B_4\overline{S}, LCL = B_3\overline{S}$
S^2		$\overline{S}^2$	$UCL = \frac{\overline{S}^2}{n-1}\chi^2_{\alpha/2,n-1}, LCL = \frac{\overline{S}^2}{n-1}\chi^2_{1-\alpha/2,n-1}$

Table 2.2 : Formulas for control charts for Variables, no Standards Given.

2.3.3 The individuals *X*-chart

It is quite frequent only one observation to be available at each sample from the observations. This situation demands some special treatment since the range, at least the way it is defined till now, can not be used to estimate the process variability.

In such situations the individuals *X*-chart is used which is especially designed for individual units is useful. The control procedure for this case uses $A=3$ and the moving ranges of pairs of successive observations to estimate the process variability. The moving range is defined as $MR_i=|X_i-X_{i-1}|$ and the control charts become as shown at the table in *Figure 2.4*. The quality factor d_2 is a constant, which is tabulated in the literature *i.e., Montgomery(2001)*.

At section *2.5* of this chapter some charts are described, that are sometimes more appropriate to use in the individuals case especially when the shift in the process characteristic is expected to be small.

individuals chart		
Chart	**Center Line**	**Control Limits**
μ,σ^2 known	μ	$\mu\pm3\sigma$
μ,σ^2 unknown	$\overline{X}$	$\overline{X}\pm3\dfrac{\overline{MR}}{d_2}$

Table 2.3 : Formulas for control charts for individual observations.

2.4 Control Charts for Attributes

A defective unit is called **non-conforming** while a defect is called **non-conformity**. Suppose that m samples of size n are available, and let D_i denote the number of non-conforming items in the ith sample. The sample fraction non-conforming $\hat{p}_i$ is defined as the ratio of the number of non-conforming items in a sample to the total number of items, thus

$$\hat{p}_i = \frac{\sum_{i=1}^{n} D_i}{n}$$

and the average of these sample fraction non-conforming is

$$\bar{p} = \frac{\sum_{j=1}^{m} p_j}{m}$$

Besides, consider the number of times a non-conformity appears and let $\bar{c}$ estimate the average number of non-conformities per sample, while $\bar{u}$ estimates the average number of non-conformities per unit.

The most frequently used charts for attributes are summarized at the table appearing in *Figure 2.5*. For a review of control chart methods based on attribute data the reader can refer to *Woodall(1997)*.

Attribute Control Charts			
Control Chart		**Center Line**	**Control Limits**
p	*fraction non-conforming*	$\bar{p}$	$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
np	*number non-conforming*	$n\bar{p}$	$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$
c	*non-conformities*	$\bar{c}$	$\bar{c} \pm 3\sqrt{\bar{c}}$
u	*average number of non conformities per unit*	$\bar{u}$	$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{n}}$

Table 2.4: Formulas for control charts for Attributes.

2.5 The Exponentially Weighted Moving Range and the Cumulative Sum charts

The previous sections of this chapter were involved with the description of control charts usually referred to as Shewhart control charts, since they are constructed under the basic fundamentals of control charting introduced by Dr. Walter Shewhart.

The major disadvantage of any Shewhart type chart is that it ignores any information given by the entire sequence of points and thus becomes relatively insensitive to small shifts in the monitored quality characteristic of the process. The following two charts namely the **Exponentially Weighted Moving Average** chart or briefly the ***EWMA*** chart, and the **Cumulative Sum** chart or briefly the ***CUSUM*** chart, try to incorporate all the information in the sequence of sample values.

2.5.1 The *EWMA* chart

The exponentially weighted moving average control chart *EWMA* which was originally proposed by *Roberts (1959)*, is designed to detect smaller shifts in the mean more quickly than the Shewhart type charts, by giving exponentially decreasing weights to past observations, the more recent data receiving the heavier weights. Suppose that the sequence of the observations $\{X_i\ ,\ i=1,2,...\}$ is independently normally distributed with mean μ_0 and variance σ^2_X.

The *EWMA* control chart uses a control statistic of the form :

$$Z_i = \lambda X_i + (1-\lambda)Z_{i-1}$$
$$i = 1,2,...,$$

where λ is a smoothing constant satisfying $0< \lambda \leq 1$ and $Z_0=\mu_0$ is the starting value. The smoothing constant λ determines the weight given to past observations. When λ is large, little weight is given to older data, but as it becomes smaller more weight is given to older observations. The variance σ^2_Z of Z_i is given by $\sigma^2_Z=var[Z_i]=[\lambda/(2-\lambda)][1-(1-\lambda)^{2i}]\sigma^2_X$ and when i is large it can be approximated by $\sigma^2_Z=[\lambda/(2-\lambda)]\ \sigma^2_X$.

The control limits of this chart are

$$\mu_0 \pm c\sqrt{\frac{\lambda}{2-\lambda}}\sigma_X$$

where $\sqrt{\lambda/(2-\lambda)}\sigma_X$ is the asymptotic standard deviation of the control statistic Z_i , and c is a constant.

When the observations are independent and the $\lambda=0.2$ using $c=2.859$ will give similar in-control behavior as the Shewhart chart with $c=3$ (see, e.g.

Crowder(1989) or *Lucas and Saccucci (1990)*). When $\lambda=1$ the *EWMA* chart reduces to a standard Shewhart chart. For details in the use and the properties of the *EWMA* charts see *Monopolis(1999)*.

2.5.2 The *CUSUM* chart

Let $\{X_i, i=0,1,2,...\}$ be a given sequence of observations. When the process is in–control, X_i has normal distribution with mean μ_0 and variance σ^2_X which can either be assumed known or can be estimated.

Suppose that the main purpose is to detect a shift of the process mean μ_0 to an out-of-control value μ_1, upwards $(\mu_1>\mu_0)$ or downwards $(\mu_1<\mu_0)$.

As mentioned before the Shewhart charts aren't very effective for small shifts, therefore a control scheme for monitoring the level of $\{X_i\}$ was proposed originally by *Page (1954)*, the Cumulative Sum Control chart *(CUSUM)*. A way to represent cusums is the **tabular *CUSUM***.

The tabular *CUSUM* works by accumulating upward deviations from μ_0 with the statistic C^+ and downwards with the statistic C^-, which are developed as follows:

$$C_i^+=max[0, x_i-(\mu_0+K)+C_{i-1}^+]$$

$$C_i^-=min[0, -(x_i-\mu_0)-K+C_{i-1}^-]$$

where the starting values are $C_0^+=C_0^-=0$ and $K>0$ is a parameter of the chart.

The tabular cusum is constructed by running simultaneously two one sided procedures C_i^+ and C_i^-. In some applications, when there is no significant problem whether the quality characteristic in question exceeds the target value, an one-sided upper cusum scheme can be used, while when there is no problem whether it drops below the target value, an one-sided lower cusum scheme is recommended.

The value of K is usually chosen close to the midway point between μ_0 and the out-of-control value of the mean μ_1. The chart signals if either C_i^+ or C_i^- exceed the decision interval $H=\pm c\sigma_x$, which usually has a value five times the process standard deviation σ_X. In particular the two-sided control chart will give an in-control *ARL* of *370.4* using $K=0.5$ and $c=4.775$ times the standard deviation of the process.

2.6 The Design of Control Charts

An important factor in control chart usage is the **design of the control chart** which includes besides the control limits for the chart, the specification of the **sample size** to use, and the **frequency of sampling**.

There some general practices for designing a control chart. A simple and thus very popular way is to follow the primary ideas of the founder of control charting Dr. W. A. Shewhart. In his early text *Shewhart (1939)* suggested *3*-sigma control limits as action limits and sample sizes of four or five. He left the interval between successive samples to be determined by the practitioner.

This simplified and very general approach ignores the specific nature of the process under consideration and often leads to the failure of the *SPC* program.

A more effective approach is the design of control charts statistically. The **statistical design** of the control charts has a stricter and more formal structure and uses the statistical properties of the chart.

Consider the situation where independent random samples of size n are taken at each sampling point, and where T_i represents some chart statistic obtained at the ith sample point. For any Shewhart type control chart the probability that the chart statistic T_i falls outside the control limit when no shift has occurred is $P(T_i > UCL \text{ or } T_i < LCL \mid \text{no shift has occurred}) = a$. This is the probability of the ***a* risk** or of the so called Type I error . The Type I error is known at the *SPC* environments as **false alarm**. The probability that the chart statistic falls within the control limits although a shift has occurred, namely the Type II error, is $\beta = P(LCL < T_i < UCL \mid \text{a shift has occurred})$, and thus the probability that this out of control condition will be detected on any subsequent sample is $1-\beta$ which is the **power** of the chart.

The desired levels of the a risk and of the power of the test are usually pre-specified and lead to the determination of the sample size and the control limits. Widening the control limits decreases the a risk and thus the false alarm rate but increases the risk of Type II error, that is the ability to indicate the out of control situation. On the other hand shortening the control limits leads to the opposite results.

A way to evaluate the decisions regarding the above choices and to determine the sampling frequency, is the **average run length (*ARL*)** of the control chart.

The **run length (*RL*)** of a process is the number of samples taken before an out of control signal is triggered. Knowledge of the run length distribution, allows the computation of the average run length *(ARL)* of a control scheme, and consequently the design of more effective control charts.

When no assignable cause has occurred and the process is in-control the average run length, denoted as ARL_0, is desired to be large so that the rate of false alarms is low, but when a mean shift occurs and the process is out-of-control the average run length, denoted as ARL_1, is desired to be small so that the detection of the shift is possible with a small number of samples.

The number of samples for a chart to signal, if in Phase II with assumed known parameters, has a geometric distribution with parameter p when the process does not change, thus the in control ARL_0 is equal to $1/a$ where a is the probability of a false alarm. The *ARL* for the out of control period that is the expected number of samples taken before a shift in the process is detected is $ARL_1 = 1/(1-\beta)$.

For the evaluation of the *ARL* of a *CUSUM* many techniques have been developed. For a one sided *CUSUM* with parameters *H*, *K*, *Siegmund(1985)* approximation is

$$ARL = \frac{exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2}$$

for $\Delta \neq 0$, where $\Delta = \delta^* - K$, $b = h + 1.166$ and δ^* represents the size of the shift that for which *ARL* is needed thus $K = (\mu_1 - \mu_0)/2$. If $\Delta = 0$, one can use $ARL = b^2$. When $\delta^* = 0$ the in-control average length ARL_0 is calculated. To obtain the *ARL* of the two-sided cusum from the *ARL's* of the one-sided charts say ARL^+, ARL^- one can use

$$\frac{1}{ARL} = \frac{1}{ARL^+} + \frac{1}{ARL^-}$$

For a good discussion of the *ARL* of the *CUSUM* chart and for useful references, see *Maravelakis (1998)* or *Hawkins and Olwell (1988)*.

A measure of detection time for process changes which occur after the control chart has been in operation for some time, is the mean of the steady state run length distribution, the **steady-state *ARL***. The steady- state run length distribution is the distribution of the number of samples from the change to the signal, computed for the case in which there are no false alarms before the change and the change occurs after the process has been running long enough for the control statistic to be in steady-state at the sample immediately before the change occurs.

Sometimes the performance of the control chart is expressed in terms of its **average time to signal *(ATS)***. When the samples are taken at fixed intervals of time that are h hours apart, then

$$ATS = ARL\ h$$

Another way of making decisions concerning the sample size the frequency of sampling and the critical region of the chart is by minimizing the average cost when a single out-of-control state (assignable cause) exists. This is the **economic design** of the chart. A brief discussion of the Duncan's cost model is stated in Chapter 10 of this thesis. For a detailed literature review concerning the economic design and applications see *Montgomery(1980)*. A review of related papers from 1950 to 1999 is available from *Dicopoulos (2000)*.

The above traditional control charts have all been constructed based on the assumption that serially generated data are normal and independently distributed. However, in practice, observations are not always independent, but are actually serially correlated. The expressions above for the *ARL* will provide incorrect values in the case of correlated data, thus it is necessary to use control limits which are adjusted for the presence of correlation. The correlation in a process can be captured using time series models.

CHAPTER 3

Time Series Models

3.1 Introduction

A (discrete) **time series** is a (discrete) set of observations generated sequentially in time. Discrete time series may arise by *accumulating* a variable over a period of time, for example the yield from a batch process which is accumulated over the batch time, or by *sampling* a continuous time series, as for example when monitoring a quality characteristic in a chemical process.

If future values of a time series are exactly determined by a mathematical function the series is said to be **deterministic**. If the future values can only be described in terms of a probability distribution the time series is said to be **statistical** time series.

A statistical phenomenon that evolves in time according to probabilistic laws or, strictly speaking, a family of random variables $\{X_t, t \in T\}$ defined on a probability space $\{\Omega, \mathcal{F}, \mathcal{P}\}$ is called a **stochastic process**. Time series can thus be regarded as a realization of a stochastic process.

In section *3.2* of this chapter an important class of stochastic models for describing time series is presented, the stationary processes, which assume that the process remains in equilibrium about a constant mean. This type of models can provide a framework for seeking statistical control when monitoring autocorrelated processes. In section *3.3* and *3.4* the linear stationary and non stationary models are described.

Section *3.5* deals with the problem of forecasting future values of time series and section *3.6* with the problem of identifying a model and of estimating its parameters. Finally in section *3.7* the steps of time series analysis are presented.

3.2 Stationary Stochastic Processes

The stochastic processes whose properties, or some of them, stay unaffected by a change of time origin are called **stationary**. Consequently the time series $\{X_t, t \in T\}$ is said to be **strictly stationary** if the joint probability distribution associated with k observations $x_{t1},\ldots, x_{tk}$ made at any set of times $t_1,\ldots,t_k$, is the same as that associated with k observations $x_{t1+h},\ldots, x_{tk+h}$ made at times $t_{1+h},\ldots,t_{k+h}$, for any $\{h \in Z\}$ (see e.g. *Box and Luceno(1997)* or *Brockwell and Davis(1996)*).

When $k=1$, the stationarity assumption implies that the probability distribution $p(x_t)$ is the same for all times t and may be written $p(x)$. The **mean** of the stationary process is thus constant and equals to

$$\mu = E[X_t] = \int_{-\infty}^{+\infty} xp(x)dx$$

and the **variance** equals to

$$\sigma_x^2 = E\left[(X_t - \mu)^2\right] = \int_{-\infty}^{+\infty} (x-\mu)^2 p(x)dx$$

If $x_1,\ldots, x_N$ are N observed values of a time series, the mean μ of the stochastic process can be estimated by the sample mean

$$\overline{X} = \frac{1}{N}\sum_{t=1}^{N} x_t$$

and the variance σ^2_x of the stochastic process by the **sample variance**

$$\hat{\sigma}_x^2 = \frac{1}{N-1}\sum_{t=1}^{N}\left(x_t - \overline{X}\right)^2$$

If $\{X_t\}$ is considered to be a stationary time series, it is useful to define the **autocovariance function** *(ACVF)* of $\{X_t\}$ at lag h as $\gamma_x(h)=Cov(X_{t+h},X_t)$, the **autocorrelation function** *(ACF)* of $\{X_t\}$ as

$$\rho_x(h) \equiv \frac{\gamma_x(h)}{\gamma_x(0)} = Cor(X_{t+h}, X_t)$$

and the **partial autocorrelation function** *(PACF)* as a function defined by

$$a(0)=1 \quad \text{and} \quad a(h)=\varphi_{hh}\,,\ h=1,2,\ldots,$$

where φ_{hh} is the last element of the matrix $\Gamma_h^{-1}\gamma_h$ where Γ is the covariance matrix $[\gamma(i-j)]_{i,j=1}^{h}$ and $\gamma_h=(\gamma(1),\ldots,\gamma(h))'$.

Some useful tools for time series analysis are the sample *ACF*, which estimates the *ACF* of $\{X_t\}$, if the data can be considered as realized values of a stationary time series $\{X_t\}$, and the sample partial autocorrelation function *PACF*. These tools are mathematically described in the literature (see e.g. *Brockwell and Davis (1996)*) and their numerical values along with their graphical representations are available in most statistical packages.

Let $x_1,\ldots,x_n$ be observations of a time series. The sample **autocovariance function** (sample *ACVF*) at lag h is

$$\hat{\gamma}(h) = n^{-1}\sum_{t=1}^{n-|h|}(x_{t+|h|}-\bar{x})(x_t-\bar{x}), \qquad -n<h<n$$

while the **sample autocorrelation function** (sample ***ACF***) is

$$\hat{p}(h) = \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)}, \quad -n<h<n$$

The **partial sample autocorrelation function** (sample ***PACF***) is given by

$$\hat{a}(0)=1 \quad \text{and} \quad \hat{a}(h)=\hat{\phi}_{hh}, h\geq 1,$$

where $\hat{\varphi}_{hh}$ is the last component of $\hat{\varphi}_h=\hat{\Gamma}_h^{-1}\hat{\gamma}_h$ and $\hat{\Gamma}_h$ is the covariance matrix $[\gamma(i-j)]_{i,j=1}^{h}$ and $\gamma_h=(\gamma(1),\ldots,\gamma(h))'$.

3.3 Linear Stationary Processes

The general linear model describes a time series which is supposed to be generated by a linear combination of random shocks. When in addition the series have the property of stationarity, an important class of models is described, the stationary linear models which are represented by the moving average, the autoregressive and the mixed moving average autoregressive models. A good description of these models is given by *Box and Luceno(1997)*.

3.3.1 Some operators

In this section and so forth some simple operators are going to be used extensively. The most commonly used is the **backward shift operator** B, which is defined by

$$Bx_t = x_{t-1},$$

$$B^k x_t = x_{t-k}$$

and the **forward shift operator** $F = B^{-1}$, given by

$$Fx_t = x_{t+1},$$

$$F^k x_t = x_{t+k}$$

Another important operator is the **backward difference operator** ∇ which is defined by

$$\nabla x_t = x_t - x_{t-1}$$

while its inverse the **summation operator** ∇^{-1} is defined by

$$\nabla^{-1} x_t = \sum_{j=0}^{\infty} x_{t-j}$$

The difference and the summation operators can be expressed in terms of the backward and the forward operators correspondingly by

$$\nabla x_t = x_t - x_{t-1} = (1-B)x_t$$

$$\nabla^{-1} x_t = \sum_{j=0}^{\infty} x_{t-j} = x_t + x_{t-1} + \ldots = (1+B+B^2+\ldots)x_t = (1-B)^{-1} x_t$$

3.3.2 The general linear model

A stochastic process can be represented as the output from a **linear filter**. A time series can be regarded as generated from series of independent shocks a_t. These shocks are random drawings from a fixed distribution. When this distribution is assumed to be normal with mean *0* and variance σ^2_a the sequence of random variables a_t is called **white noise**.

The white noise process a_t is transformed to the time series through the linear filter as shown at Figure *3.1*.

***Figure 3.1*: The generating mechanism of a time series through a linear filter.**

The linear filtering operation takes a weighted sum of previous observations, so that

$$x_t=\mu+a_t+\psi_1 a_{t-1}+\psi_2 a_{t-2}+\ldots$$

where $\{\psi_i\ ,\ i=1,2,\ldots\}$ are constants. By representing $\tilde{x}_t = x_t - \mu$ the deviation of the process from some origin or from the mean if the process is stationary, it can be written as

$$\tilde{x}_t = a_t + \sum_{j=1}^{\infty} \psi_j a_{t-j}$$

Using the backward shift operator the above equation may be written as

$$\tilde{x}_t = \left(1+\sum_{j=1}^{\infty} \psi_j B^j\right) a_t = \psi(B) a_t$$

where $\psi(B) = 1+\sum_{j=1}^{\infty} \psi_j B^j$.

The above model implies that $\tilde{x}_t$ is a weighted sum of the past values of the $\tilde{x}_t's$, plus an added shock a_t, that is

$$\tilde{x}_t = a_t + \sum_{j=1}^{\infty} \pi_j \tilde{x}_{t-j}$$

where $\{\pi_i,\ i=1,2,...\}$ are constants. The current deviation $\tilde{x}_t$ of the level μ can be regarded as some regression on past deviations $\tilde{x}_{t-1}, \tilde{x}_{t-2},...$.

Once again the above equation can be expressed in terms of the backward shift operator by

$$a_t = \left(1 - \sum_{j=1}^{\infty} \pi_j B^j\right) \tilde{x}_t = \pi(B)\tilde{x}_t$$

where $\pi(B) = 1 - \sum_{j=1}^{\infty} \pi_j B^j$.

There is a relationship between the π and the ψ weights through the backward shift operator B:

$$\pi(B)=\psi^{-1}(B)$$

Through the transfer function $\psi(B)$ the definition of stationarity is revised for the general linear model: a linear process is **stationary** if $\psi(B)$ converges on, or within the unit circle. Through $\pi(B)$ a new property arise, the invertibility: a linear process is said to be **invertible** if $\pi(B)$ converges on, or with in the unit circle.

3.3.3 The autoregressive moving average model

An extremely important parametric family of linear stationary models is introduced : the autoregressive moving average model or more briefly the *ARMA(p,q)* model

The process $\{X_t, t \in T\}$ is said to be an **autoregressive moving average *ARMA(p,q)*** process if $\{X_t\}$ is stationary and if for every t,

$$X_t = \mu(1-\varphi_1-...-\varphi_p) + \varphi_1 X_{t-1} + ... + \varphi_p X_{t-p} + \alpha_t - \theta_1 \alpha_{t-1} - ... - \theta_q \alpha_{t-q} ,$$

where $\{\alpha_{t-i}\}$ is the random error (noise) at time $t-i$ which is assumed to be *i.i.d.* normal with mean *zero* and variance σ^2_α , φ_i is the autoregressive coefficients, θ_i is the moving average coefficients and μ the mean of the process.

Sometimes, it is more convenient to use the more concise form of the above equation

$$\varphi(B)X_t=\theta(B)\alpha_t$$

where $\varphi(.)$ and $\theta(.)$ are the p^{th} and q^{th} degree polynomials $\varphi(z)=1-\varphi_1 z-\ldots-\varphi_p z^p$ $\theta(z)=1+\theta_1 z+\ldots+\theta_q z^q$ and B is the backward shift operator.

The equation $\varphi(B)X_t=\theta(B)\alpha_t$ defines a *stationary* process, provided that all the roots of the characteristic equation $\varphi(B)=0$ lie outside the unit circle and an *invertible* process when the roots of $\theta(B)$ lie outside the unit circle.

When the $\theta(z) \equiv 1$ the process

$$X_t=\varphi_1 X_{t-1}+\ldots+\varphi_p X_{t-p}+\alpha_t\ ,$$

where $\{\alpha_t\}$ is *i.i.d.* with mean *zero* and variance σ^2_a and $\varphi_1,\ldots,\varphi_p$ are constants, is called **autoregressive process** of order p or more briefly ***AR(p)***. The variance of the process may be expressed by

$$\sigma_x^2=\frac{\sigma_a^2}{1-\rho_1\varphi_1-\rho_2\varphi_2-\ldots-\rho_p\varphi_p}$$

When the $\varphi(z) \equiv 1$ the process

$$X_t=\alpha_t-\theta_1\alpha_{t-1}-\ldots-\theta_q\alpha_{t-q}\ ,$$

where $\{\alpha_t\}$ is *i.i.d.* with mean *zero* and variance σ^2_a and $\theta_1,\ldots,\theta_q$ are constants, is called **moving average process** of order q or more succinctly ***MA(q)***.

3.3.4 The *ARMA(1,1)*, *AR(1)*, *AR(2)* and *MA(1)* processes

The first order autoregressive moving average process *ARMA(1,1)* is described as follows:

$$X_t=\mu(1-\varphi)+\varphi X_{t-1}+\alpha_t-\theta\alpha_{t-1}\ ,$$

or equivalently

$$(1-\varphi B)\widetilde{X}_t=(1-\theta B)a_t$$

where X_t is the observation at time t, α_t is the random error term at time t, φ is the autoregressive parameter, θ *is* the moving average parameter, μ is the mean of the process and $\widetilde{X}_t = X_t - \mu$.

The variance of the *ARMA(1,1)* process is

$$\sigma_x^2 = \frac{1-2\varphi\theta+\theta^2}{1-\varphi^2}\sigma_a^2$$

and the process is stationary if $-1<\varphi<1$ and invertible if $-1<\theta<1$.

When $\theta=0$ the above model reduces to an *AR(1)* process, which is stationary if φ satisfies the condition $-1<\varphi<1$ and the variance of the process is

$$\sigma_x^2 = \frac{\sigma_a^2}{1-\rho_1\varphi} = \frac{\sigma_a^2}{1-\varphi^2}$$

When $\theta=0$ the *ARMA(1,1)* model reduces to an *MA(1)* process which is invertible if θ satisfies the condition $-1<\theta<1$. The variance of the process is $\sigma^2{}_x=(1+\theta^2)\sigma^2{}_a$.

The second order autoregressive process *AR(2)* is described by

$$X_t = \mu(1-\varphi_1-\varphi_2) + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \alpha_t$$

The process is stationary if the following conditions are satisfied

$$\varphi_1+\varphi_2<1$$
$$\varphi_2-\varphi_1<1$$
$$-1<\varphi_2<1$$

the parameter space defined by the above equations is a triangular region *(Box and Luceno (1997))*.

The variance of the process is

$$\sigma_x^2 = \frac{\sigma_\alpha^2}{1-\rho_1\varphi_1-\rho_2\varphi_2} = \left(\frac{1-\varphi_2}{1+\varphi_2}\right)\frac{\sigma_\alpha^2}{\{(1-\varphi_2)^2-\varphi_1^2\}}$$

3.4 Linear Non Stationary Processes

All of the processes encountered in practice are not wandering around a constant mean. There are series that exhibit explosive or evolutionary behaviour which is far from stationary, and others that exhibit some homogeneity and by supposing some suitable difference of the process can be viewed as stationary. This important class of linear non stationary models is presented in this section.

3.4.1 The Autoregressive Integrated Moving Average model

An important class of linear non stationary models are those which aren't stationary but for which the d*th difference* is a stationary autoregressive moving average model. These processes are called autoregressive integrated moving average processes.

As stated at the previous section the autoregressive moving average process is stationary if the roots of the autoregressive polynomial $\varphi(B)=0$ lie outside the unit circle and exhibits non stationary behavior if the roots lie inside the unit circle. Suppose that d of the roots are unity, and lie upon the unit circle and the remainder lie outside.

If d is a nonnegative integer, a general form of the **autoregressive integrated moving average** *ARIMA(p,d,q)* process is

$$\phi(B)z_t = \varphi(B)\nabla^d z_t = \theta(B)\alpha_t$$

where

- $\varphi(B)=1-\varphi_1 B-\varphi_2 B^2\ldots-\varphi_p B^p$ is called the *autoregressive operator* and it is assumed to be stationary, that is, the roots of $\varphi(B)=0$ lie outside the unit circle,
- $\phi(B)=\varphi(B)\nabla^d$ is called the *generalized autoregressive operator* and is a non stationary operator with d of the roots of $\phi(B)=0$ are unity, and

- $\theta(B)=1-\theta_1 B-\theta_2 B^2\ldots-\theta_q B^q$ is called the *moving average operator* and is assumed to be invertible, that is, the roots of $\theta(B)=0$ lie outside the unit circle.

If $d=0$ the model represents a stationary process .

Box and Luceno (1997) gives some explicit forms for the above model. One of them is the **difference equation form**, which expresses the current value of the process X_t in terms of previous values of X_t*'s* and current and previous values of *a's*. Thus, if

$$\phi(B)=\varphi(B)(1-B)^d=1-\phi_1 B-\phi_2 B^2\ldots-\phi_{p+d}B^{p+d}$$

the general model may be written

$$X_t=\phi_1 X_{t-1}+\ldots+\phi_{p+d}X_{t-p-d}-\theta_1 a_{t-1}-\ldots-\theta_q a_{t-q}\theta_1+a_t$$

When the objective is to express the model in terms of previous X_t's and current a_t's the **inverted form** of the model is needed. In the previous section it is stated that the model

$$X_t=\psi(B)a_t$$

can also be expressed in an inverted form as

$$\psi^{-1}(B)X_t=a_t$$

or equivalently as

$$a_t=\left(1-\sum_{j=1}^{\infty}\pi_j B^j\right)\tilde{x}_t=\pi(B)\tilde{x}_t$$

where $\pi(B)=1-\sum_{j=1}^{\infty}\pi_j B^j$ must converge on or within the unit circle.

The π weights can be derived by equating the coefficients of B in

$$\phi\,(B)X_t=\theta(B)\pi(B)$$

and if $d\geq 1$, the process may be written in the form

$$X_t=\sum_{j=1}^{\infty}\pi_j X_{t-j}+a_t$$

For many purposes, and in particular for calculating the forecasts, the difference equation is the most convenient form to employ.

3.4.2 The *ARIMA(0,1,1)* process

The *ARIMA(0,1,1)* process is defined as follows

$$\nabla z_t = \alpha_t - \theta\alpha_{t-1} = (1-\theta B)\alpha_t$$

corresponding to $p=0$, $d=1$, $q=1$, $\varphi(B)=1$, $\theta(B)=1-\theta(B)$. The *difference equation form* of the *ARIMA(0,1,1)* model is

$$X_t = X_{t-1} + \alpha_t - \theta\alpha_{t-1}$$

while the *inverted form* is

$$\pi(B)X_t = a_t$$

or equivalently

$$X_t = \sum_{j=1}^{\infty}\pi_j X_{t-j} + a_t = \overline{X_{t-1}}(\pi) + a_t$$

where $\overline{X_{t-1}}(\pi)$ is a weighted moving average of previous values of the process.

The π weights of the *ARIMA(0,1,1)* process are given by

$$\pi_j = (1-\theta)\theta^{j-1} = \lambda(1-\lambda)^{j-1},\ j\geq 1$$

and the process can thus be written as

$$X_t = \overline{X_{t-1}}(\lambda) + a_t$$

where the weighted moving average of previous values of the process

$$\overline{X_t} = \lambda\sum_{j=1}^{\infty}(1-\lambda)^{j-1}X_{t-j}$$

is an **exponentially weighted moving average** which is the recursion formula for the ***EWMA*** statistic. Thus,

$$\overline{X_t}(\lambda) = \lambda X_t + (1-\lambda)\overline{X_{t-1}}(\lambda).$$

3.5 Forecasting

Supposing that a proper model is fitted the data, predicting future values of the time series should then be carried out. The parameters of the model are never known exactly, but estimation errors will not seriously affect the forecasts unless the data set used for fitting the model is small. The properties of the $ARIMA(p,d,q)$ models may be used to forecast future values of an observed time series.

3.5.1 The Minimum Mean Square Error Forecasts

As described in section *3.4* the general form of the $ARIMA(p,d,q)$ model is $\phi(B)z_t = \theta(B)\alpha_t$ where $\phi(B) = \varphi(B)\nabla^d$.

The minimum mean square error forecasts at origin t for lead time h, may be generated from the *difference equation form* of the $ARIMA(p,d,q)$ model directly by

$$X_{t=h} = \phi_1 X_{t+h-1} + \dots + \phi_{p+d} X_{t+h-p-d} - \theta_1 a_{t+h-1} - \dots - \theta_q a_{t+h-q} + a_{t+h}$$

The minimum mean square error forecast $\hat{X}_t(h)$ for lead time h is the conditional expectation $E[X_{t+h}]$ of X_{t+h}, at origin t.

Taking conditional expectations at time t in the above equation, with [.] denoting conditional expectation:

$$[X_{t=h}] = \hat{X}_t(h) =$$
$$= \phi_1 [X_{t+h-1}] + \dots + \phi_{p+d} [X_{t+h-p-d}] - \theta_1 [a_{t+h-1}] - \dots - \theta_q [a_{t+h-q}] + [a_{t+h}]$$

To calculate the conditional expectations which occur in the expressions above, if j is a non negative integer the following are used

$$[X_{t-j}] = E_t[X_{t-j}] = X_{t-j}\ , \quad j=0,1,\dots,$$

$$[X_{t+j}] = E_t[X_{t+j}] = \hat{X}_t(j), \quad j=0,1,\dots,$$

$$[a_{t-j}] = E_t[a_{t-j}] = a_{t-j} = X_{t-j} - \hat{X}_{t-j-1}(1), \quad j=0,1,\dots,$$

$$[a_{t+j}] = E_t[a_{t+j}] = 0\ , \ j=0,1,\dots,$$

Therefore, to obtain the forecast $\hat{X}_t(h)$, one writes down the model for X_{t+h} in the difference equation and treats the terms on the right according to the following rules:

- The X_{t-j} $(j=0,1,\dots,)$, which have already happened at origin t, are left unchanged.
- The X_{t+j} $(j=0,1,\dots,)$, which have not yet happened, are replaced by their forecasts $\hat{X}_t(j)$ at origin t.

- The a_{t-j} $(j=0,1,\ldots,)$, which have happened, are available from $X_{t-j} - \hat{X}_{t-j-1}(1)$.
- The a_{t-j} $(j=0,1,\ldots,)$, which have not yet happened, are replaced by zeroes.

The **forecast error** for lead time h is

$$e_t(h)=a_{t+1}+\psi_1 a_{t+h-1}+\ldots+\psi_{t-1}a_{t+1}$$

while using the above equation the **one step ahead forecast error** is

$$e_t(1) = X_{t+1} - \hat{X}_t(1) = a_{t+1}$$

which makes clear that the sequence of the residuals a_t which generate the process and are assumed to be independent random variables are the one step ahead forecast errors.

Besides, it is useful to note that for a minimum mean square error forecast, the one step ahead forecast errors are uncorrelated.

3.5.2 Forecasting an *ARIMA(0,1,1)* process

The *ARIMA(0,1,1)* model is

$$\nabla z_t = \alpha_t - \theta_1 a_{t-1} = (1-\theta_1 B)\alpha_t$$

and using the *difference equation* approach at time $t+h$, it may be written

$$X_{t+h}=X_{t+h-1}+a_{t+h}-\theta a_{t+h-1}$$

taking conditional expectations at origin t,

$$\hat{X}_t(h) = \hat{X}_t - \theta\alpha_t$$
$$\hat{X}_t(h) = \hat{X}_t(h-1), h \geq 2$$

It is obvious that for all lead times, the forecasts at origin t will follow a straight line parallel to the time axis. Using the fact that

$$X_t = \hat{X}_{t-1}(h) + a_t$$

the above equations can be written in two forms.

The first of these is

$$\hat{X}_t(h) = \hat{X}_{t-1}(h) + \lambda\alpha_t$$

where $\lambda = 1 - \theta$. This implies that, having seen that the previous forecast $\hat{X}_{t-1}(h)$ falls short of the realized value by α_t, it is adjusted by an amount $\lambda\alpha_t$.

The second form is

$$\hat{X}_t(h) = \lambda X_t + (1-\lambda)\hat{X}_{t-1}(h),$$

and this way of writing the forecasts implies that the new forecast is a linear interpolation at argument λ between the old forecast and the new observation.

The forecasts can also be expressed as a weighted average of previous observations

$$\hat{X}_t(h) = \lambda X_t + \lambda(1-\lambda)X_{t-1} + \lambda(1-\lambda)^2 X_{t-2} + \ldots +$$

which makes it obvious that for the *ARIMA(0,1,1)* model the forecast for all future time is an *exponentially weighted moving average* of current and past X_t's.

3.6 Model Identification and Estimation

An issue involved in time series modeling is the **selection of the appropriate model** to fit the residuals. The determination of the appropriate *ARMA(p,q)* model includes the choice of p,q (order selection), the estimation of the coefficients $\{\varphi_i, i=1,\ldots,p\}$, $\{\theta_i, i=1,\ldots,q\}$, the variance of the white noise and the mean of the process.

3.6.1 Order selection

Once the data has been transformed to the point where the transformed series can be fitted by a *zero* mean *ARMA(p,q)* model, the problem of selecting appropriate values for the orders p and q arises.

It might appear at first sight that the higher the values chosen for p and q the better the resulting fitted model will be. However, the estimation of a number of parameters introduces errors that affect the use of the fitted model for prediction.

A practical suggestion for selecting the proper *ARMA(p,q)* time series model is to examine the sample *ACF* and the *PACF* plots of the data, and see how many of their values are clearly outside the bounds. The potential orders *p* and *q* will be up to those values.

The final decision will be based on one or more of the criteria existing in bibliography for selecting the order of the *ARMA* model, which include a penalty term to discourage the fitting of too many parameters.

There are many criteria to determine the appropriate order of an *ARMA* model. Some of those are the Theil's Residual Variance criterion *(RVC)*, the Final Prediction Error criterion *(FPE)*, the Akaike's Information criterion *(AIC)*, the Bayesian Information criterion *(BIC)*, the Likelihood Ratio Tests *(LR)*, the Parzen's *CAT* Criterion, the Hannan and Quinn's Criterion *(φ_a or HC)* and the Bayesian Estimation criterion *(BEC)*. A brief discussion of these criteria can be found in bibliography *i.e.*, : in *Box and Luceno(1997)*, in *Priestley(1981)*, in *Brockwell & Davis (1996)* and elsewhere.

A major criterion for the selection of the orders *p, q* is the minimization of the ***AICC*** statistic. The *AICC* criterion, introduced by *Hurvich & Tsai (1989)*, is a bias-corrected version of the of the *AIC* criterion, namely the criterion of *Akaike (1973)*. To apply this criterion:
Chose p, q, φ_p and θ_q, to minimize

$$AICC=-2\ln L(\varphi_p,\theta_q,S(\varphi_p,\theta_q)/n)+2(p+q+1)n/(n-p-q-2)$$

where $L(\varphi_p,\theta_q,\sigma^2{}_X)$ is the likelihood of the data under the Gaussian *ARMA* model with parameters $(\varphi_p,\theta_q,\sigma^2{}_X)$ and $S(\varphi_p,\theta_q)$ is the residuals sum of squares defined:

$$S(\hat{\phi},\hat{\theta})=\sum_{j=1}^{n}\left(X_j-\hat{X}_j\right)^2/j-1$$

3.6.2 Estimation of the parameters

Once a model has been found that minimizes the *AICC* criterion, it is necessary to check the model for goodness of fit (essentially by checking whether the residuals come from an *i.i.d* sequence), to estimate the variance of the white noise sequence of the residuals and to estimate the coefficients of the model.

At this paragraph the parameters p and q are considered known and the objective is to estimate the coefficients $\varphi=(\varphi_1,\dots,\varphi_p)'$, $\theta=(\theta_1,\dots,\theta_q)'$ of the zero mean *ARMA(p,q)* model as well as the variance σ.

When p and q are known, good estimators of φ and θ can be found by considering that the data as being observations of a Gaussian time series and maximizing the likelihood with respect to the $p+q+1$ parameters φ, θ, σ.

The maximum likelihood algorithm which is going to be described needs some initial parameter values. These preliminary values can be estimated by various procedures such as the *Yule-Walker* and the *Burg* estimators for the pure autoregressive models, and the *Innovations* or the *Hannan-Rissanen* algorithms for the mixed autoregressive moving average models. A presentation of these algorithms can be found in *Brockwell & Davis (1996)*.

The **Maximum Likelihood Estimators** $\hat{\phi}$,$\hat{\theta}$ and $\hat{\sigma}$ are the solution of the following procedure:

$$\hat{\sigma}^2 = n^{-1} S(\hat{\phi},\hat{\theta})$$

where

$$S(\hat{\phi},\hat{\theta}) = \sum_{j=1}^{n} \left(X_j - \hat{X}_j\right)^2 / r_{j-1}$$

and $\hat{\phi}$,$\hat{\theta}$ are the values of φ, θ that minimize

$$l(\phi,\theta) = ln(n^{-1} S(\phi,\theta) + n^{-1} \sum_{j=1}^{n} ln\, r_{j-1}$$

The initial values for φ and θ can be found from one of the preliminary algorithms and the mean squared errors r_j from the innovations algorithm

3.7 Basic Steps in Time Series Analysis

The first step in the analysis of any time series, is to plot the data and examine the main features of the graph checking in particular if there is

- a trend component,
- a seasonal component,
- any apparent sharp changes in behavior,
- any outlying observation

If there are any sharp changes in the series, it may be advisable to analyze the series by first breaking it into homogenous segments. If there are outlying observations, they should be checked carefully and if possible explained. Then the trend and seasonal components should be removed to get stationary residuals which will be used for further analysis. To achieve this goal, it may sometimes be necessary to apply a preliminary transformation to the data, after which the elimination of trend and seasonal components could be pursuit :

(a) by using the **classical decomposition model** $X_t=m_t+s_t+Y_t$ where m_t is a function known as a trend component, s_t is a function with known period referred to as a seasonal component, and Y_t is the random noise component (residuals), which may turn out to be stationary.

(b) by repeatedly **differencing** the series $\{X_t\}$ until the differences of the observations resemble a realization of some stationary time series.

The sample autocorrelation function can be calculated for any time series data set $\{x_1,\ldots,x_n\}$ and can be useful as an indicator of non-stationarity. For data still containing a trend $|\hat{\rho}(h)|$ will exhibit slow decay as the lag h decreases and for data with a substantial periodic component $|\hat{\rho}(h)|$ will exhibit similar behavior with the same periodicity.

After having produced a time series with no apparent deviations from stationarity, the next step is to model the estimated noise sequence $\{Y_t,\ t=1,2,\ldots,\}$.

If there is no dependence between the estimated noise sequence, it could be regarded as coming from an independent and identically distributed sequence, and there is no further modeling to be done, except from estimating its variance and mean. There are some simple tests to check this hypotheses. The most important of those are the following:

(a) **The Sample Autocorrelation Function** : For the residuals $y_1,\ldots,y_n$ to be a realization of an *i.i.d.* sequence, *95%* of the sample autocorrelations should fall between the bounds $\pm 1{,}96\sqrt{n}$. Thus if sample autocorrelations up to lag *40* are computed, no more than two or three values should fall outside the bounds.

(b) **The Portmanteau test** : Consider the single statistic

$$Q = n\sum_{j=1}^{h}\hat{p}^2(j)$$

where $\hat{p}$ is the sample autocorrelation function, and reject the *i.i.d.* hypotheses for the residuals $y_1,\ldots,y_n$ at significance level α if $Q>X^2_{1-\alpha}(h)$, where $X^2_{1-\alpha}(h)$ is the $1\text{-}\alpha$ quantile of the chi-squared distribution with h degrees of freedom.

However, if there is a significant dependence between the residuals further analysis is needed to fit the data with a more complex stationary time series model that accounts for the dependence.

CHAPTER 4

Autocorrelated Processes

4.1 Introduction

A basic assumption in traditional application of *SPC* techniques is that the observations from the processes under investigation are normally and independently distributed. When these assumptions are satisfied, conventional control charts may be applied. However, the independence assumption is often violated in practice. In discrete as well as in continuous production process data often shows some autocorrelation, or serial dependence.

Section *4.2* of this chapter describes how correlated data can be generated and section *4.3* how autocorrelation can effect the performance of traditional control charts. In section *4.4* the objectives of monitoring an autocorrelated process are stated along with the basic ideas for dealing with non independent data.

4.2 The Genesis of Autocorrelated Processes

Autocorrelation is present in the data generated by most continuous and batch process operations since the value of the particular parameter under monitoring is dependent on the previous value of that parameter. Continuous product manufacturing operations such as the manufacture of food, chemicals, paper and other wood products often exhibits serial correlation. This phenomenon can also be present in monthly series of survey quality data.

Autocorrelation is more apparent for data collected with frequent sampling but can also be due to the dynamics of the process. For instance, observations from automated test and inspection procedures where every quality characteristic is measured on every unit in time order of production, or measurements of process variables from tanks, reactors and recycle streams in chemical processes are often highly correlated.

The following example from *Montgomery (1997)* demonstrates the above mechanism. A simple process system, as shown at *Figure 4.1* bellow, consisting of a tank -or reactor, column and so forth- with a single input stream and a single output stream is used.

Assume that the tank has volume V and flow rate f. Let z_t represent the concentration of a certain material in the input stream at time t and x_t the corresponding concentration in the output stream at time t. Assuming homogeneity within the tank and defining $T=V/f$ the time constant of the system, the relationship between the input and output is

$$x_t = z_t - T\frac{dx_t}{dt}$$

Suppose that at time $t=0$ a step change of z_0 occurs at the input stream then at time t the concentration at the output stream is

$$x_t=z_0(1-e^{-t/T})$$

In practice x_t is not observed continuously, but only at small, equally spaced intervals Δt. Thus,

$$x_t= x_{t-1}+(z_t-x_{t-1})(1-e^{-\Delta t/T})=\theta z_t+(1-\theta)x_{t-1}$$

where $\theta=1-e^{-\Delta t/T}$.

The input stream concentration z_t, and the sampling interval Δt influence the properties of the output concentration x_t. Assuming that z_t are uncorrelated random variables, the autocorrelation between x_t and x_{t-1} is given by

$$\rho=1-\theta=e^{-\Delta t/T}$$

It is obvious from the above equation, that the correlation between successive values of x_t depends on the sampling interval. If Δt is much greater than T, ρ almost equals *0*. Which means that if the sampling interval in the output is much longer than the time constant T, the observations on output concentration will be uncorrelated.

However if $\Delta t \leq T$ there will be significant autocorrelation present in the observations. For example, sampling one time per time constant ($\Delta_t/T=1$) results in autocorrelation between x_t and x_{t-1} of $\rho=0,37$, while sampling ten times per constant ($\Delta_t/T=0,1$) results in autocorrelation between x_t and x_{t-1} of $\rho=0,9$

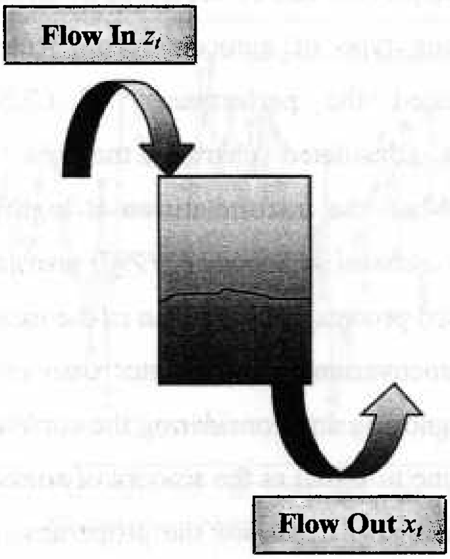

Figure 4.1 **: An example of the mechanism that generates correlated processes.**

4.3 The Effects of Autocorrelation to Control Charts

Traditional control charts have statistical properties that are developed under the assumption of independence and do not work very well if the quality characteristic exhibits even low levels of autocorrelation. Autocorrelation between successive observations as small as *0,25* can have big effects on the statistical properties of conventional control charts.

Many authors have considered the effect of autocorrelation on the performance of *SPC* charts. *Johnson & Bagshaw (1974)* and *Bagshaw & Johnson (1975)* derived approximate run length distribution for the *CUSUM* chart when the process follows an *AR(1)* or a *MA(1)* model. They stated that incorrect conclusions can be drawn by using conventional *CUSUM* schemes in the presence of data correlation. *Harris & Ross (1991)* discussed the impact of autocorrelation on the performance of *CUSUM* and *EWMA* charts, and showed that the average and median run lengths of these charts were sensitive to the presence of autocorrelation. *Alwan(1992)* discussed the masking effect of special causes by the autocorrelation of the data, and demonstrated that in the presence of even moderate levels of autocorrelation, an out of control point of the chart did not necessarily indicate a process change. *Padgett et al. (1992)* investigated Shewhart charts when the correlation structure of the process can be described by an *AR(1)* model plus a random error and found that this type of autocorrelation effects the false alarm rate. *Yashchin(1993)* evaluated the performance of *CUSUM* charts applied to autocorrelated data. He considered charting the raw data directly when the autocorrelation is low. When the autocorrelation is high he considered the use of transformed observations. *Schmid and Schone (1997)* proved theoretically that the run length of the autocorrelated process is larger than in the case of independent variables provided that all the autocovariances are greater than or equal to *zero*. *Prybutok (1997)* found that not diagnosing and considering the correlation in the data leads to a decrease in the average time to signal as the amount of correlation increases.

Many others have also discussed the properties of standard control charts applied to correlated data as for example *Goldsmith and Whitfield(1961)*, *MacGregor and Harris(1990)*, *Alwan and Radson(1992)*, *Maragah and Woodall(1992)*.

The main effect of autocorrelation in the process data to *SPC* schemes is that it produces control limits that much tighter than desired. This causes a substantial increase in the average false alarm rate and a decrease in the ability of detecting changes on the process. Consequently, the *ARL* performance of the control charts is degraded. Early detection of the occurrence of assignable causes ensures that necessary corrective action can be taken before a large quantity of non-conforming product is manufactured. Thus, when there is autocorrelation in the process data standard control charts should not be applied. The following application shows the disastrous effect of correlation to traditional control charts.

4.3.1 An application

A data set of *100* observations was generated from an *ARMA(1,1)* time series model by simulating the sequence of the random errors a_t's. The process parameters used for this simulation were $\varphi=0,7$, $\theta=0,15$ and $\sigma_a=0,9$.

Using the *MINITAB* statistical software an *ARMA(1,1)* time series model is fitted to the observations resulting $\varphi=0,6675$, $\theta=0,1152$ and $\sigma_a=0,8756$. According to the plots of Figure B.1 (Appendix B) the *ARMA(1,1)* model is as expected appropriate for the data. The mean of the process was $\mu=-0,099$ and was subtracted from the data to set its target value to *zero*.

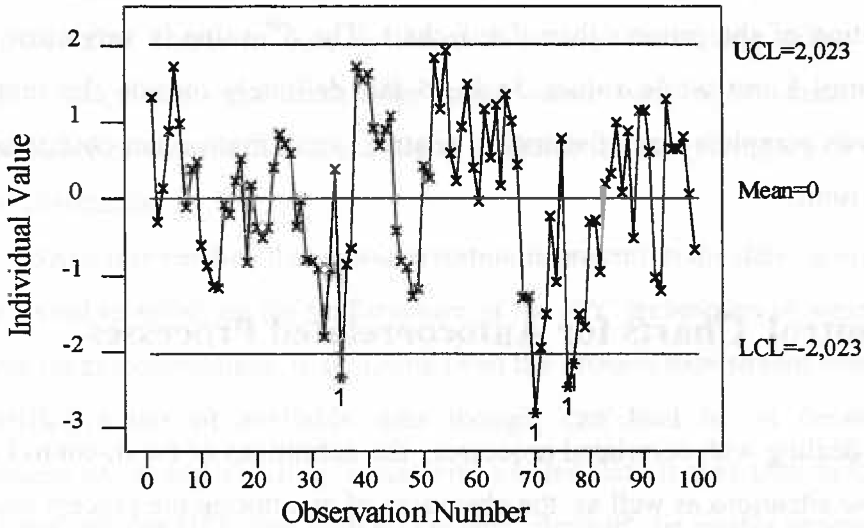

***Figure 4.2:* *X*-chart of the observations from a simulated In-Control *ARMA(1,1)* process.**

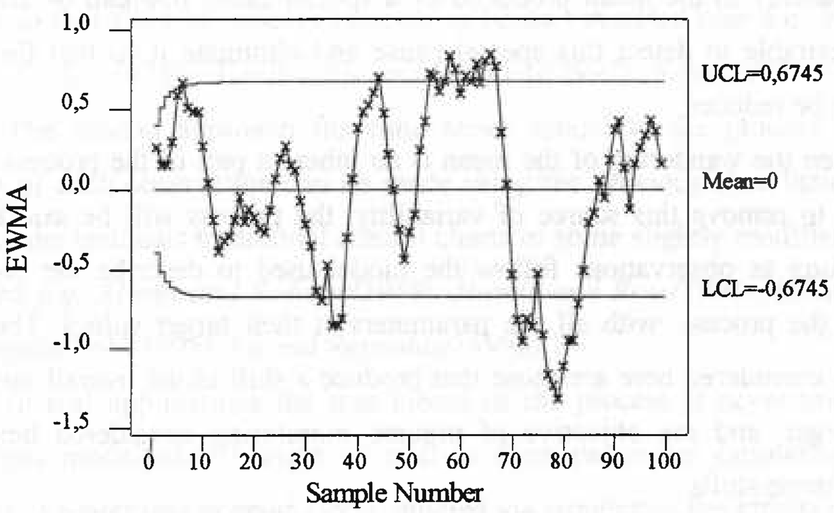

***Figure 4.3:* *EWMA* chart of the observations from a simulated In-Control *ARMA(1,1)* process with $\lambda=0,2$.**

A *X*-chart of the observations is presented at *Figure 4.2*. The variance of the process is calculated without taking into consideration the autocorrelation, using moving ranges. The estimate of σ_x is $MR/d_2=0{,}674$ and the *3*-sigma control limits are $\pm 2{,}023$. Although the process is in-control the observations *35,70,76,77* fall outside the control limits. These false alarms are the result of the autocorrelation of the data.

At *Figure 4.3* an *EWMA* chart of the observations is presented with the *EWMA* parameter being $\lambda=0{,}2$. This chart seem to be more affected by the autocorrelation of the process than the *X*-chart. The 6^{th} value is very close to the Upper Control Limit while values *34,35,36* fall definitely outside the limits. The process shows complete lack of statistical control , since many other observations fall outside the limits.

4.4 Control Charts for Autocorrelated Processes

In dealing with correlated processes, the definitions of the in-control and the special cause situations as well as the objectives of monitoring the process need to be clearly stated.

In statistical process control the usual assumption is that the observations are independent with a constant mean and the objective of monitoring the mean is to detect special causes which produce changes in the mean from the target value. If there is variability in the mean produced by a special cause that can be eliminated, then it is desirable to detect this special cause and eliminate it so that the process variance can be reduced.

When the wandering of the mean is an inherent part of the process and it is not feasible to remove this source of variability, the process will be said to be in-control as long as observations follow the model used to describe the correlation structure of the process, with all the parameters at their target values. The special causes to be considered here are those that produce a shift in the overall mean away from the target, and the objective of process monitoring considered here is the detection of those shifts.

Since there is a real possibility that the autocorrelations are small and the apparent drifts in the process average quite large, causing the systematic variation, it is useful to check whether the autocorrelation of the process remains and is an inherent part of the process after having made every effort to remove the systematic variation. A possible indication of serial correlation might be the observation of more out-of-control points in a process then warranted and expected. This suspicion can be verified by computing the sample autocorrelation function and testing for the presence of significant autocorrelation at lags $h=1,2,\ldots$.

Another way to recognize positive correlation in the process is by using Shewhart control charts in conjunction with several sensitizing rules. *Balkin and Lin(2001)* stated that the sensitizing rules work well when there are strong autocorrelative relationships, but are not as effective in recognizing small to moderate levels of correlation.

When it is verified that autocorrelation is present in the data, action should be taken to avoid its effect on the performance of the *SPC* techniques. A simple idea that can break up autocorrelation is sampling from the process data stream less frequently. The inefficient use of available data though, can lead to a decrease of the performance of control charting since with limited data it may take much longer to detect a real process shift than with all the data. Beyond the simple approach of using less frequent samples two general approaches for constructing control charts in the case of correlated processes are developed.

The first approach uses standard control charts, but adjusts the control limits to account for the autocorrelation and adjusts the method of estimating the process variance so that the true process variance is being estimated (see e.g. *Vassilopoulos and Stamboulis (1978)*, *VanBrackle and Reynolds(1997)* , *Schmid(1995)*).

The second approach fits time series model to the process data so that forecasts of each observation can be made using the previous observations and then applies to the residuals traditional control charts or some slightly modified versions of those (see e.g. *Alwan and Roberts(1988)*, *Harris and Ross(1991)*, *Montgomery and Mastrangelo(1991,1995)*, *Lu and Reynolds(1999a)*).

In real applications the true model of the process is never known and the errors from model identification as well as from parameter estimation are hardly avoidable. *Kramer and Schmid(1997)* studied via simulation the effects of estimating the parameters of the process and found that charts perform much better when the parameters are precisely estimated. They suggested the use of the recommendations in

Kramer and Schmid(1996) of how to estimate the parameters in the context of control designs. Similarly, when a set of historical data is used for fitting the model, *Zhang (1997)* and *Lu and Reynolds (1999a)* recommended a size at least equal to *100* if it is possible. If a control chart must be constructed using a small data set, then signals from this chart should be interpreted with caution.

The rational of using residuals charts is that assuming that the correct time series model is fitted to the data, the residuals will be independently and identically distributed random variables. All the assumptions of traditional quality control will then be met, and thus any of the traditional *SPC* charts can be applied. Once a change of the mean and/or variance in the residuals process is detected, it is concluded that the mean and/or variance of the process itself has been changed. Thus, plotting the residuals on a control chart provides a mechanism for detecting a process change. However, many people seem to agree that the residuals charts do not have the same properties as the traditional charts *i.e.*, the charts for the original observations and that the ability of a chart to detect a mean shift depends on the model that is assumed to describe appropriately the data.

This was first demonstrated in print, as mentioned in *Ryan(2000)*, by *Longnecker and Ryan (1991)* and *Ryan(1991)* where an *AR(1)* model is used. Additional models were considered in *Longnecker and Ryan (1992)* where the performance of a *X*-chart of the residuals from an *AR(1)*, *AR(2)* and *ARMA(1,1)* model is investigated. They pointed out that the *X*-chart of the residuals may have poor capability to detect the process mean shift and showed that a residuals chart has high probability of detecting a mean shift as soon as it occurs, but if it fails to detect the shift immediately, there is low probability that the shift will be detected later especially for an *AR(1)* process with positive autocorrelations. To the same conclusion came *Wardell et al(1994)* after deriving the run length distribution of residuals chart and more recently *Zhang(1997)* , *Lu and Reynolds (1999a)* and others.

Harris & Ross(1991) also studied the response of residuals of *ARIMA(0,1,1)* and *AR(1)* processes to step shift in process mean and concluded that the residual analysis is insensitive to the mean when the process is positively autocorrelated and recommended traditional charting methods for residuals monitoring.

Another important issue in applying traditional control charting techniques to residuals is that the mean of the residuals is not constant once a step in the mean occurs. This is mentioned in *Lu and Reynolds (1999a,2001)* where the expectations of the residuals before and after a shift in the mean are studied. Besides, *Yang and Makis(2000)* developed a procedure for studying residual response and found that residuals are always independent, with *zero* means when the process is in-control but when a disturbance occurs the residuals means are determined by the disturbance sequence and the autocorrelation structure of the process.

The rationale of this approach is the following: if the correct time series model is fitted to the data, the residuals will be independently and identically distributed random variables, and the assumptions of the classical quality control will then be met, and then any of the traditional SPC charts can be applied. Once a change of the mean and/or variance in the residuals' process is detected, it is concluded that the mean and/or variance of the process itself has been changed. Thus, plotting the residuals on a control chart provides a mechanism for [illegible] process. However, some people seem to agree that [illegible] the [illegible] to detect a mean shift depends on [illegible] describe accurately the data.

This was first demonstrated [illegible] Longnecker and Ryan (1991) and [illegible]. Additional models were considered [illegible] performance of a X-chart of the [illegible] model is investigated. They pointed out that the X-chart of the residuals has poor capability to detect the process mean shift and showed that a residuals chart has high probability of detecting a mean shift as soon as it occurs [illegible] the shift immediately, there is low probability that the shift will be detected later, especially for an AR(1) process with positive autocorrelations. To the same conclusions came Wardell et al (1994) after deriving the run length distribution of residuals charts and more recently Zhang (1997), [illegible] (1998), and others.

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Another important issue in applying [illegible] control charting techniques to residuals is that the mean of the residuals is not constant after a step in the mean

CHAPTER 5

Control Charts for Monitoring the Mean

5.1 Introduction

In this chapter, some of the control charts for monitoring the mean which attempt to deal with the problem of autocorrelation are presented and applied to simulated correlated data. In section *5.2* traditional control charts are implemented to the original observations with modified control limits to account for the autocorrelation. In section *5.3* the charting of the residuals from the fit of a time series model to the data is discussed along with some modifications of this procedure. Finally, in section *5.4* modified control limits and other techniques for use with processes that can be described by an *AR(1)* plus a random error model are presented.

5.2 Control Charts based on the observations

A simple but sometimes effective way to deal with autocorrelated data is to implement traditional control charts to the observations using the correct variance of the process when calculating the control limits. First the appropriate time series model must be fitted to the data. The variance must then be estimated using the equations presented at *Box and Luceno (1997)*.

5.2.1 A first Approach

A first approach to adapting traditional control charts to account for autocorrelation was made by *Vassilopoulos and Stamboulis (1978)* who had modified the control limits of traditional charts. They considered the case of several samples of size *n*.

They described the process whose control is under consideration by

$$X_t=\mu+\varepsilon_t$$

where μ is a constant, and the error term is coming from a *zero* mean second order autoregressive process, *AR(2)*

$$X_t= \mu(1-\varphi_1-\varphi_2)+ \varphi_1 X_{t-1}+\varphi_2 X_{t-2}+\alpha_t$$

which is stationary when the following conditions are satisfied

$$\varphi_1+\varphi_2<1$$

$$\varphi_2-\varphi_1<1$$

$$-1<\varphi_2<1$$

The variance of the process, σ^2_X, as described by *Box and Luceno (1997)* is given by

$$\sigma_X^2=\frac{\sigma_\alpha^2}{1-\rho_1\phi_1-\rho_2\phi_2}=\left(\frac{1-\phi_2}{1+\phi_2}\right)\frac{\sigma_\alpha^2}{\left\{(1-\phi_2)^2-\phi_1^2\right\}}$$

and the variance of the sample mean in terms of the autocovariance function γ_k is given by

$$\sigma_{\bar{X}}^2=\frac{1}{n}\left[\sigma_X^2+2\sum_{t=1}^{n-1}\left(1-\frac{t}{n}\right)\gamma_t\right]$$

which after substituting the corresponding expression for the autocovariance of the *AR(2)* process becomes

$$\sigma_{\bar{X}}^2=\frac{\gamma_0}{n}\left[\frac{G_1(1-G_2^2)}{(G_1-G_2)(1+G_1G_2)}\lambda(G_1,n)-\frac{G_1(1-G_1^2)}{(G_1-G_2)(1+G_1G_2)}\lambda(G_2,n)\right]=\frac{\gamma_0}{n}\lambda(\varphi_1,\varphi_2,n)$$

where $\lambda(G,n)=\dfrac{1+G}{1-G}-\dfrac{2G(1-G^n)}{n(1-G)^2}$ and G_1, G_2 are the real and distinct roots of the characteristic polynomial of the *AR(2)* process, $\varphi(z)=1-\varphi_1(z)-\varphi_2^{\,2}(z)$.

If $\overline{\overline{X}}$ denotes the grand mean over several subgroups of size n, the control limits for monitoring the mean when the standard deviation is known are

$$\overline{\overline{X}} \pm \sqrt{\lambda(\phi_1,\phi_2,n)}\frac{3}{\sqrt{n}}\sigma_X$$

Vassilopoulos and Stamboulis (1978) have extended their results for G_1, G_2 being complex conjugates where more complicated calculations are needed. They have also modified the control limits for monitoring the mean when the standard deviation has to be estimated, and for monitoring the variance.

5.2.2 The modified Shewhart X-chart

To monitor the stability of a process $\{X_t, t \in T\}$ at time t, the individuals Shewhart chart compares observation X_t with the in-control mean equal to the target value μ_0. If the absolute value of the difference at time t is considered large then the process is said to be out-of-control *i.e.*, $E(X_t) \neq \mu_0$. Otherwise the process is said to be in-control and the next observation is taken and examined. Statistically speaking, large is always defined in terms of the standard deviation and thus the process is said to be out-of-control if

$$|X_t - \mu_0| > c\sigma_X$$

where σ_X is the standard deviation of the process.

The main idea behind the modified X-chart is that the deviation of the observation from the target value of the mean is compared with the standard deviation of the time series model that describes the process. This standard deviation for a particular $ARMA(p,q)$ model can be calculated following the suggestions of *Box and Luceno (1997)* using some previous observations from the time the process was supposed to be in-control.

The other issue concerning the implementation of the Shewhart chart to the observations of a correlated process is the selection of the appropriate value of c. This can be done in terms of the desired average run length ARL of the chart. According to *Kramer and Schmid (1997)* for autocorrelated processes setting the in-control *ARL* equal to a specified value, the quantity c does not only depend on this value but on the parameters of the process as well. For the case of modeling the process by an *AR(1)* time series model *Schmid(1995)* gives tables of c for an in-control *ARL* of *500*. *Kramer and Schmid (1997)* suggest the use of this table when the level of correlation is large. When the correlation is low or moderate they recommend the choice of c as in the *i.i.d.* case.

Wardell et al. (1992) have considered an individuals chart for a sequence of autocorrelated observations $\{X_t\}$ when the correlation structure is modeled by an *ARMA(1,1)* time series model. Since the process is autocorrelated, its standard deviation σ_X depends on the parameters of the time series model, and as stated at *Box and Luceno (1997)* equals to

$$\sigma_X = \sqrt{\frac{1-2\phi\theta+\theta^2}{1-\phi^2}}\sigma_a$$

where σ_a is the standard deviation of the random error terms a_t and φ, θ are the model parameters. The control limits for the X-chart are

$$\mu_0 \pm c\sigma_X$$

where c is a constant. When the process is uncorrelated φ, θ equal to *zero*, thus $\sigma_X=\sigma_a$ and the above limits are the traditional Shewhart individuals chart.

Wardell et al. (1992) recommended choosing c equal to *3* as in the *i.i.d.* case. For achieving the desired in-control *ARL* value of *370,4 Wardell et al. (1994)* used simulation to set the limits and found that c should range from *±2,45* to *±3,03* standard deviations of the observations to adjust for autocorrelation.

5.2.3 The modified *EWMA* and *CUSUM* charts

The same approach can be used to construct the control limits for a traditional *EWMA* chart of the observations.

The *EWMA* control chart based on the original observations is defined by $Z_t=\lambda X_t+(1-\lambda)Z_{t-1}$, $t=1,2,\ldots$, and the control limits are

$$\xi \pm c\sqrt{\frac{\lambda}{2-\lambda}}\sigma_X$$

where σ_X is the standard deviation of the observations, calculated taking into consideration the correlative structure of the process.

In the same way a *CUSUM* chart can be constructed using the correct standard deviation of the process for setting the decision interval $H=c\sigma_X$.

The analytical treatment of modified control charts is more complicated than just correcting the variance. The determination of the control limits includes the choice of c which is more extensive since it should depend on the parameters of the underling times series model.

Wardell et al. (1992) implemented the *EWMA* chart to an *ARMA(1,1)* process adjusting the variance to the calculation of the control limits. They determined the width of the control limits through simulation using as criterion that ARL_0 should be around *380* and found that the control limits should range from $\pm 0,97$ to $\pm 7,38$ standard deviations of the observations. They also noted that these wide control limits may affect the *ARL* performance of the *EWMA* chart especially for detecting large shifts.

Schmid*(1997a,b)* also gave several tables for the critical values of the modified *EWMA* and *CUSUM* schemes for the *AR(1)* model.

5.2.4 An *EWMA* chart for Stationary Processes

Zhang (1998) proposed an extension of the traditional *EWMA* control chart for monitoring step shifts in the mean applied to any stationary process data, the so called ***EWMAST*** chart (generalization given *by Jiang,Tsui and Woodall(2000)*). He assumed that only one observation of the process $\{X_t, t \in T\}$ is available at each sample, and showed that an *EWMA* of a stationary process is asymptotically a stationary process and determined the limits of the *EWMAST* by the process variance and autocorrelation.

He also assumed that $\{X_t\}$ is a discrete stationary process with constant mean and autocovariance function. It can be showed that the *EWMA* of X_t defined as $Z_t=\lambda X_t-(1-\lambda)Z_{t-1}$ is asymptotically stationary, and that the variance σ^2_z of Z_t is

$$\sigma_z^2 = [\lambda/(2-\lambda)]\sigma_X^2$$
$$\times\left\{1+2\sum_{k=1}^{M}\rho(k)(1-\lambda)^k \times [1-(1-\lambda)^{2(M-k)}]\right\}$$

and that for large t its approximation is

$$\sigma_z^2 = var[Z_t] = [\lambda/(2-\lambda)]\sigma_X^2$$
$$\times\left\{1-(1-\lambda)^{2t} + 2\sum_{k=1}^{t-1}\rho(k)(1-\lambda)^k \times [1-(1-\lambda)^{2(t-k)}]\right\}$$

where M is an integer and $\rho(k)$ is the autocorrelation of X_t at lag k.

The *EWMAST* chart is constructed by charting Z_t. The limits are

$$\mu \pm c\sigma_z$$

where σ_z is the standard deviation of Z_t. Assuming no change of the autocorrelation of $\{X_t\}$, the *EWMAST* will signal changes of the process mean.

In practice, μ and σ^2_z are estimated based on some historical data of X_t when the process is in control, and by replacing μ by the sample mean and σ^2_X and $\rho(k)$ by their sample estimates. *Zhang (1998)* suggests that the size of the historical data set should contain at least *100* observations and that for $\lambda \geq 0,2$ M should be *25*.

The *EWMA* could be considered as a special case of the *EWMAST*, chart. When $\{X_t\}$ is an *i.i.d.* sequence, $\rho(k)$ equals to *0* and the variance of Z_t as well as its approximation calculated from the equations above, are the same as the variance calculated for an *EWMA* chart for independent observations.

Zhang(1998) also considered the effect of the *EWMA* parameter λ to the performance of the *EWMAST* control chart. He concluded that a reasonable choice of λ is *0,2* or *0,1*.

5.2.5 An application

The data used for this application is the same with the simulated data used in application *4.3.1*.

The process is supposed to be in statistical control for the first *80* observations but after the *80^th^* observation a shift in the mean will be introduced. To avoid the effect of the shift in the mean after the 80^{th} observation, the first *80* observations are used to fit the appropriate time series model. Using the *MINITAB* statistical software an *ARMA(1,1)* time series model is fitted to the observations resulting to $\varphi=0,7081$, $\theta=0,1613$ and $\sigma_a=0,8812$. By examining the plots of Figure B.2 (Appendix) it seems that this model fits well the data. Using the equations of *Box and Luceno (1997)* which are presented in paragraph *3.3.4* of chapter *3*, the true variance of the process is estimated to be $\sigma^2_X=1,242$ and the standard deviation $\sigma_X=1,1145$.

Suppose now that due to a special cause a shift in the mean of magnitude $\delta=1\sigma_X$ occurs after the 80^{th} observation. This is accomplished by adding a constant of *1,1145* to the last *20* observations.

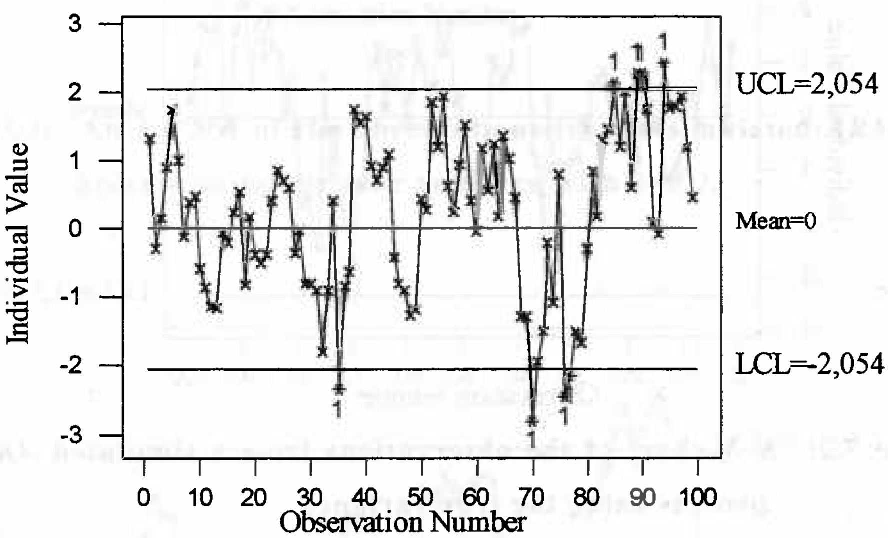

Figure 5.1: ***X*-chart of the observations from a simulated *ARMA(1,1)* process.**

To show the effect of the autocorrelation a traditional *X*-chart of the observations is applied to the data as shown at *Figure 5.1*. The variance of the process is calculated without taking into consideration the autocorrelation, using moving ranges. The estimate of σ_X is $MR/d_2=0,685$ which is considerably below the actual $\sigma_X=1,1145$. Using three-sigma limits with this estimate of σ_X results to control limits of $\pm 2,054$. This chart detects the shift in the mean soon after it occurs, at observation *84*, but the problem is that it also signals four times at the first *80* in-control observations *(Nos.*

35,70,76,77). The cause of these false alarms is of course the failure of this chart to account for the autocorrelation of the process.

A *X*-chart of the observations *(Figure 5.2)* is applied to the data that uses for the calculation of the control limits the true standard deviation of the process $\sigma_X=1,1145$ and results to *3*-sigma control limits of $\pm 3,344$. These limits seem to be a little too wide. There aren't any signals before observation *80*, but neither are there any afterwards, when the shift occurs. Of course after observation *80* a trend is apparent which applying run-rules indicates the lack of statistical control.

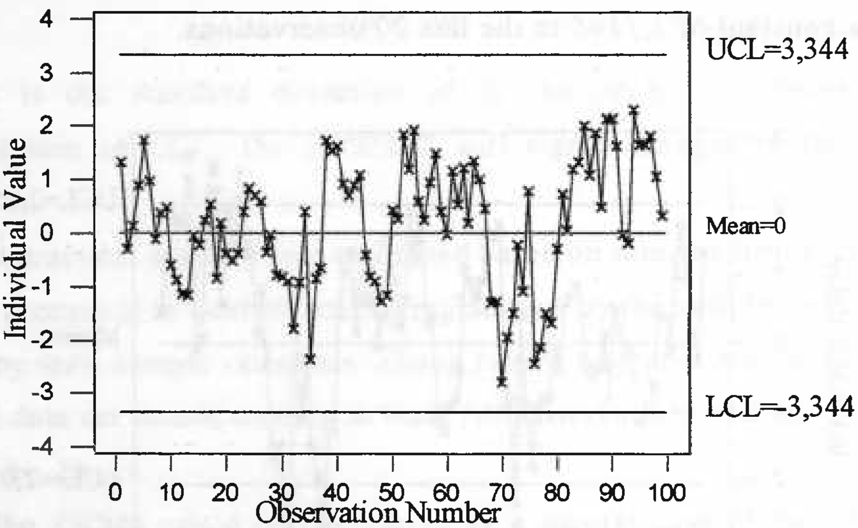

***Figure 5.2:* A *X*-chart of the observations from a simulated *ARMA(1,1)* process using the true variance.**

Taking now into consideration that the observations are correlated and using the correct standard deviation the *3*-sigma limits of the *EWMA* chart using $\lambda=0,2$ are $\pm 1,115$. This chart *(Figure 5.3)* performs much better. The shift is captured again at observation *87* but the false alarms are considerably less.

Using the methods of *Zhang (1998)* the *EWMAST* chart *(Figure 5.4)* is applied to the data . The standard deviation used is *0.353* and the *3*-sigma control limits are $\pm 1,059$. The performance of this chart is similar to the performance of the *EWMA* chart of the observations with control limits calculated using the true variance of the autocorrelated process. This is expected because *Zhang(1998)* actually estimates correctly the variance of the process.

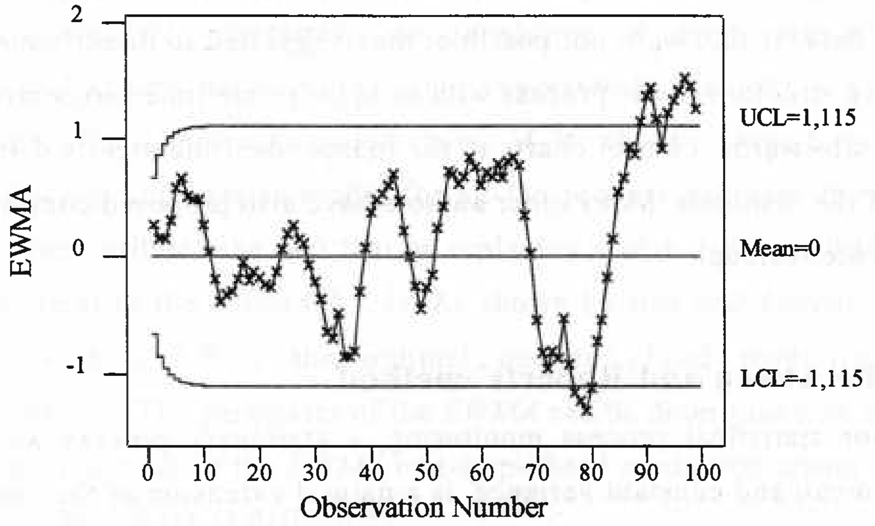

Figure 5.3: **An *EWMA* of the observations from a simulated *ARMA(1,1)* process using the true variance with $\lambda=0,2$.**

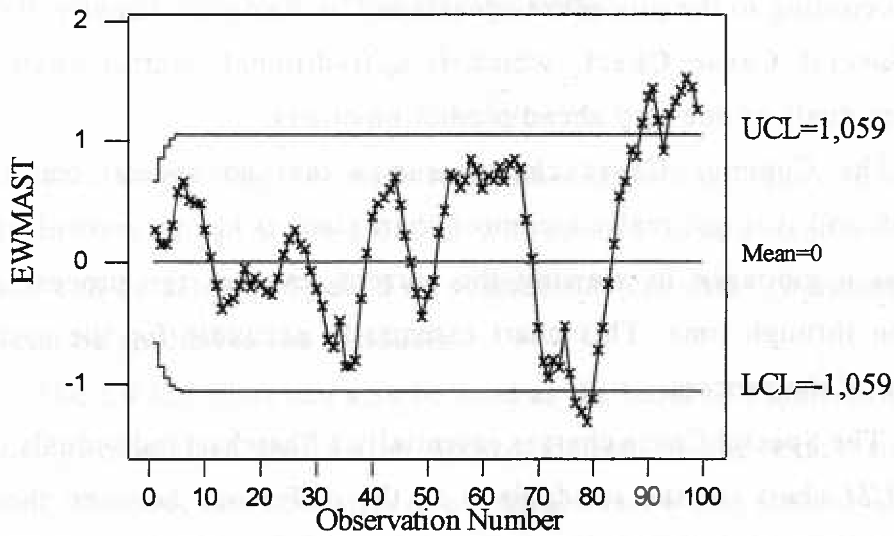

Figure 5.4: **The *EWMAST* of the observations for an *ARMA(1,1)* process with $\lambda=0,2$.**

5.3 Control charts based on the residuals

Alwan and *Roberts (1988)* first tried to remove the autocorrelation from the data. If this were not possible, they suggested to directly model the correlative structure of the process with an appropriate time series model and to apply afterwards control charts to the independent identically distributed stream of the residuals. Many other authors have also proposed control charts based on the residuals.

5.3.1 The Alwan and Roberts' method

For statistical process monitoring, a stationary process which has constant mean and constant variance, is a natural extension of the case of an *i.i.d.* sequence, thus the appropriate model should be a stationary time series model. *Alwan and Roberts (1988)* suggested the implementation of two basic charts rather than one:

I. **Common Cause Chart**, which is a chart of forecasted values that are determined by fitting the correlated process with an *ARIMA* model, according to the procedures developed by *Box and Luceno (1997)*.

II. **Special Cause Chart**, which is a traditional control chart of the residuals or one step ahead prediction errors.

The Common Cause chart assumes that no special causes have occurred, and it is not really a control chart since it has no control limits. It serves as a guidance in viewing the current level of the process and its evolution through time. This chart essentially accounts for the systematic variation of the process.

The Special Cause chart is essentially a Shewhart individuals, *EWMA* or *CUSUM* chart for the residuals (*i.e.*, the difference between the actual process values and their forecasts). A Shewhart individual chart of the residuals is a time-ordered plot of the residuals with $c\sigma$ control limits, and will be referred to as residuals chart, *X*-chart of the residuals. Since the residuals which are the data used for the Special Cause chart are *i.i.d.* random variables all supplemental guidelines, such as run rules, are applicable.

5.3.2 Approximate *EWMA* procedures

In practice, the time series modeling needed to implement the above procedure might be awkward. To tackle this difficulty *Montgomery* and *Mastrangelo(1991)* suggested the modeling of every process with an *ARIMA(0,1,1)* time series model. They suggested the use of a *X*-chart for the residuals, for simplicity called here ***M-M* chart**, which assumes an *ARIMA(0,1,1)* time series model for all the process and uses the prediction errors. They utilized the fact that by replacing λ with $1-\theta$ the *EWMA* statistic is equivalent to the *ARIMA(0,1,1)*. As shown by *Box and Luceno (1997)* the *EWMA* with $\lambda=1-\theta$ is the optimal one-step-ahead prediction for the *ARIMA(0,1,1)*. The parameter of the *EWMA* can be determined by minimizing the sum of squares of the *EWMA* one-step-ahead prediction errors.

The *EWMA* is defined by

$$Z_t=\lambda X_t+(1-\lambda)Z_{t-1}\,,\ t=1,2,\ldots$$

if $\hat{X}_{t+1}(t)$ is the forecast for the observation in period $t+1$ made at the end of period t, then the optimal forecast is the value of the *EWMA* calculated in period t, and the one step ahead residuals are:

$$\hat{X}_{t+1}(t) = Z_t$$
$$e_t = X_t - \hat{X}_t(t-1)$$

If the underlying process is really an *ARIMA(0,1,1)* the one step ahead prediction errors given above are *i.i.d.* with mean *zero* and standard deviation σ, which can be estimated based on some historical data. Therefore, control charts can be applied to the residuals.

The *EWMA* chart can also be used as the basis of a statistical process monitoring procedure that is an approximation of the exact time-series approach. The procedure consists of a control chart where the one-step-ahead prediction errors, is plotted, and a run chart of the original observations on which the *EWMA* forecast is superimposed.

The residuals chart detects unusual random shocks but is not as sensitive to slow trending shifts in the mean. Therefore, *Montgomery and Mastrangelo (1991)* provided a modification of the above procedure that combines information about the state of statistical control and process dynamics on a single control chart. The modified procedure actually monitors simultaneously the forecast and the forecast errors. Since the *EWMA* statistic is a suitable one step ahead predictor, Z_t could be used as the centerline on a control chart for period $t+1$ with control limits at

$$CL=Z_t \pm 3\sigma$$

and the observation X_{t+1} will be compared to these limits to test for statistical control. This procedure is called moving centerline *EWMA* control chart or for simplicity ***MCEWMA***.

The *MCEWMA* is an exact procedure for the *ARIMA(0,1,1)* process, but is a good approximation as well for many other time series, provided that that the value of λ is selected appropriately. It is a good approximation, especially for a process in which the mean exhibits slow drifting behavior and the observations at low lags appear to be positively autocorrelated.

The *MCEWMA* accounts for the autocorrelative structure of the data and does not indicate any unnecessary out-of-control conditions. When the process does change it rather tends to track a trend instead of indicating an out-of-control process. Since the *MCEWMA* chart plots the actual autocorrelated observations, run rules and other supplemental guidelines can not be used to capture any out-of-control conditions. To overcome this limitation and provide trend detection and enhance other types of shift detection, *Mastrangelo and Montgomery (1995)* have supplemented the *MCEWMA* chart with tracking signals. They introduced the cumulative error tracking signal $T_c(t)$, and the smoothed error tracking signal $T_s(t)$.

The cumulative error tracking signal is

$$T_C(t)=\left|\frac{Y(t)}{\hat{\Delta}(t)}\right|$$

where $Y(t)=\sum_{j=1}^{t} e_j$ is the sum of the forecast errors up to time t and the quantity $\hat{\Delta}(t)=a|e(t)|+(1-a)\hat{\Delta}(t-1)$ is the mean absolute deviation with a being a smoothing constant, typically selected between $0,05$ and $0,15$. At each sampling point t the statistic $T_c(t)$ is compared to a critical value K_c, where $4 \leq K_c \leq 6$, and if the tracking signal exceeds K_c there is indication that an out of control signal has occurred.

Similarly, the smoothed error tracking signal is

$$T_s(t) = \left| \frac{Q(t)}{\hat{\Delta}(t)} \right|$$

where $Q(t)=2ae_t+(1-a)Q(t-1)$ with a still being a smoothing constant selected between $0,05$ and $0,15$. The statistic $T_s(t)$ is now compared to a constant K_s, where typically $0,2 \leq K_s \leq 0,5$, resulting to an out of control indication if it exceeds it.

To analyze monthly series of survey quality data *Hapuarachchi et al.(1997)* proposed a modification of the Alwan and Roberts's procedure that resembles the *MCEWMA* chart.

Hapuarachchi et al.(1997) superimposed residuals charts on the time series plots of the original observations and their forecast values to enable easier detection of process level changes. They used data up to a predefined date to fit an appropriate *ARIMA* time series model and they used this model to calculate control limits based on its confidence interval and to make forecasts of its subsequent values. On the same graph they plotted the original data, the control and warning limits, the forecasts and the trend. They draw a vertical line to divide the original series and the forecasts and they compared the original values after the predefined date with the forecasts and the control and warning limits. *Hapuarachchi et al.(1997)* used this method to analyze several series of quality data and in many cases they found assignable causes for processes behavior.

5.3.3 The Common Cause chart with control limits

Wardell et al.(1992) considered an extension of the *Alwan and Roberts'* Common Cause chart for a process $\{X_t\}$, which can be fitted by a first order autoregressive moving average process (*ARMA (1,1)*) time series model.

As stated before, the Common Cause chart is the chart of the fitted values or forecasts obtained when the process is modeled by some time series model, and has no limits. Sometimes forecasts signal when an out of control condition arise, before the residuals indicate the change. Therefore, it might be useful to provide the Common Cause chart with limits.

To construct the limits for the Common Cause chart, *Wardell et al.(1992)* derived the mean and the variance of the forecasts. The one step ahead forecast which minimizes the mean squared error for the *ARMA(1,1)* model is given by

$$\hat{X}_{t+1}=(1-\phi)\mu+(\phi-\theta)X_t+\theta\hat{X}_t$$

where $\hat{X}_{t+1}$ is the forecast made at time t for period $t+1$, and μ is the mean of the process which is assumed to be constant while φ is the autoregressive parameter, and θ is the moving average parameter.

The forecast can be expressed as a weighted sum of series of independent error terms generating the process by

$$\hat{X}_{t+1}=\mu+(\phi-\theta)\sum_{k=0}^{t-1}\phi^k\alpha_{t-k}++(\phi^t-\theta^t)\sum_{k=t}^{\infty}\phi^{k-t}\alpha_{t-k}$$

This equation shows that the mean of $\hat{X}_{t+1}$ is μ since the error terms $\{\alpha_t\}$ are independent with mean 0. The variance of the forecasts σ^2_f is given by

$$\sigma_f^2=Var(\hat{X}_{t+1})=\frac{(\phi-\theta^t)^2}{(1-\phi^2)}(1-2\phi^t\theta^t+\theta^{2t})\sigma_\alpha^2$$

where σ^2_α is the variance of the error terms for the appropriate *ARMA(1,1)* model. When t is sufficiently large the steady state variance of the forecasts, which is given by

$$\sigma_f^2=\frac{(\phi-\theta)^2}{(1-\phi^2)}\sigma_\alpha^2$$

can be used.

The control limits for the Common Cause chart can now be constructed and be

$$\mu\pm c\sigma_f$$

where c is a constant usually equal to 3. When the steady state variance is used for the above limits, they do not vary with time.

5.3.4 A *CUSUM* chart for the residuals from an *AR(p)* process

Runger et al. (1995) applied an one sided *CUSUM* chart on the residuals of an autoregressive process of order p ($AR(p)$).

They considered that the observations $\{X_t, t=1,2,...,\}$ follow an $AR(p)$ time series model with constant mean μ

$$X_t=(1-\varphi_1-...-\varphi_p)\,\mu+\varphi_1 X_{t-1}+...+\varphi_p X_{t-p}+\alpha_t\,,$$

where $\{\alpha_t\}$ is the random error (noise) at time t which is assumed to be *i.i.d.* with mean *zero* and variance σ^2_a while φ_i are the autoregressive coefficients. The residual e_t at time t is the difference between X_t and the prediction of X_t based on the previous data.

The one-sided *CUSUM* accumulates upward derivations from μ_0 with the statistic C, which is developed as follows:

$$C_t=[C_{t-1}+e_t-(\mu_0+K)]^+=max\{0,\ [C_{t-1}+e_t-(\mu_0+K)]\}$$

where the starting values are $C_0=0$ and $K>0$ is a parameter of the chart.

The value of K is usually chosen close to the midway point between μ_0 and the out of control value of the mean μ_1.

When the shift in the process mean is a single step change of size δ (where δ is measured in terms of the standard deviation of the process), for the special case of an $AR(1)$ process, the mean of the first residual after the shift is δ, and the mean of subsequent residuals is $\delta(1-\varphi)$.

Therefore, since every residual but the first has expected value $\delta(1-\varphi)$ *Runger et al. (1995)* proposed the modified guideline $K=\sigma(1-\varphi)/2$ which takes explicit account of the autocorrelation.

The chart signals if C_t exceeds the decision interval H which usually has a value five times the process standard deviation σ.

5.3.5 An application

The data set of application *4.3.1* is used to show how the modeling of the autocorrelation of the process by time series works.

As previously, for the first *80* observations the process is in control but after the *80*th observation a shift in the mean occurs. The first *80* observations are used to calculate the parameters. The standard deviation of the process is calculated to be $\sigma_X = 1{,}1145$. A shift in the mean of magnitude $\delta = 1\sigma_X$ occurs after the *80*th observation by adding a constant of *1,1145* to the last *20* observations.

The method of *Alwan and Roberts (1988)* is applied to the data. A time series model is fitted. The Special Cause chart is constructed which is actually a *X*-chart of the residuals, along with the Common Cause chart *i.e.*, the fitted values charted without control limits *(Figure 5.5)*. The fitted values that are charted without limits, come from the *ARMA(1,1)* model using the estimated autoregressive and moving range parameters

$$\text{Fitted } X_t = 0{,}7081X_{t-1} + a_t - 0{,}1613a_{t-1}$$

The standard deviation used for the *3*-sigma limits of the *X*-chart of the residuals, is the standard deviation of the residuals $\sigma_a = 0{,}8812$ and the limits are $\pm 2{,}644$. The shift at the mean is detected quickly at observation *89* but there is also a false alarm at observation *76*.

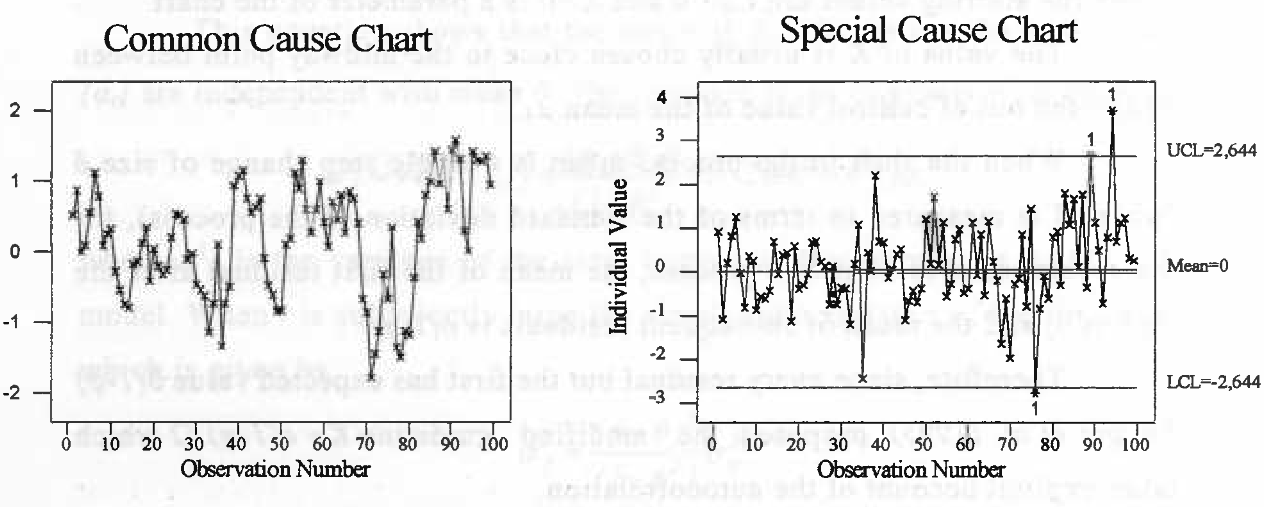

Figure 5.5: **Implementation of the Alwan and Roberts' method to a simulated *ARMA(1,1)* process.**

The Common Cause chart takes advantage of the fact that the process is correlated to make forecasts of future quality. Each point is an estimate of the local level of the process and can be used as a signal that a corrective action is needed. To be more specific, suppose that the desired level of the above process is *zero* and that upwards or downwards deviations from this level lead to economic loss from not acceptable product. Suppose that recentering the process at any time is possible at some known cost. Economic calculations can then be made to balance the expected loss of bad product over some period of time, against the cost of recentering and consequently to define action limits for both bellow and above *zero*.

The Common Cause chart provided with control limits as suggested from *Wardell et al. (1992)* is implemented to the data *(Figure 5.6)*. The standard deviation of the fitted values calculated as described above is $\sigma_f=0,6824$ and the control limits using $c=3$ are $\pm 2,805$.

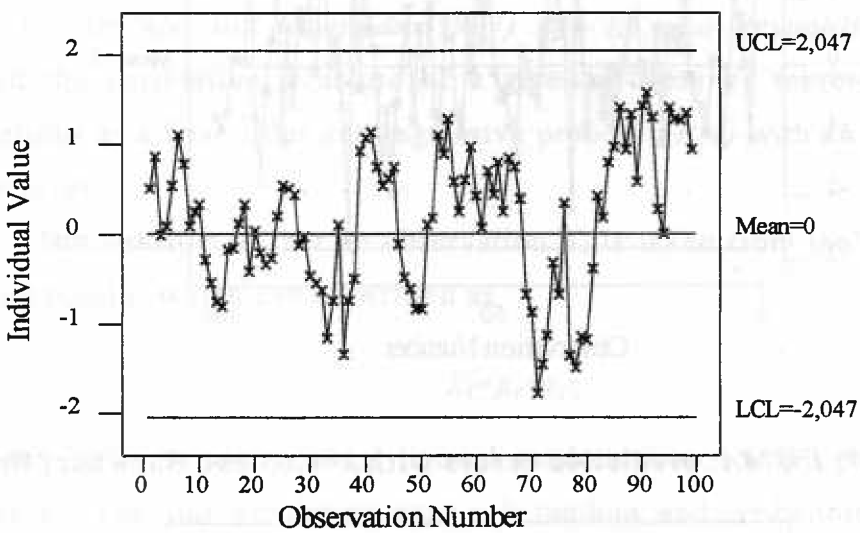

***Figure 5.6* : A *X*-Chart of the fitted values for an *ARMA(1,1)* process .**

This chart doesn't seem to be more helpful than the simple Common Cause chart without control limits of *Alwan and Roberts (1988)*. It does give useful information about the level of the process, but it doesn't indicate the shift in the process mean before the residuals chart. Further more this chart doesn't signal at all, since no observation falls outside the control limits. After observation *80* though, it is obvious that the process shows an upwards drift.

The same data set is used to apply the *M-M* chart *(Figure 5.7), i.e.*, the control chart introduced by *Montgomery and Mastrangelo (1991)*. The *EWMA* parameter λ was selected to be *0,65* by minimizing the sum of squares of the *EWMA* one step ahead prediction errors using the set of data 1-80. The standard deviation of the prediction errors is *0,95* and the *3*-sigma control limits are $\pm 2,805$. This chart does not signal any false alarm, but it doesn't signal either any out of control situations after the shift of the mean at the 80^{th} observation. The modification of the *M-M* chart, the *MCEWMA* chart *(Figure 5.8)* seems to perform better.

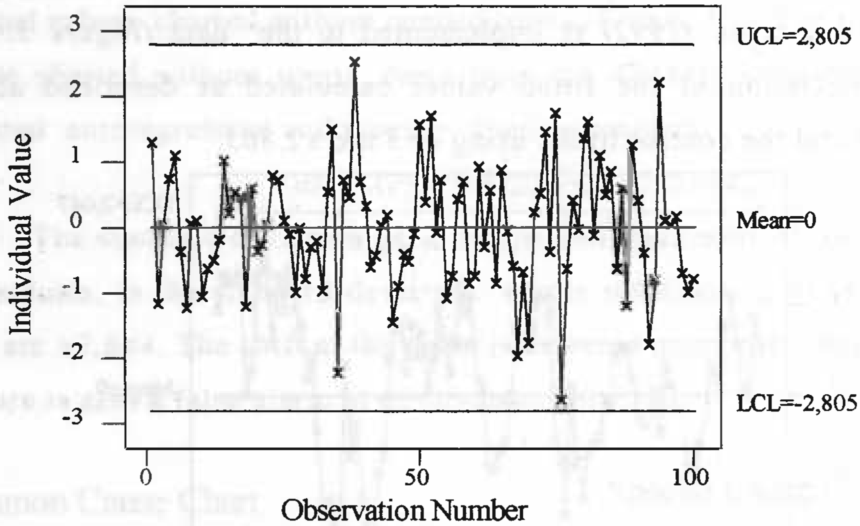

***Figure 5.7: EWMA* prediction errors with λ=0,65 and Shewhart limits.**

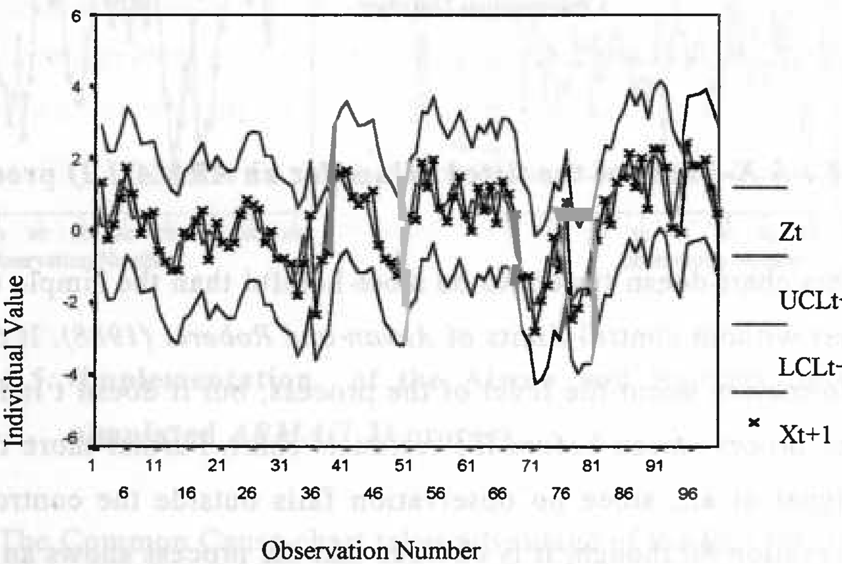

***Figure 5.8:* Moving centerline *EWMA* chart with λ=0,65.**

5.4 Control Charts for the *AR(1)* plus a random error model

The *AR(1)* plus a random error model is a natural way to view the autocorrelation as an inherent characteristic of the process. In the first paragraph of this section this model is described. In the following two paragraphs some control charts especially designed and evaluated for this model are discussed. Last, an implementation of these charts to autocorrelated data is presented.

5.4.1 The *AR(1)* plus a random error model

VanBrackle and Reynolds (1997) and *Lu and Reynolds (1999a)* modeled the correlative structure of a given process by representing the observations as a first order autoregressive process *AR(1)* with an additional random error.

They considered that an observation X_t is taken from the process at sampling point t, which can be written as

$$X_t=\mu_t+\varepsilon_t\,,$$

where the ε_t's are independent normal random errors with mean 0 and variance σ^2_ε. The time wandering mean μ_t is random, and can be interpreted as the random process mean at time t following an *AR(1)* process with mean ξ, namely $\mu_t=(1-\varphi)\xi+\varphi\mu_{t-1}+\gamma_t$, $t=1,2,\ldots$, where γ_t are independent and normally distributed random errors with mean 0 and variance σ^2_γ while φ is the autoregressive parameter satisfying $|\varphi|<1$.

The objective of the statistical control in this case is to detect a change in the overall mean $\xi=E\,(\mu_t)$.

If it is assumed that the starting value μ_0 follows a normal distribution with mean ξ and variance $\sigma^2_\mu=\sigma^2_\gamma\;/(1-\varphi^2)$, then this implies that the distribution of X_t is constant with mean μ and variance $\sigma^2_x=\sigma^2_\mu+\sigma^2_\varepsilon$ for all $t=1,2,\ldots,$.

Besides, ψ can be defined as the proportion of the process variance that is due to the *AR(1)* process:

$$\psi = \frac{\sigma_\mu^2}{\sigma_x^2} = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2}$$

The covariance between X_{t-i} and X_t is $\varphi^i \sigma^2_\mu$ for $t \geq i$, and the correlation between X_{t-1} and X_t is $\varphi\psi$.

The above model is a possible model for processes in which the variability of observations has both short and long term components. One can think of σ^2_μ as representing long term variability and σ^2_ε as representing a combination of short term variability and measurement error. If $\sigma^2_\varepsilon = 0$ the model reduces to the simple *AR(1)* model.

Although the *AR(1)* plus a random error model is a natural way to represent the autocorrelation as an inherent characteristic of the process, sometimes for technical reasons, it is convenient to express it in terms of an *ARMA(1,1)* time series model.

The *AR(1)* process with additional random error is equivalent to an *ARMA(1,1)* process. The *ARMA(1,1)* process can be written as

$$X_t - \varphi X_{t-1} = (1-\varphi)\xi + \alpha_t - \theta\alpha_{t-1}$$

where the α_t's are independent normal random variables with mean 0 and variance σ^2_α, θ is the moving average parameter, and φ is the same autoregressive parameter as in the *AR(1)* model used before to describe the wandering of the mean.

There are equations for expressing the parameters φ, θ, σ^2_a in the *ARMA(1,1)* model in terms of the parameters φ, σ^2_γ in the *AR(1)* plus random error model and vice versa as shown in *Box et al. (1994)*. In particular, if $\varphi > 0$ and $\sigma^2_\varepsilon > 0$, then the *ARMA(1,1)* model parameters can be derived from the *AR(1)* plus random error parameters by

$$\theta = \frac{\sigma_\gamma^2 + (1+\phi^2)\sigma_\varepsilon^2}{2\phi\sigma_\varepsilon^2} - \frac{1}{2}\sqrt{\frac{\sigma_\gamma^2 + (1+\phi^2)\sigma_\varepsilon^2}{\phi\sigma_\varepsilon^2} - 4}$$

and $\sigma_\alpha^2 = \dfrac{\phi\sigma_\varepsilon^2}{\theta}$.

Alternately, if $\varphi>0$ then the *AR(1)* plus error model parameters can be obtained from the *ARMA(1,1)* model parameters using the following equations

$$\sigma_{\gamma}^{2} = \frac{\sigma_{\alpha}^{2}(\phi-\theta)(1-\phi\theta)}{\phi}$$

$$\sigma_{\varepsilon}^{2} = \frac{\theta\sigma_{\alpha}^{2}}{\phi}$$

One extension of the above model is to the case where more than one observations *(n>1)* are taken at each sample point. In this case, if X_{ti} is the i^{th} observation at time t, then

$$X_{ti}=\mu_t+\varepsilon'_{ti}$$

where the ε'_{ti}'s are *i.i.d.* following $N(0,\sigma^2_{\varepsilon'})$ and μ_t is the mean at time t following as previously an *AR(1)* process.

The sample means are now used to construct control charts for monitoring the mean of the process. Let

$$\overline{X}_t = \frac{\sum_{i=1}^{n} X_{ti}}{n}$$

be the sample mean for the sample at time t. Then

$$\overline{X}_t = \mu_t + \overline{\varepsilon}_t$$

where $\overline{\varepsilon}_t = \frac{\sum_{i=1}^{n}\varepsilon_{ti}}{n}$, and the variance of $\overline{X}_t$, say $\sigma^2_{\overline{X}}$, is $\sigma^2_{\overline{X}} = \frac{\sigma^2_{\gamma}}{1-\phi^2}+\frac{\sigma^2_{\varepsilon}}{n}$ and μ_t follows as previously an *AR(1)* time series model.

Lu and Reynolds (1999a,2000) studied using simulation, the effects of estimating the parameters of the process and found that the charts perform much better when the parameters are precisely estimated. They proposed that a large data set *(>100)* should be used in the process of fitting a model for the process observations and estimating the parameters.

5.4.2 Control Charts based on the observations

For the problem of detecting shifts in the overall mean, traditional control charts are frequently applied whether or not the observations are independent.

The individuals *X*, the *EWMA* and the *CUSUM* charts based on the original observations can be used in a modified form for monitoring the mean of autocorrelated processes.

Reynolds et al.(1996) following the ideas of *Stamboulis and Vassilopoulos(1978)* modified the *X*-chart to account for the autocorrelation present in the *AR(1)* plus a random error model. They considered the case where only one observation is available at each sample, and the mean of the process is at its target value ξ_0.

The individuals *X*-chart for monitoring the process mean using the original observations has control limits at

$$\xi_0 \pm c\sigma_X$$

where σ_X is the standard deviation of the observations calculated as previously taking into consideration the correlative structure of the process, and c is a constant. *Reynolds et al.(1996)* gives values for c to achieve an in-control *ARL* of *370.4*. For low to moderate level of autocorrelation these values are very close to *3*, which is the value which gives in–control *ARL* of *370,4* for the case of independent observations.

VanBrackle and Reynolds (1997) introduced the modification of the *EWMA* control chart when only one observation is available at each sample, and the mean of the process is at its target value ξ_0.

The *EWMA* control chart based on the original observations is defined by

$$Z_t=\lambda X_t+(1-\lambda)Z_{t-1},\ t=1,2,\ldots,$$

and the control limits are

$$\xi \pm c\sqrt{\frac{\lambda}{2-\lambda}}\sigma_X$$

where $\sigma^2_X\ \lambda/(2-\lambda)$ is the asymptotic variance of Z_t under the assumption that the observations(or for $n>1$ the successive sample means) are independent but σ_X is the standard deviation of the observations, calculated taking into consideration the correlative structure of the process.

The above limits are based on the assumption that the observations (or the successive sample means) in the *EWMA* are independent, although the correct standard deviation of the observations (or sample means) is used. *VanBrackle and Reynolds (1997)* proved that the true asymptotic variance of the *EWMA* statistic is

$$\frac{\lambda}{2-\lambda}\left[\left\{\frac{1+\phi(1-\lambda)}{1-\phi(1-\lambda)}\right\}\sigma_{\mu}^{2}+\frac{\sigma_{\varepsilon}^{2}}{n}\right]$$

but as they stated expressing the control limits in terms of the above variance does not seem to help in making c independent of the level of autocorrelation.

VanBrackle and Reynolds(1997) provided a table for the design of the *EWMA* chart of the observations, taking into consideration the autocorrelation of the process. This table is appearing as TABLE A.1 at Appendix A and gives the in control *ARL* and steady-state *ARL* for several shifts $\delta=0,5$, $1,0$, $2,0$, $4,0$ (where δ is measured in terms of the standard deviation of the process), for various choices of control limits for some combinations of φ, ψ and λ. It gives the *ARL* of the *EWMA* for $\lambda=0,1$ and $0,2$.

TABLE A.1 can be used in conjunction with FIGURE 1 in *Lu and Reynolds(1999a)* which gives values of c for achieving an ARL_0 of $370,4$ for various values of φ, ψ and λ for the *EWMA* chart of the observations.

The optimal values of λ are also considered for detecting specific shifts in the mean. These values are presented at TABLE A.3 for combinations of $\psi=0,5$, $0,9$ $\varphi=0,4$, $0,8$ and $\delta=0,5$, $1,0$, $2,0$. Of course these values are useful for designing a chart when the shift to be detected is known. In real applications though, the a-priori knowledge of the magnitude of the shifts is rarely possible. Therefore, the selection of λ should aim to ensuring a good performance over a large range of shifts. When the level of autocorrelation is low, choosing $\lambda=0,2$ would provide a satisfying overall performance. The value of λ should be increased as the level of autocorrelation increases especially when detecting large shifts is important. Increasing λ in the *EWMA* chart of observations improves performance for large shifts, but does not hurt that much for small shifts.

VanBrackle and Reynolds (1997) and *Lu and Reynolds(2001)* discussed the modification of the *CUSUM* control chart based on the observations for detecting a shift in the mean from its target value ξ_0. A *CUSUM* chart is usually obtained by using two one-sided charts simultaneously.

The one-sided *upper CUSUM* chart based on the original observations uses at its t^{th} observation (or sample for $n>1$) the control statistic

$$C_t^+ = max\{0, C_{t-1}^+ + (X_t - \xi_0 - K)\}$$

where the starting value C^+_0 is a constant usually taken to be *zero*.

The one-sided *lower CUSUM* chart based on the original observations uses at its t^{th} observation (or sample for $n>1$) the control statistic

$$C_t^- = min\{0, C_{t-1}^- + (X_t - \xi_0 - K)\}$$

where the starting value C^-_0 is a constant usually taken to be *zero* unless a head starter is defined. .

The constant $K>0$ is the reference value and is a parameter of the chart. In *Lu and Reynolds(2001)* is expressed as $r\sigma_X$. A signal is given for the upper *CUSUM* if C_t^+ falls above an upper control limit $c\sigma_X$ and for the lower *CUSUM* if C_t^- falls bellow a lower control limit - $c\sigma_X$ where c is a constant.

The two-sided *CUSUM* chart uses both the C^+_t and the C^-_t statistics simultaneously and signals if either statistic signals.

If it is desirable to detect a shift from ξ_0 to ξ_1 then choosing $r=\delta/2$ will minimize the *ARL*, where

$$\delta = \frac{|\xi_1 - \xi_0|}{\sigma_X}.$$

Designing a *CUSUM* chart requires the specification of the parameters of the chart. *VanBrackle and Reynolds(1997)* provided a table for the design of the *CUSUM* chart of the observations, taking into consideration the autocorrelation of the process. This table is appearing as TABLE A.2 at Appendix A and gives the in-control *ARL* and steady-state *ARL* for several shifts δ=*0,5, 1,0, 2,0, 4,0* for various choices of control limits for some combinations of φ, ψ and K. It gives the *ARL* of the *CUSUM* chart for $K=0,25$ and $0,5$.

TABLE A.2 can be used in conjunction with FIGURE 1 in *Lu and Reynolds (2001)* which gives values of c for an ARL_0 of $370,4$ for various values of φ, ψ and r for the *CUSUM* chart of the observations.

When the observations used in the chart are correlated, the optimal choice of the reference value r is not always the same as in the case of independent observations. *Lu and Reynolds (2001)* discussed the choice of r over a range of shifts in the mean that may occur. These optimal values of r are summarized in TABLE A.3 for $\delta=0,5$, $1,0$ and $2,0$ for some combinations of φ and ψ. From the table it is obvious that for shifts of small magnitude *i.e.*, $\delta=0,5$ and $1,0$ the optimal choice for r is close to $\delta/2$. For bigger shifts in the mean *i.e.*, $\delta=2,0$ the optimal values for r are larger than $\delta/2$.

Since the amount of the shift to be detected is not known a priori, it is reasonable before deciding on the value of r, to consider the performance of a chart over a range of shifts of interest. *Lu and Reynolds (2001)* found that for relatively low levels of autocorrelation a good choice for r is $0,5$ while for relatively high levels of autocorrelations a choice of r such as $1,0$ is advisable.

5.4.3 Control charts based on the residuals

Lu and Reynolds (1999) used the *AR(1)* model with an additional random error presented in section *5.8.1* to describe the autocorrelation of the process. They developed an individuals Shewhart and an *EWMA* control chart based on the residuals from the forecast values of the model for monitoring the mean of the process. They assumed that each sample contains only one observation and that the mean of the process is at its target value ξ_0. *Lu and Reynolds (2001)* on the other hand developed a *CUSUM* chart for the residuals.

When the process is in control the minimum mean square error forecast made at time t-1 for time t is for the *AR(1)* model plus a random error, or the equivalent *ARMA(1,1)* model

$$\hat{X}_t = \xi_0 + \phi(X_{t-1} - \xi_0) - \theta e_{t-1}$$

where $e_t = X_t - \xi_0 - \varphi(X_{t-1} - \xi_0) + \theta e_{t-1}$ is the residual at time t. The mean of the residuals is constant and thus to achieve an in-control *ARL* of *370,4* the methods for independent observations can be used.

When there is a shift in the mean of the process the mean of the residuals varies with time. Suppose that there is a step change from ξ_0 to ξ_1 in the process mean between time $t=T-1$ and T. The expectations of the residuals for various times are

$$E(e_t) = \begin{cases} 0 & t = T-1, T-2, \ldots, \\ \xi_1 - \xi_0 & t = T \end{cases}$$

and for $t=T+l$, $l=1,2,\ldots,$

$$E(e_t) = \left[\theta^l + (1-\phi)\sum_{i=1}^{l}\theta^{i-1}\right](\xi_1 - \xi_0) = \frac{\theta^l(\phi-\theta) - \phi + 1}{1-\theta}(\xi_1 - \xi_0)$$

The residual immediately after the shift has its largest mean which decreases afterwards and asymptotically gets to

$$E(e_\infty) = \frac{1-\phi}{1-\theta}(\xi_1 - \xi_0)$$

The residuals are uncorrelated and normally distributed with variance σ^2_a.

A Shewhart individuals chart based on the residuals plots the residuals e_t and uses their standard deviation σ_a as appearing in the *ARMA(1,1)* model for constructing the control limits:

$$\pm c\sigma_a$$

where c is a constant. When the process is in control the residuals are independent and have constant mean and thus using $c=3$ gives an in-control *ARL* of *370,4*.

An *EWMA* chart of the residuals for monitoring the process mean uses the control statistic

$$Z_t = \lambda e_t + (1-\lambda)Z_{t-1}, \quad t=1,2,\ldots,$$

and the control limits are of the form

$$\pm c\sqrt{\frac{\lambda}{2-\lambda}}\sigma_a$$

where c is a constant and σ_a is the standard deviation of the residuals from the *ARMA(1,1)* model. The residuals are independent when the process is in control and have constant mean, thus the control limits of the *EWMA* statistic can be determined using methods for independent observations. In particular, using $c=2,859$ will give $ARL_0=370,4$ when $\lambda=0,2$.

Lu and Reynolds(1999a) investigated the optimal choice of the smoothing parameter λ of the *EWMA* and concluded as expected that small values of λ are better for detecting small shifts while large values of λ are better for detecting large shifts. When the autocorrelation is at its lowest level a relatively small value of λ, such as $\lambda=0,2$, would work well across a wide range of shifts. When the level of autocorrelation becomes higher very small values of λ are optimal for detecting small shifts, but these values of λ perform poorly for large shifts.

The optimal values of λ are presented at TABLE A.3 for combinations of $\psi=0,5,\ 0,9$ $\varphi=0,4,\ 0,8$ for detecting shifts of magnitude $\delta=0,5,\ 1,0,\ 2,0$.

Lu and Reynolds (2001) presented the *CUSUM* chart based on the residuals from the *AR(1)* plus a random error model. The two-sided *CUSUM* chart of the residuals plots simultaneously the

$$C_t^+ = max\{0, C_{t-1}^+ + (e_t - r\sigma_a)\},$$

$$C_t^- = min\{0, C_{t-1}^- + (e_t - r\sigma_a)\}$$

where σ_a is the standard deviation of the residuals of the equivalent *ARMA(1,1)* model and r is the reference value. The chart signals if either of the statistics exceeds the decision interval $H=c\sigma_a$.

The statistics are the same as those of the two-sided *CUSUM* chart of the observations, except that they are based on the residuals e_t instead of the observations X_t. The mean of the residuals is *zero* so the in-control mean ξ_0 is omitted from the above equations.

Lu and Reynolds (2001) discussed the choice of the parameters for the *CUSUM* chart of the residuals. The residuals are independent thus the optimal value of r should depend on the expected values of the residuals after the shift. After the shift though, the mean of the residuals is not constant and although the residuals are independent, the values of the parameters of the chart can not be determined using the methods for independent observations.

For the case of a small shift the optimal value of r is close to half the standardized asymptotic mean of the residuals. This is reasonable because a small shift is not easily detected and thus many residuals will be plotted before the chart signals. This way the mean of the residuals will be close to its asymptotic mean $E(e_\infty)$.

At TABLE A.3 the optimal values of r are presented for various combinations of φ, ψ and δ. A reasonable compromise for good overall performance of a *CUSUM* chart of the residuals is a moderate value of r such as 0.5.

5.4.4 An application

Using the *MINITAB* statistical software an *ARMA(1,1)* time series model has been fitted to the first 80 in-control observations of application *5.3.2* resulting to $\varphi=0,7081$, $\theta=0,1613$ and $\sigma_a=0,8812$. The true variance of the process has been $\sigma^2_X=1,242$ and the standard deviation $\sigma_X=1,1145$.

This model is equivalent with an *AR(1)* plus a random error model with $\varphi=0,7081$, $\sigma_\gamma=0,729$, $\sigma^2_\varepsilon=0,1769$ and $\psi=0,86$. This implies that *86%* of the variability of this process is due to its correlation structure, and that the correlation between adjacent observations is $\rho=\varphi\psi=0,61$.

An *EWMA* chart is constructed, as shown at *figure 5.9*, following the methods of *Lu and Reynolds (1999a)* with the *EWMA* parameter being $\lambda=0,2$. Using $\varphi=0,7081$ and $\psi=0,86$ at FIGURE 1 of *Lu and Reynolds(1999a)* results to $c=4,5$. The *4,5* sigma control limits using the true standard deviation $\sigma_X=1,1145$ are $\pm1,672$. There is no point out of the control limits but there is an obvious shift after observation *80*.

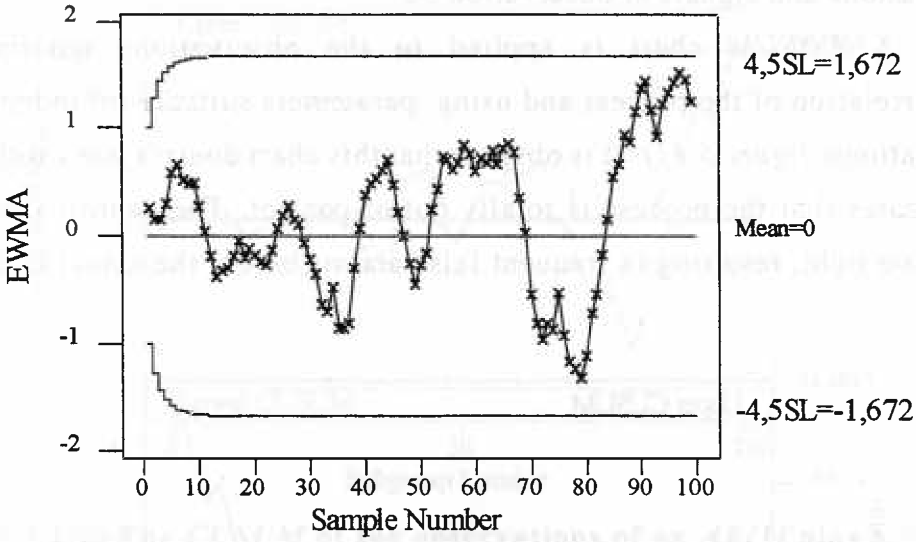

Figure 5.9: **The *EWMA* of the observations for an *AR(1)* plus a random error process with λ=*0.2* and *4.5* sigma limits.**

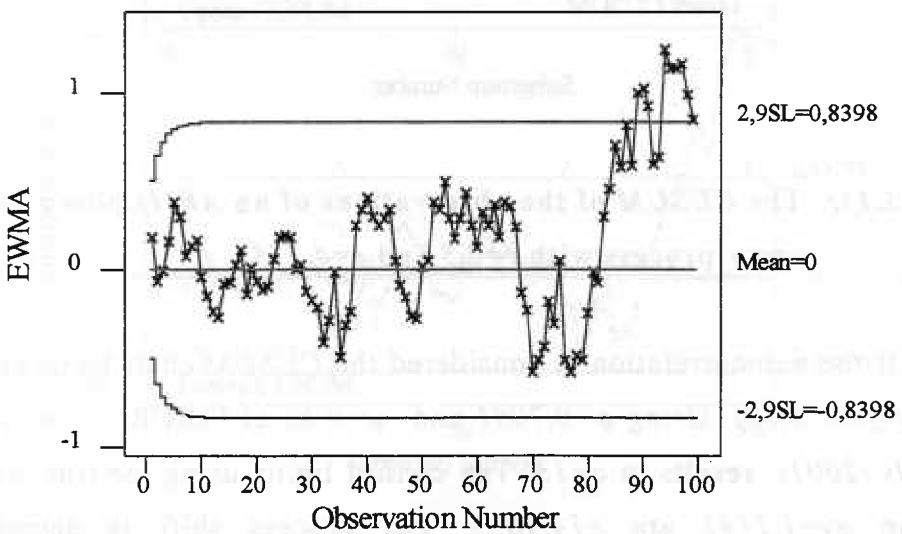

Figure 5.10: **The *EWMA* of the residuals from an *AR(1)* plus a random error process with λ=*0.2*.**

An *EWMA* chart of the residuals *(figure 5.10)* is also applied. The *EWMA* parameter is selected to be λ=*0,2* and the *2,859* control limits are

$\pm 0,8398$. This chart shows an increase in the mean immediately after the 80^{th} observations and signals at observation *88*.

A *CUSUM* chart is applied to the observations ignoring the autocorrelation of the process and using parameters suitable for independent observations *(figure 5.11)*. It is obvious that this chart doesn't work well since it indicates that the process is totally out-of-control. The control limits are much too tight, resulting in frequent false alarms before the actual shift.

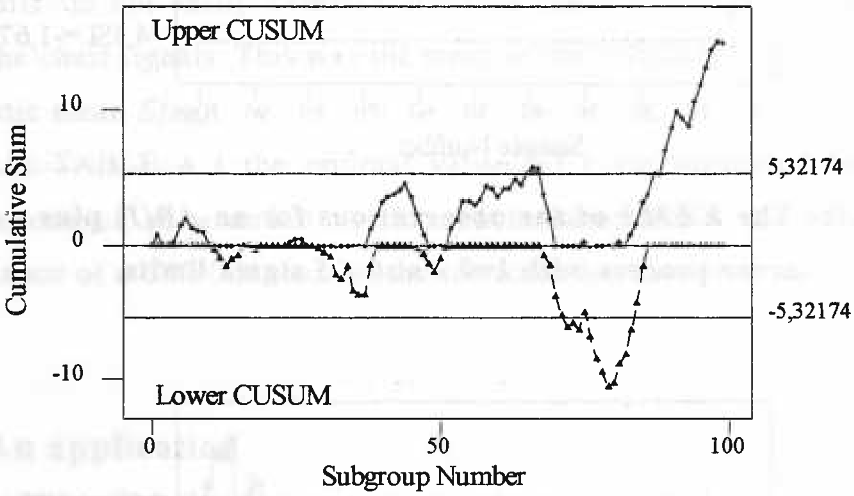

***Figure 5.11:* The *CUSUM* of the observations of an *AR(1)* plus a random error process with *r=0.5* and *c=4.775*.**

If the autocorrelation is considered the *CUSUM* chart behaves much better *(figure 5.12)*. Using $\varphi=0,7081$ and $\psi=0,86$ at FIGURE 1 of *Lu and Reynolds (2001)* results to $c=13$. The control limits using the true standard deviation $\sigma_X=1,1145$ are $\pm 14,4855$. The process shift is detected at observation *98*

At *figure 5.13* the *CUSUM* chart of the residuals is plotted. The parameters for designing a chart of independent observations is used. The control limits using $r=0,5$, $c=4,775$ and $\sigma_X=1,1145$ are $\pm 4,20773$. This chart performs well. The shift is detected at observation *88*.

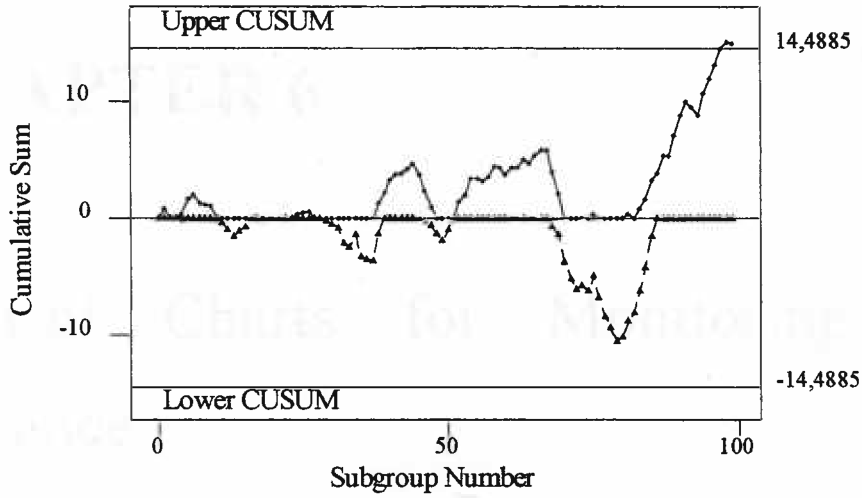

Figure 5.12: **The *CUSUM* of the observations of an *AR(1)* plus a random error process with *r=0.5* and *c=13* .**

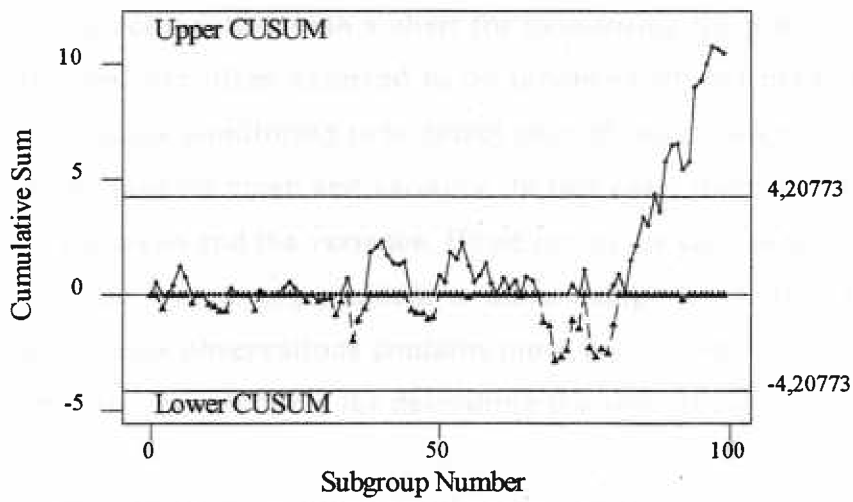

Figure 5.13: **The *CUSUM* of the residuals of an *AR(1)* plus a random error process with *r=0.5* and *c=4.775.***

CHAPTER 6

Control Charts for Monitoring the Variance

6.1 Introduction

In traditional chart methodology, a chart for monitoring the mean is usually used in conjunction with a chart for monitoring the process variance. The observations are often assumed to be independent and normal, and the objective of process monitoring is to detect special causes which can produce a change in the process mean and variance. In this case, there are two process parameters: the mean and the variance. However, in the case of autocorrelated observations, the monitoring problem is more complicated since the model used for the process observations contains more parameters.

There are three models for describing the type of correlation existing in the data:

a) a model for correlation within samples *(model I)*,

b) a model for correlation between samples *(model II)*

c) a model for correlation both within and between samples *(model III)*.

In section *6.2* correlation within samples is discussed and R, S^2 charts are designed for data from an *AR(1)* process. The following sections deal with correlation between samples and control charts for data modelled by the *AR(1)* plus a random error process.

6.2 The *R* and S^2 Charts for the *AR(1)* process

Amin et al. (1997) considered the effects on control charts of autocorrelation within samples, because as they stated, the effect of correlation within samples appears to be more important in charts when the objective is to monitor the variability of the process. They constructed R and S^2 Shewhart type charts based on estimating the variance correctly, but using constants for independent data.

An *AR(1)* time series model has been considered to describe the correlation structure within samples *(Model I)*

$$X_{i,j}= \mu(1-\varphi)+ \varphi X_{i,j-1}+\alpha_{i,j}$$

where $X_{i,j}$ denotes the jth observation in the ith sample, $|\varphi|<1$ and the errors α's are $i.i.d.\ N(0,\sigma^2)$.

The autocovariance function of a stationary *AR(1)* process at lag h is given by

$$\gamma_h = Cov(X_t, X_{t+h}) = \frac{\sigma_0^2}{1-\phi_1^2}\phi_1^{|h|}$$

for $h \in Z$.

The variance σ^2_X of the *AR(1)* process is the autocovariance γ_0 at lag 0, thus

$$\sigma_X^2 = \gamma_0 = \frac{\sigma_0^2}{1-\phi_1^2}$$

The S^2-chart is based on the sample variances

$$S_i^2 = \frac{\sum_{j=1}^{n}(X_{i,j}-\overline{X_i})^2}{n-1}$$

where $\overline{X_i}$ is the sample mean within sample i.

In the case of the one-sided chart, assuming independence but using the correct variance, the S^2–chart signals if:

$$S^2_i>\sigma^2_X\chi^2_{a,n-1}/n-1$$

where $\chi^2_{a/2,n-1}$ denote the upper $\alpha/2$ percentage points of the chi-square distribution with $n-1$ degrees of freedom and σ^2_X denotes the true variance of the process.

Similarly, in the one-sided R-chart for the $AR(1)$ process the average range is a function of the autocovariance function at lag 0, γ_0, and the chart signals if

$$R_i > \sigma_X C_{R,a}$$

where the constant $C_{R,a}$ is the upper a-quantile of the relative range.

Amin et al. (1997) showed that the in-control ARL of the S^2-chart with an upper control limit, based on estimating the variance correctly but using constants for independent data, is quite robust for certain values of n and relative small levels of autocorrelation. When it is not possible to use those samples sizes and when autocorrelation is strong, the use of modified control limits, such as those in *Vasilopoulos and Stamboulis(1978)* is necessary.

Amin et al. (1997) presented modified control limits for both S^2 and R charts for different sample sizes n, different values of the autoregressive parameter φ and different in-control ARL values. These limits are presented correspondingly at TABLE A.4, TABLE A.5 and can be used to design effective control charts, which correspond to the specific level of autocorrelation of the process.

6.3 The MacGregor and Harris's Approach

MacGregor and Harris (1993) considered two control charts for monitoring the process variance: one based on an exponentially weighted mean squared deviation from the target, the *EWMS*, and another based on an exponentially weighted moving variance in which the current process mean is estimated using an *EWMA* chart of the observations, the *EWMV*.

To monitor the variance of autocorrelated processes, they considered the situation in which only one observation X_t is taken from the process at sampling point t and the correlative nature of the process is described by the *AR(1)* plus a random error model and its equivalent *ARMA(1,1)* form which were presented here in paragraph *5.4.1* of the previous chapter.

The ratio of the variance σ^2_ε of the random errors to the total variance of the process σ^2_X will be needed later and is expressed by

$$\sigma_\varepsilon^2 \Big/ \sigma_X^2 = 1-\rho_1/\phi = 1-\frac{(1-\phi\theta)(\phi-\theta)}{\phi(1-\theta^2-2\phi\theta)}$$

where ρ_1 is the first lag autocorrelation of the X_t process and φ, θ the autoregressive and the moving average parameters correspondingly of the *ARMA(1,1)* model.

It should be noted that as the φ parameter approaches unity, the time varying mean μ_t becomes non stationary, and behaves as a random walk, and the observed X_t behaves as an integrated moving average process *ARIMA(0,1,1)*.

6.3.1 The Exponentially Weighted Mean Square chart

The exponentially weighted mean square chart, or briefly the ***EWMS*** chart, plots the statistic

$$S_t^2 = \sum_{k=1}^{t} r(1-r)^{t-k}[X_t-\mu]^2 + (1-r)^t S_0^2 = (1-r)S_{t-1}^2 + r[X_t-\mu]^2$$

where X_t is an individual observation of the process taken at sample time t, S_0^2 is an initial estimate of the mean squared error (usually taken to be the historical in-control value), r is a weight $(0<r\leq 1)$ that controls the rate of exponential discounting of past data. The sum of weights is given by

$$r\sum_{k=1}^{t} r(1-r)^{t-k} + (1-r)^t = 1$$

A modification of the *EWMS* control chart is the exponentially weighted root mean square chart, or briefly the ***EWRMS*** chart, which is actually the square root of the *EWMS* and plots the statistic S_t.

When the process mean is on target and the in-control variance is σ_0^2 (where σ_0^2 is obtained from historical data) the control limits for the *EWRMS* are

$$(\sigma_0 - \chi^2_{1-a/2}(v)\,\sigma_0\,,\ \sigma_0 + \chi^2_{a/2}(v)\sigma_0)$$

where $\chi^2_{1-a/2}(v)$ and $\chi^2_{a/2}(v)$ are the *(1-a/2)100%* and *(a/2)100%* percentiles correspondingly, of the chi-square distribution with v degrees of freedom. The degrees of freedom v for an autocorrelated process which can be described by the above model, are given at *Table 6.1* for various values of the parameters φ and $\sigma^2_\varepsilon/\sigma^2_X$ when the weight $r=0,05$.

v:degrees of freedom					
	φ				
$\sigma^2_\varepsilon/\sigma^2_X$	0.10	0.25	0.50	0.75	0.90
1.00	*39.0*	*39.0*	*39.0*	*39.0*	*39.0*
0.90	*39.0*	*39.0*	*38.8*	*38.1*	*36.6*
0.50	*38.8*	*37.8*	*33.7*	*24.8*	*14.6*
0.10	*38.4*	*35.3*	*25.9*	*13.6*	*6.10*

Table 6.1: Degrees of freedom for the *EWRMS* chart for various values of the autoregressive parameter φ and various shifts of the process variance.

6.3.2 The Exponentially Weighted Moving Variance chart

The exponentially weighted moving variance chart, briefly the ***EWMV*** chart, is obtained by replacing the overall mean of the process μ with the time varying μ_t and thus plots the statistic

$$S_t^2 = (1-r)S_{t-1}^2 + r[X_t - \mu_t]^2$$

The control limits for the *EWMV* control chart when the process follows the model described by *MacGregor and Harris (1993)* are given by

$$(\sigma_0 - C_7\,\sigma_0\,,\ \sigma_0 + C_8\,\sigma_0)$$

where the constants C_7 and C_8 are given at *Table 6.2* for the case when $r=0,05$, $\varphi=0,9$ and $\sigma^2_\varepsilon/\sigma^2_X=0,5$.

a	**0.05**	**C_7**	*0.68*
		C_8	*1.08*
	0.01	**C_7**	*0.63*
		C_8	*1.14*

Table 6.2: Constants for the *EWMV* chart for *r=0,05*, φ=0,9 and $\sigma^2_\varepsilon/\sigma^2_X$=0,5.

6.4 The Lu and Reynolds's method

Lu and Reynolds (1999b) investigated the problem of monitoring the process variance when there is correlation between samples, and the sample size is $n=1$. They also considered the problem of simultaneously monitoring the process mean and variance. Similarly to *MacGregor and Harris (1993)*, they used the *AR(1)* plus a random error and the equivalent *ARMA(1,1)* models to describe the correlation structure of the process, for the case of positive autocorrelation.

6.4.1 The variance of the residuals

Lu and Reynolds (1999b) used the *AR(1)* with an additional random process to model the correlation structure of the process. This model is described in *5.4.1*.

The variance of the time wandering mean using the equations in *Box and Luceno (1997)* is $\sigma^2_\mu = \sigma^2_\gamma/(1-\varphi^2)$ and the variance of the process is

$$\sigma_X^2 = \sigma_\mu^2 + \sigma_\varepsilon^2 = \frac{\sigma_\gamma^2}{1-\phi^2} + \sigma_\varepsilon^2$$

A change in the process variance σ^2_X could be caused by an increase in σ^2_μ and/or σ^2_ε. When the autoregressive parameter φ in the *AR(1)* plus a random error model is considered fixed, the increase in σ^2_μ is caused by an increase in σ^2_γ. Equivalently, when both the autoregressive and the moving average parameter in the *ARMA(1,1)* model are considered constant a change in σ^2_a would change both σ^2_ε and σ^2_γ through the parameter relationships given in section *5.4.1*.

Lu and Reynolds(1999b) supposed that between samples *T-1* and *T*, σ^2_γ increases from the in-control value $\sigma^2_{\gamma 0}$ to $\sigma^2_{\gamma 1}$ and σ^2_ε increases from the in-control value $\sigma^2_{\varepsilon 0}$ to $\sigma^2_{\varepsilon 1}$.

The residuals after the shift are correlated normal random variables with mean *zero* and variance at $t=T$

$$Var(e_T) = \sigma^2_{\alpha 0} + (\sigma^2_{\varepsilon 1} - \sigma^2_{\varepsilon 0}) + (\sigma^2_{\gamma 1} - \sigma^2_{\gamma 0})$$

and at $t=T+l$

$$Var(e_{T+l}) = \sigma_{a0}^2 + \left[1+(\phi-\theta_0)^2 \sum_{i=0}^{l} \theta_0^{2(l-i-1)}\right]$$

$$\times\left(\sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 0}^2\right) + \sum_{i=0}^{l} \theta_0^{2(l-i)}\left(\sigma_{\gamma 1}^2 - \sigma_{\gamma 0}^2\right), l = 1,2,3,\ldots$$

where θ_0 is the value of the θ parameter when the process is in-control.

After the shift, the smallest variance, at T, is

$$\sigma^2_{a0} + (\sigma^2_{\varepsilon 1} - \sigma^2_{\varepsilon 0}) + (\sigma^2_{\gamma 1} - \sigma^2_{\gamma 0})$$

but then the variances continually increase to the limit

$$\sigma_{a0}^2 + \frac{\phi^2 - 2\phi\theta_0 + 1}{1-\theta_0^2}\left(\sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 0}^2\right) + \frac{\sigma_{\gamma 1}^2 - \sigma_{\gamma 0}^2}{1-\theta_0^2}$$

It is interesting to note that the effect of an increase in σ^2_γ and/or σ^2_ε is to increase the variances of the residuals, while the means of the residuals remain constant at *zero*. On the other hand, a shift in the overall mean of the process changes the means but not the variances of the residuals with the largest change occurring at $t=T$ immediately after the shift.

6.4.2 The *EWMA* of the Logs of the Squared Residuals chart

An *EWMA* chart of the residuals for monitoring the process variance uses a control statistic based on the logs of the squared residuals

$$Z_t = max((1-\lambda)Z_{t-1} + \lambda\, ln(e^2_t),\ ln(\sigma^2_{a0}))$$

where the starting value $Z_0 = ln(\sigma^2_{a0})$ and λ is a smoothing parameter satisfying $0<\lambda\leq 1$. This *EWMA* statistic is one-sided, and resets to the target $ln(\sigma^2_{a0})$ at the next sample whenever the statistic drops below this target. As *Reynolds and Lu (1999b)* stated, a two-sided version of this control chart could be used, but in most applications, the primary interest is in detecting an increase in the process variance. The sequence of the residuals e_t is normally distributed with mean *zero*, thus e^2_t follows *gamma* distribution with shape parameter equal to ½ and a scale parameter equal to $1/(2Var(e_t))$. If $W = ln(e^2_t)$ then W has a log-gamma distribution with density of

$$f_W(w) = \frac{1}{\Gamma\left(\frac{1}{2}\right)} exp\left[\frac{1}{2}(w - ln(2Var(e_t))) - exp\{w - ln(2Var(e_t))\}\right],$$

$-\infty < w < \infty$.

It is obvious that the scale parameter in the gamma distribution becomes the location parameter in the log-gamma distribution. Because the *EWMA* is primarily designed for detecting location shifts in a process, it is reasonable to use the logs of squared residuals in the *EWMA* chart to monitor shifts in the process variance. When the process is in-control $Var(e_k)=\sigma^2_{a0}$ and using the mean and the variance of the log-gamma distribution the in-control mean and variance of W are $E(W) \approx ln(\sigma^2_{a0})-6/5$ and $Var(W) \approx 64/15$ respectively. The variance of the one-sided *EWMA* statistic based on the logs of the squared residuals, does not equal the variance of the two-sided *EWMA*, but the control limits are expressed in this form for convenience. Thus, the control limit of the *EWMA* chart of the logs of the squared residuals is

$$ln(\sigma^2_{a0}) + c\sqrt{\frac{64\lambda}{15(2-\lambda)}}$$

where c is a smoothing constant. The integral equation approach can be used to obtain the in-control ARL_0 and simulation to obtain the ARL when σ^2_γ or σ^2_ε shifts.

6.4.3 Simultaneously monitoring the Mean and the Variance of correlated processes

When the objective is to simultaneously monitor the mean and the variance of a process *Lu and Reynolds (1999b)* considered six monitoring schemes.

a. A Shewhart chart of residuals and an *EWMA* chart for residuals.

b. A Shewhart chart of residuals and an *EWMA* chart of observations

c. A Shewhart chart of residuals alone

d. An *EWMA* chart of logs of squared residuals and an *EWMA* chart of observations.

e. An *EWMA* chart of logs of squared residuals and an *EWMA* chart of residuals.

f. A Shewhart chart of forecasts and a Shewhart chart of residuals.

To decide which control scheme performs better for a specific type and magnitude of process change *Lu and Reynolds (1999b)* derived the *ARL* of the above schemes.

The process shifts considered were the combinations of $(\xi-\xi_0)/\sigma_{X0}=0,1,2,3$ and of $\sigma^2_X/\sigma^2_{X0}=1,2,3,10$, where σ^2_{X0} is the in-control variance of the observations. The values of $\psi_0=\sigma^2_{\mu 0}/\sigma^2_{X0}$ were $0,1$ and $0,9$ and those of the autoregressive parameter φ $0,4$ and $0,8$. All charts or combinations of charts were adjusted to give in- control *ARL* of approximately *370,4*. The above *ARL* results are presented in TABLE A.6 and TABLE A.7 where the increase in σ^2_X is caused by σ^2_γ and σ^2_ε correspondingly.

Obviously, schemes *a* and *b* perform well across most situations while scheme *c* does not detect efficiently shifts in the mean alone, and scheme *d* performs well except for large shifts in the mean alone. Schemes *d*, *e* and *f* have similar performance.

Lu and Reynolds(1999b) recommend the use of scheme *b* in practice, since it runs simultaneously a Shewhart chart of the residuals and an *EWMA* chart of the observations, is relatively easy to interpret and is reasonably effective in detecting shifts in the mean and/or the variance for most cases, especially for the case of low to moderate autocorrelation. They recommend though that λ should be appropriately chosen, to provide fast detection when there is a change in the process.

6.5 An application

A data set of *100* observations was generated from an *AR(1)* time series model plus a random error by simulating the sequence of the random errors γ_t*'s* and ε_t*'s*. The process parameters used for this simulation were $\varphi=0,9$, $\sigma_\varepsilon=0,5$ and $\sigma_\gamma=0,5$.

Using the *MINITAB* statistical software an *AR(1)* time series model is fitted to the observations resulting to $\varphi=0,825$ and $\sigma^2_\gamma=0,26$. According to the checking plots of the residuals at Figure B.3 the *AR(1)* model describes the data satisfactorily as expected. Using the equations presented at *5.4.1*, ψ is calculated to be *0,8*. This model is equivalent to an *ARMA(1,1)* model with $\varphi=0,825$, $\theta=0,347$ and $\sigma^2_a=0,6$. The true variance of the data is $\sigma^2_X=1$.

Two types of changes in the process variance are investigated on the same basic sequence of γ_t's and ε_t's.

For the first process change to be investigated, consider an increase in the process variance caused by an increase in σ^2_γ. Suppose that due to special cause immediately after observation *60*, σ^2_γ increases from *0,26* to *0,9* resulting to an increase of σ^2_X from *1* to *3*. This is accomplished by multiplying the last *40* observations by $\sqrt{0,9/0,26}=1,85$.

The performance of six different control charts is examined. These charts are shown at the first column of *Figure 6.1*. The first four charts are charts that are designed to monitor changes in the process mean. The last two are the *EWMA* chart of the Logs of Squared Residuals and the *EWRMS*, which are especially designed for monitoring the variance of correlated processes.

First a *X*-chart of the observations is applied to the data. The true variance of the process is used resulting to *3* sigma control limits of ± 3. This chart captures a change at observation *97*.

Then an *EWMA* of the observations with modified control limits is applied. Using $\psi=0,8$ and $\varphi=0,825$ at FIGURE 1 of *Lu and Reynolds (1999a)* results to $c=5,2$. The *5,2* sigma control limits using the true standard deviation $\sigma_X=1$ are $\pm 1,73$. This chart signals that the process is out-of-control only at observation *100*.

A residuals *X*-chart is also applied to the data. The *3*-sigma limits are $\pm 2,335$ and the first signal comes at observation *70*.

The next chart applied is an *EWMA* of the residuals with *2,859* control limits at $\pm$ *0,7414*. This chart does not perform as well as the previous one since it only detects a change at observation *99*.

The fifth chart that is applied to the data is the *EWMA* of the Logs of the Squared Residuals with $c=0,944$. The lower control limit is $LCL=0,51$ and the upper control limit $UCL=0,14$. This chart shows increased variability after the increase in σ^2_γ and signals at observation *91*. It also indicates a false alarm at observation *6*.

The last chart is the *EWRMS* chart. Using $r=0,05$ and $\sigma_X=1$ the constants C_5 and C_6 are *0,55* and *1,45* correspondingly at significance level $a=0,001$. At this chart the increase in the variability is obvious after observation 60 but a signal is available only at observation *99*.

The next process change to be investigated is a change in σ^2_ε. After observation *60*, σ^2_ε increases from *0,25* to *1,28*, while σ^2_γ remains constant, resulting to an increase of σ^2_X from *1* to *2*. This is accomplished by multiplying the last *40* observations by $\sqrt{1{,}28/0{,}25} = 2{,}283$.

The observations corresponding to the increase in σ^2_ε were applied to the six control charts under investigation. These charts are shown at the second column of *Figure 6.3*.

Neither the *X*-chart nor the *EWMA* chart of the observations signal a state of lack of statistical control. The residuals *X*-chart signals immediately after the change occurs, at observation *65*, while the *EWMA* chart of the residuals signals at observation *80*. The *EWMA* of the Logs of the Squared Residuals performs much better for this type of change in the process variability since it signals sooner at observation *65*. The *EWRMS* chart also seems to perform better for this type of change since it shows increased variability immediately after the increase in σ^2_ε. It signals though again only at value *99*.

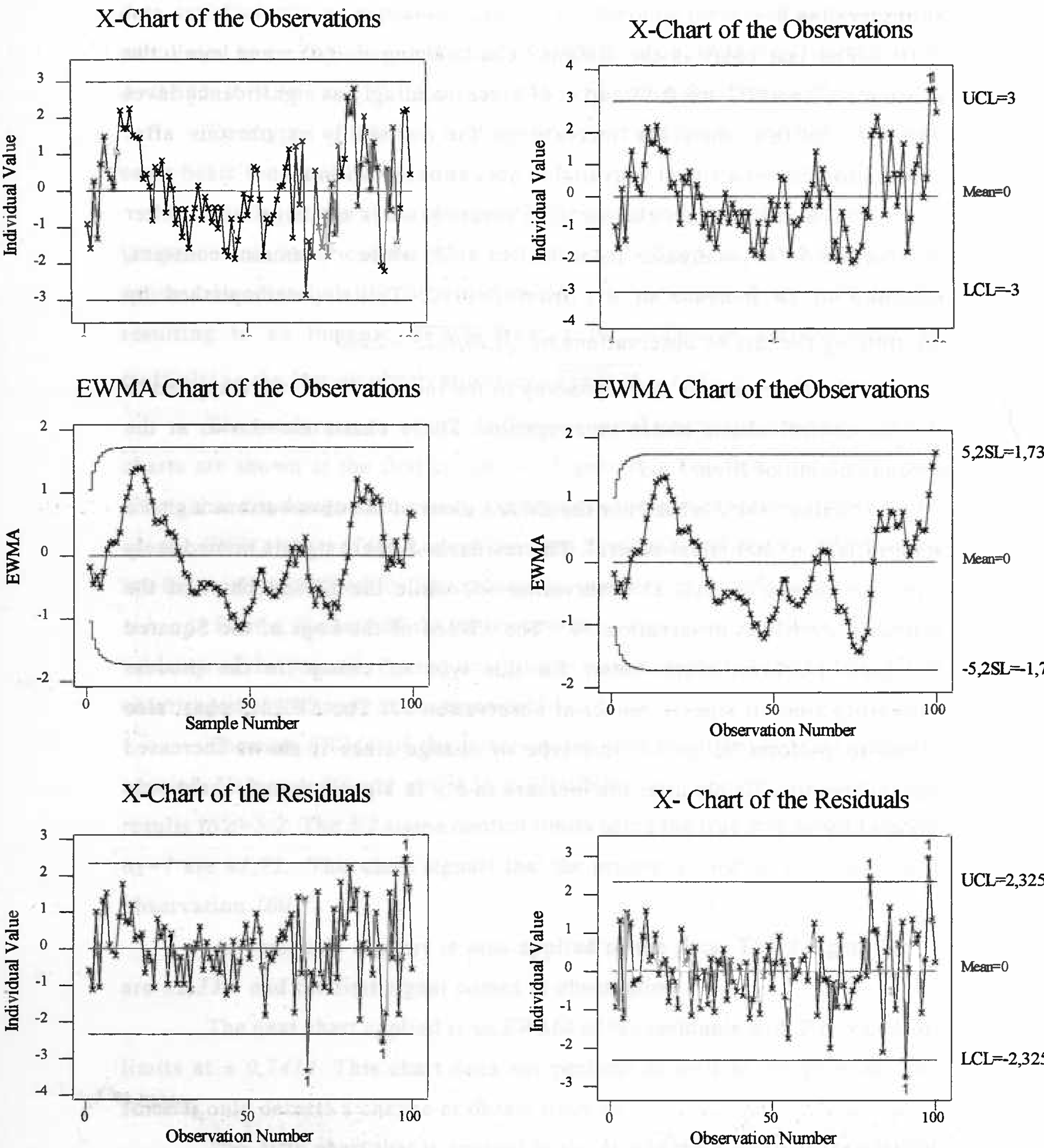

Figure 6.1: **Control Charts for monitoring the Variance of an *ARMA(1,1)* process. The first column corresponds to changes in $\sigma^2\gamma$ and the second in σ^2_ε.**

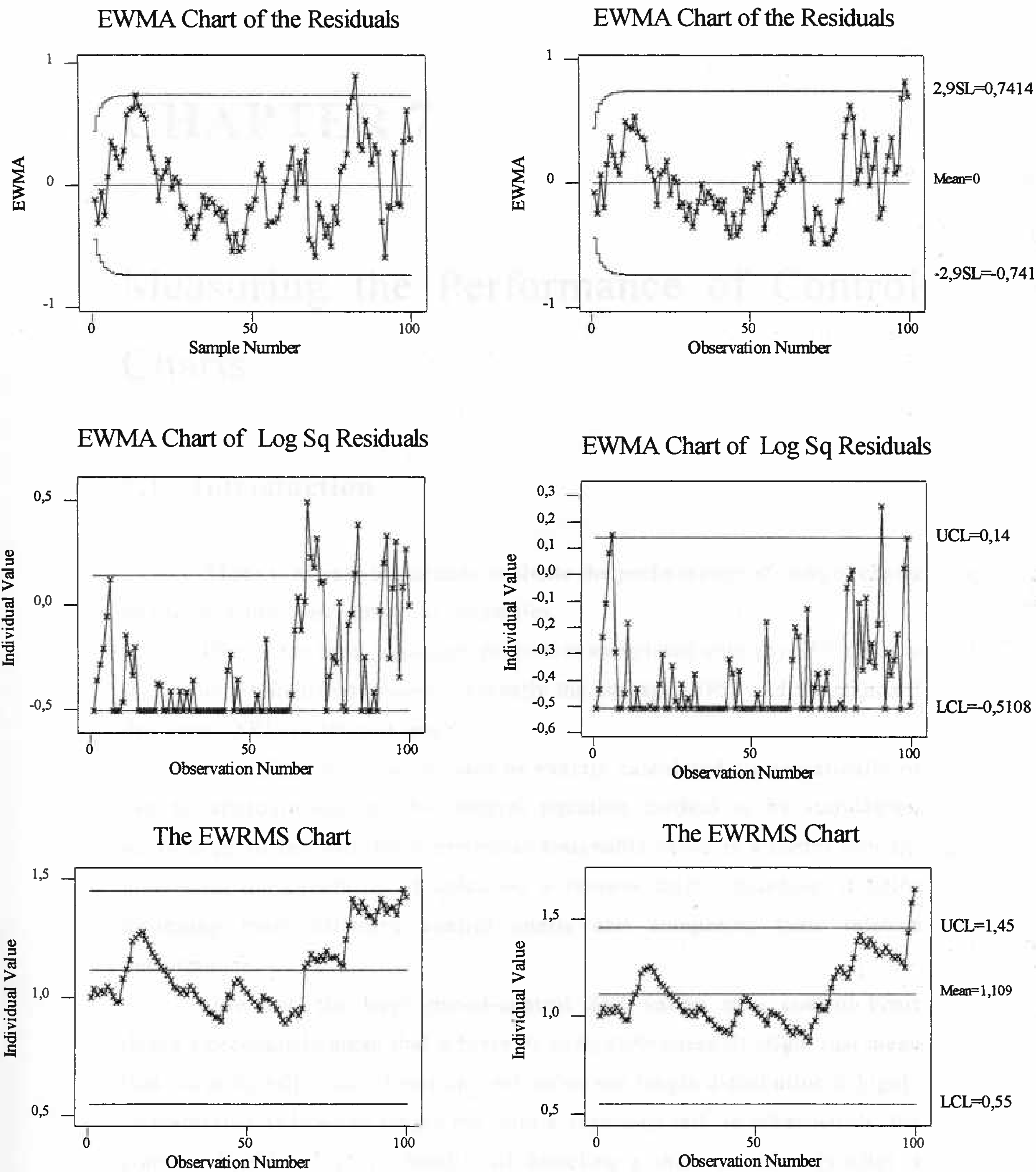

Figure 6.1: **Control Charts for monitoring the Variance of an *ARMA(1,1)* *(continued)* process. The first column corresponds to changes in $\sigma^2\gamma$ and the second in σ^2_ε. second in σ^2_ε .**

X-Chart of the Observations
EWMA Chart of the Observations
Observation Number

CHAPTER 7

Measuring the Performance of Control Charts

7.1 Introduction

There is always the need to evaluate the performance of control charts and to measure their statistical properties.

One of the most important properties associated with any *SPC* chart is its run length distribution and eventually the average *(ARL)* and the standard deviation *(SRL)* of the run length.

The *ARL* can in some cases be exactly calculated mathematically or can be approximated by the integral equation method or by simulation. Knowledge of the *ARL* for a particular assignable cause is a useful tool for measuring the capability of detecting a process shift. Therefore, it helps designing more effective control charts and comparing their relative performance.

However, the large out-of-control *ARL* values of a control chart doesn't necessarily mean that it loses its competitiveness. It might just mean that the probability mass function *(pmf)* of its run length distribution is highly concentrated at low run length but with a very long tail. In other words, the control chart has high probability of detecting a shift immediately after it occurs which decays afterwards.

Early detection makes the cause of the signal easier to identify and thus results in the fastest rate of continuous quality improvement. Therefore, besides the actual *ARL* values of a control chart, an attentive look at its run-length distribution as well as the development of any other measures of its detection capability and of the way it reacts to the shifts, are very helpful to actually understand its potential.

In section *7.2* some measures of performance of Shewhart type charts are presented while in section *7.3* a method for approximating the *ARL* of the *EWMA* and *CUSUM* charts and for deriving the run length distribution of one sided *CUSUM* charts is given. A useful tool to compare the effectiveness of various control charts designed to monitor autocorrelated processes the Dynamic Step Response Function is presented in section *7.4*.

7.2 Measures of Performance for Shewhart control charts

For the special case of the residuals *X*-chart the run length distribution is determined mathematically as shown in *7.2.1* and some detection capability indices are developed in section *7.2.2*.

7.2.1 The Run Length distribution of the *X*-chart for the residuals

Wardell et al.(1994) determined mathematically the run length distribution of the Shewhart *X*-chart of residuals proposed by *Alwan and Roberts(1998)* for a particular assignable cause attributable to a single shift in the mean of a process $\{X_t, t=0,1,2,\dots,\}$ which can be represented by the mixed autoregressive moving average processes of order *p* and *q* $(ARMA(p,q))$ time series model.

The $ARMA(p,q)$ model is given by

$$X_t = \mu(1-\varphi_1-\dots-\varphi_p)+\varphi_1 X_{t-1}+\dots+\varphi_p X_{t-p}+\alpha_t-\theta_1\alpha_{t-1}-\dots-\theta_q\alpha_{t-q}\ ,$$

where μ is the mean of the process, $\{a_{t-i}\}$ is the random error at time t-i which is assumed to be *i.i.d.* with mean *zero* and variance σ^2_α while φ_i and θ_i are the autoregressive and the moving average coefficients.

The one-step-ahead forecast equation for the *ARMA(p,q)* model that minimizes the mean squared error, assuming with out loss of generality, that the mean of the process is *zero* is according to *Box and Luceno(1997)*

$$F_t=\varphi_1 x_{t-1}+\ldots+\varphi_p x_{t-p}-\theta_1 e_{t-1}-\ldots-\theta_q e_{t-q}$$

where F_{t-1}=*one-step-ahead forecast* of the process at time t-i and e_{t-i}= *the one-step-ahead forecast errors* or residuals of the process at time t-i that is,

$$e_{t-i}=x_{t-i}-F_{t-i}.$$

Combining the definition of the *ARMA(p,q)* process and the above equations, gives the general recursive expression for the residuals e_t at time t:

$$e_t=\mu_t-\varphi_1\mu_{t-1}-\ldots-\varphi_p\mu_{t-p}+\alpha_t-\theta_1\alpha_{t-1}-\ldots-\theta_q\alpha_{t-q}+\theta_1 e_{t-1}-\ldots-\theta_q e_{t-q}$$

The recursive nature of the above equation can be removed by expressing e_t in terms of only past values of μ and α_t using the z transform *(Drake 1967)*, thus

$$e_t=\sum_{k=0}^{\infty}c_k\mu_{t-k}+\alpha_t$$

where c_k is a constant defined by

$$c_k=\begin{cases}0 & k<0\\ 1 & k=0\\ \theta_1 c_{t-1}+\ldots+\theta_q c_{t-q}-\varphi_k & k>0\end{cases}$$

where φ_k is 0, if $k>p$.

As mentioned before the run length is the number of observations required to obtain an observation outside of the control limits for a given shift in the mean. Since the residuals calculated as previously depend only on the most recent error term, the probability that the most recent observation will exceed the control limits can be independently determined, thus:

$$Pr(|R_t|>3\sigma_\alpha)\equiv Pr_t=1-F\left(3\sigma_\alpha-\sum_{k=0}^{\infty}c_k\mu_{t-k}\right)+F\left(-3\sigma_\alpha-\sum_{k=0}^{\infty}c_k\mu_{t-k}\right)$$

where F is the cumulative distribution function *(cdf)* of the random error α_t, which is usually assumed to be the cumulative normal distribution. When the shift in the process mean is a single step change of size δ (where δ is measured in terms of the standard deviation of the process) it can be expressed as

$$\mu_i = \begin{cases} 0, & i \leq 0 \\ \delta, & i > 0 \end{cases}$$

and Pr_t can be expressed as

$$Pr(|R_t| > 3\sigma_\alpha) \equiv Pr_t = 1 - F\left(3\sigma_\alpha - \delta \sum_{k=0}^{\infty} c_k\right) + F\left(-3\sigma_\alpha - \delta \sum_{k=0}^{\infty} c_k\right)$$

Defining now Y as a discrete random variable representing the run length of the residuals chart, the distribution of Y is defined by the above probabilities, and its probability mass function *(pmf)* is given by

$$Pr\{Y = y\} = Pr_y \prod_{i=0}^{y-1} (1 - Pr_i), \quad z = 1,2,...,$$

where $Pr_0 = 0$. If there is no shift in the mean, Pr_t is constant in t and its *pmf* follows the geometric distribution, which is the run length distribution of the standard Shewhart control chart.

The run length distribution for the special case of an *ARMA(1,1)* process is given by

$$Pr_t = 1 - F\left\{3\sigma_a - \delta\left[1 + \frac{\theta - \varphi}{1 - \theta}\left(1 - \theta^{t-1}\right)\right]\right\} + F\left\{-3\sigma_a - \delta\left[1 + \frac{\theta - \varphi}{1 - \theta}\left(1 - \theta^{t-1}\right)\right]\right\}$$

where φ and θ are the parameters of the model, F is the cumulative distribution function *(cdf)* of the random error α_t and σ_α is its standard deviation.

After having derived the *pmf* of the run-length of the residual chart for a process which follows an *ARMA(p,q)* time series model, any of its moments can be determined, including the first and second moments, which can be used to find the *ARL* and *SRL*.

7.2.2 Detection Capability Indices and the *ARL* of the *X*-chart for the residuals

Zhang(1997) developed a measure for the detection capability of the residuals chart for the general stationary process A*R(p)*. He also established the relationship between the detection capability and the *ARL* of the residuals *X*-chart and provided the conditions under which the latter performs better than an individuals chart of the observations.

Suppose that the process $\{X_t, t=1,2,...,\}$ follows an *AR(p)* time series model

$$X_t = \mu(1-\varphi_1-...-\varphi_p)+\varphi_1 X_{t-1}+...+\varphi_p X_{t-p}+\alpha_t,$$

where μ is the mean of the process, $\{\alpha_t\}$ is the random error (noise) at time t which is assumed to be *i.i.d.* with mean *zero* and variance σ^2_a and φ_i are the autoregressive coefficients.

Suppose now that a mean shift of size $\delta\sigma_X$, where σ_X is the standard deviation of the process, occurred at time $t=T$, namely

$$E[X_t]=\mu,\ when\ t<T$$
$$E[X_t]=\mu+\delta\sigma_X,\ when\ t\geq T$$

the residuals at time t can be given by

$$e_t = X_t - \sum_{i=1}^{p}\phi_i X_{t-i} - \left(1-\sum_{i=1}^{p}\phi_i\right)\mu$$

$$E[e_T]=\delta\sigma_X$$

Zhang(1997) showed that at $t=T+j(1\leq j<p)$:

$$E[e_t] = \left(1-\sum_{i=1}^{j}\phi_i\right)\delta\sigma_X$$

and that at $t=T+j(j\geq p)$:

$$E[e_t] = \left(1-\sum_{i=1}^{p}\phi_i\right)\delta\sigma_X$$

The variance of the residual is the same as the variance of the process white noise α_t thus

$$Var[e_t] = \sigma_a^2 = \left(1-\sum_{i=1}^{p}\phi_i\rho_i\right)\sigma_X^2$$

where ρ_i $(i=1,\ldots,p)$ is the autocorrelation at lag i. The detection capability indices are defined for $t=T$ as

$$f(T)=\frac{1}{\left(1-\sum_{i=1}^{p}\phi_i\rho_i\right)^{1/2}}$$

for $t=T+j(1\leq j<p)$ as

$$f(t)=\frac{\left|1-\sum_{i=1}^{j}\phi_i\right|}{\left(1-\sum_{i=1}^{p}\phi_i\rho_i\right)^{1/2}}$$

and for $t=T+j(j\geq p)$ as

$$f(t)=\frac{\left(1-\sum_{i=1}^{p}\phi_i\right)}{\left(1-\sum_{i=1}^{p}\phi_i\rho_i\right)^{1/2}}$$

When $f(t)>1$, the detection capability of the residuals chart at time t is greater than that of the traditional X-chart in other words the residual chart increases the detection capability of a mean shift of size $\delta\sigma_X$ at t. Conversely, when $f(t)<1$ the residuals chart reduces the detection capability at t. When the process is an independent sequence which is the case of $\varphi_i=0$, $i=1,\ldots,p$, then $f(t)=1$.

For a given process when $t>T$, the comparison can be made between the denominator and the numerator in the above equations to determine whether $f(t)>1$ or $f(t)<1$.

Zhang(1997) related the *ARL* of the residuals X-chart with $k\sigma_X$ limits to detect shift of size $\delta\sigma_X$ of an $AR(p)$ process with the above detection capability indices with

$$ARL = 1+\sum_{j=1}^{p-1}\left\{j\left[1-\beta(T+j)\right]\prod_{i=1}^{j}\beta(T+i-1)\right\}+$$
$$\left[\prod_{i=1}^{p}\beta(T+i-1)\right]\left[p-(p-1)\beta(T+p)\right]\Big/\left[1-\beta(T+p)\right]$$

with $\beta(t)=P(-k-\delta f(t)<Z<k-\delta f(t)$, $t\geq T$ and Z is a standard normal random variable.

When $\{X_t\}$ follows an $AR(1)$ stationary process the detection capability index of the residuals chart at the time T when the shift occurs is given by

$$f(T)=1/(1-\varphi^2)^{1/2}$$

and for $t>T$

$$f(t)=(1-\varphi)/(1-\varphi^2)^{1/2}$$

The ARL of the residuals X-chart for $t\geq T$ is

$$ARL=1+\beta(T)/[1-\beta(T+1)]$$

When $\{X_t\}$ follows a stationary $AR(2)$ process, the detection capability of an individuals chart applied to the residuals of the process for T and $T+1$ is

$$f(T)=1/(1-\varphi_1\rho_1-\varphi_2\rho_2)^{1/2}$$

$$f(T+1)=|1-\varphi_1|/(1-\varphi_1\rho_1-\varphi_2\rho_2)^{1/2}$$

and for $t\geq T+2$

$$f(t)=(1-\varphi_1-\varphi_2)/\ (1-\varphi_1\rho_1-\varphi_2\rho_2)^{1/2}$$

The ARL of the residuals X-chart for $t\geq T$ is

$$ARL=1+\beta(T)\{\ 1+\beta(T+1)\}/[1-\beta(T+2)]$$

Zhang(1997) proved that for a stationary $AR(p)$ process, $f(T)>1$, which means that the detection capability of the residuals chart at the time of the occurrence of the mean shift is larger than that of the X-chart of the observations.

He also proved that for a stationary $AR(1)$ process

(a) $f(T)>1$

(b) For $t>T$, $f(t)>1$ *if and only if* $\varphi<0$

The condition in *(a)* stands for every stationary autoregressive process, the condition in *(b)* states that the residuals chart detects the shift in the mean after the time it first occurred, faster than the X-chart of observations only for negatively autocorrelated *AR(1)* processes. Besides, when an *AR(1)* process is near non-stationarity the detection capability of the residuals chart becomes very large.

For a stationary *AR(2)* process *Zhang(1997)* proved that

a) $f(T)>1$

b) $f(T+1)<1$ *if and only if*

c) $\varphi_1<\{1-\varphi_2+[(1-\varphi_2^2)(1-2\varphi_2)]^{1/2}\}/2$ *and* $\varphi_1>\{1-\varphi_2-[(1-\varphi_2^2)(1-2\varphi_2)]^{1/2}\}/2$

$f(T+2)>1$ *if and only if*

d) $\varphi_1+\varphi_2-\varphi_2^2<0$

He also proved that assuming that the *AR(2)* process has normally distributed white noise when

a) $\varphi_1<0$ *and or* $\varphi_1+\varphi_2-\varphi_2^2<0$ or

b) $0<\varphi_1<1$ *and* $\varphi_1+\varphi_2-\varphi_2^2<0$ *and* $\varphi_1<\{1-\varphi_2-[(1-\varphi_2^2)(1-2\varphi_2)]^{1/2}\}/2$ or

c) $1<\varphi_1<2$ *and* $\varphi_1+\varphi_2-\varphi_2^2<0$ *and* $\varphi_1>\{1-\varphi_2+[(1-\varphi_2^2)(1-2\varphi_2)]^{1/2}\}/2$

the out-of-control *ARL* of the residuals chart is uniformly smaller than that of the X-chart.

7.3 Measures of performance for the *EWMA* and the *CUSUM* charts

To examine the properties of the *EWMA* and *CUSUM* charts when observations come from the *AR(1)* model with an additional random error described at the previous section, *VanBrackle and Reynolds(1997)* developed an integral equation to evaluate their *ARL* when the process is in control and when there has been a step shift in the overall mean of the process.

Assuming without loss of generality that the target value of the overall mean of the process ξ_0 is zero, *VanBrackle and Reynolds(1997)* derived an equation $N(z_0,\mu_0)$ for the *ARL* of the *EWMA* conditional on z_0, μ_0 where z_0 is the *EWMA* statistic at time $t=0$ and μ_0 is the value of the process mean

$$N(z_0,\mu_0)=1+\int_{-H}^{H}\int_{-\infty}^{\infty}N(z_1,\mu_1)f(z_1,\mu_1\mid z_0,\mu_0)d\mu_1 dz_1$$

where $f(z_1,\mu_1|\ z_0,\mu_0)$ is the joint density of (Z_1,μ_1) conditional (Z_0,μ_0). Under the assumption that the error terms in the $AR(1)$ model with an additional random error, ε_t and α_t ,are normal it follows that this conditional density is bivariate normal with parameters

$E(Z_1|z_0,\mu_0)=(1-\lambda)z_0+\lambda E(\mu_1+e_1|z_0,\mu_0)=(1-\lambda)z_0+\lambda((1-\varphi)\xi+\varphi\mu_0)$

$E(\mu_1|z_0,\mu_0)=(1-\varphi)\xi+\varphi\mu_0$

$Var(Z_1|z_0,\mu_0)=\lambda^2\ Var(\mu_1+e_1|z_0,\mu_0)=\lambda^2(\sigma^2_a+(\sigma^2_e/n))$

$Var(\mu_1|z_0,\mu_0)=\sigma^2_a$

and $Cov(Z_1|z_0,\mu_0)=Cov(1-\lambda)z_0+\lambda(\mu_1+e_1),\mu_1=\lambda\ \sigma^2_a$

The solution of this integral equation has been approximated using Gauss-Legendre quadrature. Since μ_0 is assumed to be a random variable the *ARL* conditional only on z_0 can be obtained by taking expectation with respect to the distribution of μ_0, thus

$$N(z_0)=\int_{-\infty}^{\infty}N(z_0,\mu_0)f(\mu_0)d\mu_0$$

where $f(\mu_0)$ is a normal density function with mean ξ and variance σ^2_μ. This integral can be approximated using Gause-Hermite quadrature.

The same approach was used to develop an integral equation for the one-sided *CUSUM* chart.

Yashchin (1993) approximated the run length distribution of an one sided upper CUSUM scheme applied to a given serially correlated stationary process provided that the variability explained by the presence of correlation is no more than 25% of the total variability of the process. He showed that for moderate correlated processes one can replace the sequence of observations by an *i.i.d.* sequence for which the run length distribution is approximately the same. The results of this replacement are exact for non correlated processes and approximations of good quality when the level of correlation is not too high especially for Gaussian processes.

7.4 Measures for the Relative Performance of Control Charts

When someone wants to explain why one chart is preferred to another he usually compares their *ARL* values and study their run length distribution if it is available. *Wardell et al.(1992)* developed another useful tool to determine the relative performance of different charts when the process is autocorrelated, the Dynamic Step Response Function *(DSRF)*. The *DSRF* describes how the chart would dynamically react to a shift in the process mean if there were no noise in the process.

They formed the *DSRF* for the *X*-chart and the *EWMA* of the observations with control limits calculated taking into consideration the true variance of the process, the residuals *X*-chart or Special Cause chart *(SCC)*, and the Common Cause chart *(CCC)* with additional control limits when the process follows an *ARMA(1,1)* time series model. Since the *EWMA* and *ARMA* forecasts depend on past data and on the values of the autoregressive and the moving average parameters φ, θ as well as on the smoothing parameter λ of the *EWMA* chart, they do not step up immediately to a new value when there is a shift in the process mean. Instead, they react dynamically to the shift and gradually converge to a new steady state value. Assuming that the process is completely deterministic and that the shift in the mean can be expressed by $\delta\sigma_X$ the *DSRF* for each of the above charts when j is the number of observations since the step change occurs, is

- for the Shewhart chart

$$DSRF_x=\delta/c_x$$

- for the *EWMA* chart

$$DSRF_{EWMA}(j)=\frac{\sigma}{c_{EWMA}}\left[1-(1-\lambda)^j\right]$$

- for the *CCC* chart

$$DSRF_{CCC}(j) = \frac{\delta}{c_{CCC}}(\phi - \theta)\left(\frac{1-\theta^j}{1-\theta}\right)$$

- for the residuals chart *SCC*

$$DSRF_{SCC}(j) = \frac{1}{c_{SCC}}\left[\delta - \delta(\phi - \theta)\left(\frac{1-\theta^j}{1-\theta}\right)\right]$$

In all cases, the *DSRF* has been normalized by the value of the upper control limit, so a normalized response of *1* or larger indicates that the mean of the statistic has exceeded its upper limit.

Knowing the dynamic response of each chart to a shift in the process mean, the difference in the *ARLs* of the control charts under investigation can be explained. *Wardell et al. (1992)* mention that this function is relatively easy determined for any *ARMA* model and for different types of shifts in the process mean. A spreadsheet can be used to chart the *DSRF* and to view the effect of changing parameters such as the smoothing constant in the *EWMA* model.

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7.4 Measures for the Relative Performance of Control Charts

[illegible] be usually compared [illegible] it is available [illegible] determine the [illegible] autocorrelated [illegible] describes how the chart [illegible] mean of these two [illegible] They [illegible] observations with [illegible] variance of the [illegible] and the Common [illegible] process follows a [illegible] forecasts depend on [illegible] moving average parameters [illegible] the *EWMA* chart, they do not [illegible] is a shift in the process mean [illegible] gradually converge [illegible] completely deterministic and that the shift [illegible] $\delta\sigma_X$ the *DSRF* for each of the above charts [illegible] observations since the step change [illegible]

- for the Shewhart chart [illegible]
- for the *EWMA* chart [illegible]

DSRF [illegible]

CHAPTER 8

Performance of Control Charts for Monitoring the Mean

8.1 Introduction

Many researchers have investigated the properties of the proposed charts for autocorrelated processes, determine their relative performance, and compare their performance to the performance of the traditional charts.

Although the use of traditional control charts to autocorrelated processes seems to be disastrous, some insist on preferring them to the residuals charts under some circumstances, stating that the residuals charts do not have the same properties as the traditional charts.

In section *8.2* a discussion about the performance of the residuals *X*-chart and the *X*-chart of the observations is stated. In section *8.3* the performance of *CUSUM* and *EWMA* schemes for autocorrelated processes charts is presented while in section *8.4* the relative performance of control charts dealing with autocorrelation.

8.2 The performance of the Shewhart *X*-chart

Following the methods discussed in section *7.3*, *Wardell et al.(1994)* derived the run length distribution of the residuals *X*-chart for a particular assignable cause attributable to a single shift in the process mean for the general *ARMA(p,q)* model. In addition, they calculated theoretically the *ARL* and *SRL* of the *ARMA(1,1)* process. The *ARL* and *SRL* results, verified by the authors via simulation, along with the first lag autocorrelations are shown at TABLE A.8 for various combinations of the autoregressive and moving average parameters (factors φ and θ) of the *ARMA(1,1)* model.

A general inference that can be drawn from the *ARL* and *SRL* results is that the in-control *SRL* and *ARL* are the same for all the combinations of the parameters φ and θ.

This is expected because when there is no shift in the mean, there is no dynamic response and hence the residuals do not depend on the two model parameters. The probability that the residuals exceed the limits is constant, and thus the *pmf* of the run length distribution of the residuals X- chart is a geometric distribution, equivalently to the traditional X-chart of the observations.

When a shift occurs and the process is negatively autocorrelated the *SRL* is smaller than the *ARL*, but when the process is positively autocorrelated the *SRL* is usually greater that the *ARL*. The large values of the *SRL* mean that the time at which the signal will actually be detected is not precise.

Another remark is that when the first lag autocorrelation is positive, the residuals chart has larger *ARL* than when its first lag autocorrelation is negative.

This doesn't necessarily imply that the residuals chart performs poorly for positive autocorrelated processes. In fact, even though the residuals X-chart doesn't detect shifts quickly on the average, the probability of detecting shifts early is high but the *ARL*'s are affected by the long tails of the run length *pmf*.

An example of this behavior is illustrated in *Figure 8.1*, where the *pmf* of the run length distribution of a residuals X-chart truncated at run length *10*, for $\varphi=0,95$, $\theta=0,45$ and $\sigma_a=0,7$ and a mean shift of one standard deviation is plotted. From TABLE A.8 it is obvious that the X-chart of the residuals performs poorly for this particular combination of parameters *(ARL=274.69)*, but in *Figure 8.1* it is shown that there is a probability of *13%* to detect the shift at run length *1*. This probability decays very quickly and thus inflates the average run length.

Wardell et al. (1994) stated that the residuals chart has high probability of detecting a mean shift as soon as it occurs, but if it fails to detect the shift immediately, there is low probability to detect it later. This can be explained by the fact that when a shift first occurs there is a large discrepancy between the forecasted and the actual value, but in the next instance the forecasted values adjust to the shift. In particular, when the process is negatively autocorrelated and the process mean shifts, the one-step-ahead forecast moves in the opposite direction of the shift and consequently the residual becomes very large and the shift is detected earlier.

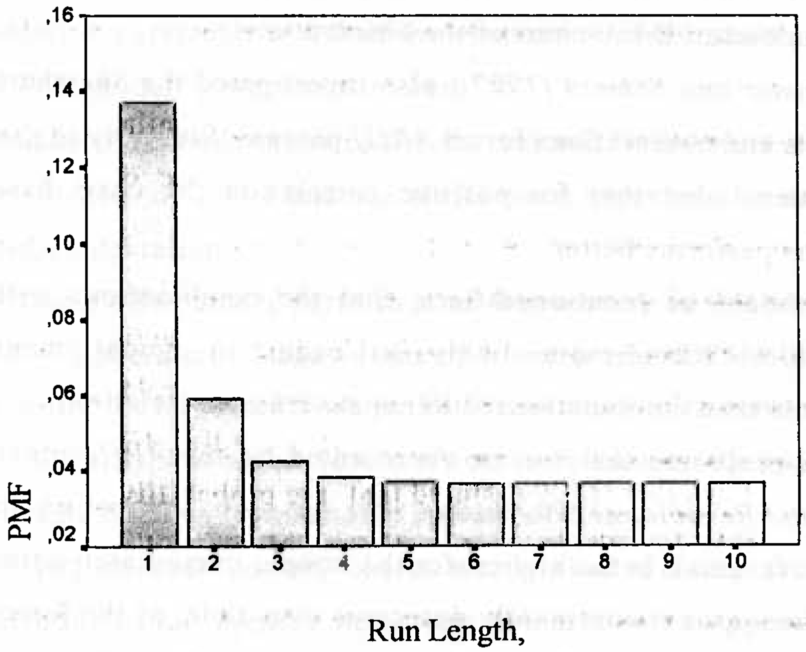

Figure 8.1: **A portion of the pmf of the Run Length of the residuals *X*-chart for an *ARMA(1,1)* Process with $\varphi=0,95$, $\theta=0,45$, $\sigma_a=0,7$ and a shift of one Standard Deviation**

The above results are consistent with the findings of *Zhang(1997)* who mathematically proved, as shown in section *7.4*, that for every stationary autoregressive process *AR(p)* the residuals *X*-chart has a great likelihood of detecting the change of the mean at the time it occurs. The probability of detecting the change of the mean at some future time depends on the parameters of the process.

For the stationary region of the *AR(1)* process the residuals *X*-chart detects the shift in the mean after the time it first occurred, faster than the *X*-chart of observations only when the process is negatively autocorrelated.

When the process is near non-stationarity, the detection capability of the residuals chart becomes very large.

Assuming that the error term in the *AR(1)* process is normally distributed, *Zhang(1997)* concluded that when the process is negatively autocorrelated, the in-control *ARL* of the residuals *X*-chart is uniformly smaller than that of the *X*-chart of the observations. When $0<\varphi<0,8$, 3σ limits are used and the magnitude of the shift δ (in terms of standard deviation) is less or equal to *3*, the in-control *ARL* of the residuals *X*-chart is also uniformly smaller than that of the *X*-chart of the observations.

Kramer and Schmid (1997) also investigated the Shewhart charts of the residuals and observations for an *AR(1)* process. Similarly to *Zhang(1997)* they also concluded that for positive correlation the chart based on the observations performs better.

It should be mentioned here, that the combinations with positive autocorrelation seem more likely to occur in actual manufacturing environments than combinations with negative autocorrelation.

For processes that can be represented by the *AR(1)* plus a random model *Lu and Reynolds (1999a)* stated that the probability of a signal by a *X*-chart of the residuals is the highest for the sample immediately after the shift, and that afterwards it continually decreases over time, as the forecast adapts to the shift. They derived the above conclusion after observing that the expectation of the residual after the shift is a decreasing function of the time after the shift.

Zhang(1997) also investigated the performance of the *X*-charts of the residuals and the observations for the *AR(2)* process. When the process is near non stationarity the detection capability of the residuals chart becomes very large, while for $\varphi_1 \geq 0$ and $\varphi_2 \leq 0,5$ the residuals *X*-chart has smaller probability to detect the shift at run length *1*, than the *X*-chart of the observations.

When $\varphi_1 < -0,25$, the in-control *ARL* of the residuals chart is uniformly smaller than that of the traditional *X*-chart. When a small shift occurs (*i.e.*, a shift of magnitude $\delta \leq 0,5$), $\varphi_1 + \varphi_2 < 0,9$, and the residuals chart has smaller probability than the *X*-chart to detect the shift at run length *1* and *2*, the out-of-control *ARL* of the residuals chart is larger than that of the *X*-chart of the observations. Besides, when $\varphi_1>0$, $\varphi^2_1+4\varphi_2>0$, $\varphi_1+\varphi_2<0,8$ and $\delta\leq 3$, the *ARL* of the residuals chart is uniformly larger than that of the *X*-chart.

8.3 The performance of the *EWMA* and *CUSUM* charts

Lu and Reynolds (1999a) compared the adjusted *EWMA* charts of the observations to the *EWMA* chart of the residuals for a positively correlated process described by an *AR(1)* model with additional error. Various values of the autoregressive parameter φ and of the proportion of the process variance due to the *AR(1)* model ψ were considered.

To make the comparisons easier the control limits were adjusted to have in-control *ARL* of *370,4* and the shifts were expressed in units of the process standard deviation.

The two *EWMA* charts perform similarly when φ is small and/or ψ is close to *0*. For moderate or large values of φ and ψ the *EWMA* chart of the residuals is better for large shifts while the *EWMA* chart of the observations is better for small shifts. For higher levels of autocorrelation neither chart would detect very small shifts in a reasonable amount of time.

The *ARL* of both control charts was evaluated using the integral equation method, which applies numerical integration to approximate the integral. The Markov chain method which replaces the continuous control statistic by a discretized version which is a Markov chain was also implemented. To provide a check on the accuracy of the individual methods and to provide results for those combinations of parameter values for which the other two methods doesn't work, simulations were performed.

The steady state *ARL* results for both charts, when both charts use $\lambda=0,2$ are presented in TABLE A.9.

In addition to the *ARL* , they obtained using integral equation methods the cumulative distribution function *(cdf)* of the run length. The *EWMA* chart of the residuals usually has higher probabilities of detecting a shift in the first few sample points after the shift and can detect large shifts faster than the *EWMA* chart of the observations when correlation is high.

Runger et al. (1995) evaluated the one sided *CUSUM* chart of the residuals for *AR(p)* processes. They developed approximate analytical models for the evaluation of the *ARL*, when the shift in the process mean is a single step change of size δ (where δ is measured in terms of the standard deviation of the process).

For the special case of an *AR(1)* process, numerical results for several levels of autocorrelation φ and shift δ and several choices of the reference value *K* of the *CUSUM* chart were developed. To have results more comparable with the two sided control charts the decision interval *H* must be selected to provide in control *ARL* of *740,8*.

The main result from the above studies is that the *CUSUM* chart for the residuals is much more sensitive to process shifts than Shewhart charts of the residuals especially when the reference value is chosen to be $K=(1-\varphi)/2$.

8.4 The Relative Performance of Control Charts

This section deals with the relative performance of the *X* and *EWMA* charts of the observations with control limits constructed taking into consideration the true variance of the process, the residuals *X*-chart and other charts designed for application to correlated processes.

Wardell et al. (1992) compared the *X*-chart of the residuals chart to the adjusted *X*-chart and *EWMA* of the observations as well as to the Common Cause chart *(CCC)* with additional control limits. They considered processes that follow *ARMA(1,1)* and consequently *AR(1)* or *MA(1)* models.

Wardell et al. (1992) used the Dynamic Step Response Function *(DSRF)* described in section *7.2* to see how every chart reacts dynamically to a shift in the process mean and determined the relative performance of the different charts when the process is autocorrelated. They derived the *ARL* of the charts via simulation, and designed an experiment over the entire stationary region of the *ARMA(1,1)* model and over two values of the parameter λ of the *EWMA* statistic to derive the location of regions in the (φ,θ) plane, in which a particular chart is superior to the other charts.

A general conclusion that can be drawn from their studies is that certain charts are equivalent under specific circumstances. Namely, the *X*-chart of the observations and the *CCC* have similar *ARL* performancefor the *AR(1)* process while the residuals and the Common Cause charts for the *MA(1)* process.

It is also clear that for a shift of one standard deviation the *EWMA* chart has a smaller *ARL* than the other three charts over most of the stationary region. When the shift in the mean is larger *i.e.*, three standard deviations, the Common Cause chart dominates most of the region, with the *EWMA* still being superior when the autoregressive parameter is negative and the moving average parameter positive. Further, the *X*-chart of the observations rarely achieves an *ARL* lower than those obtained by other charts.

The relative *ARL* performance of the adjusted *X* and *EWMA* charts of the observations and the residuals *X*-chart was also considered by *Wardell et al. (1994)*. From this study can be concluded that for positively autocorrelated processes, the residuals *X*-chart performs relative poorly in terms of the *ARL*, when compared to traditional charts.

The *ARL* of the residuals chart was determined through its run length distribution while the *ARL* of the *EWMA* and the *X*-chart of the observations through simulation. The control limits were adjusted, sometimes substantially, to have in-control *ARL* of around *370,4*.

Zhang(1998) considered the *ARL* performance of the residuals *X*-chart as did *Wardell et al. (1994)*. He additionally implemented the *EWMAST* chart to *AR(1)*, *AR(2)* and *ARMA(1,1)* processes and made comparisons of its *ARL* performance with the *X*-chart of observations, and the *X*-chart of residuals. When he made comparisons for various *AR(1)* processes, he also considered the performance of the chart of *Montgomery and Mastrangelo (1991)(M-M)*.

For the *AR(1)* processes, it appears that when the autocorrelations vary from weak to medium levels *(φ≤0,75)*, the *EWMAST* chart performs better since it has larger in-control and smaller out-of control *ARL's* than both the residuals and the individuals *X*-chart. This is more obvious for mean shifts from small to medium.

When there are very strong positive autocorrelations *(i.e.,* $\varphi=0,95$*)* the residuals chart has smaller out-of control *ARL's* especially when the mean shifts are large. Neither chart performs well for detecting small shifts.

For the *M-M* chart when $\varphi=0,5$ and $0,75$ the in-control *ARL* and the out-of-control *ARL's* are almost the same for small mean shifts. Even when the mean shifts are medium or large, the out-of-control *ARL's* of the *M-M* chart are much larger than those of the other charts. When $\varphi=0,95$ the *M-M* chart performs better than the *EWMAST* chart but not better than the residuals chart.

For the *ARMA(1,1)* model the *EWMAST* chart performs better than, or equal to the residuals chart except for the cases in which the processes have strong positive autocorrelations. For the other cases, the *EWMAST* has larger in-control *ARL* than that of the residuals chart and moreover when the mean shifts are small the *EWMAST* chart has smaller *ARL's* than those of the residuals chart but when the shifts are large, the *EWMAST* chart performs better than or equal to the residuals chart. Except from the case where the process has strong positive autocorrelations or the mean shift is large the *EWMAST* perform better than the individuals *X*- chart.

When the process follows an *AR(2)* time series model and the process is not nearly non-stationary , with $\varphi_1+\varphi_2$ not near to *1*, the *EWMAST* chart performs better than the residuals chart, especially when the mean shift is not large. When the processes are near non-stationarity and have very strong positive autocorrelation the residuals chart has smaller out-of-control *ARL's* than those of the *EWMAST* chart. For the near non-stationary processes but not with strong positive autocorrelation the *EWMAST* chart has much larger in-control *ARL's* than those of the residuals chart, much smaller out-of-control *ARL's* than those of the *X*- chart and comparable with those of the residuals chart. Concluding, the residuals chart performs better than the *EWMAST* chart only when the process is nearly non-stationary with strong positive autocorrelations.

The *ARL's* of the residuals chart for step mean shifts were calculated using the formulas developed by *Zhang(1997)*. To obtain the *ARL's* of the *EWMAST* chart for $\lambda=0,1$, $0,2$ simulation was conducted. The above *ARL* results are shown at TABLE A.10 for various shifts of the mean and various values of the autoregressive parameter and the moving average parameters for the *ARMA(1,1)*, *AR(1)* and *MA(1)* time series models along with the results presented by *Wardell et al. (1994)*.

Schmid (1997a) compared the performance of several control schemes for process assumed to be represented by the *AR(1)* time series model by their *ARL* values. He concluded that the *EWMA* and *CUSUM* charts are more suitable to detect small changes while the Shewhart chart is more appropriate for large shifts. He also found that for positively correlated processes it is preferable to use modified control charts of the observations but elsewhere the residuals charts should be preferred.

For processes modelled by the *AR(1)* plus a random error *Lu and Reynolds(2001)* compared the *ARL* performance of the *CUSUM* Chart of the observations, the *CUSUM* chart of the residuals, the *EWMA* chart of the observations, and the *EWMA* chart of the residuals. The steady state *ARL* values of the Shewhart chart of the observations, and the Shewhart chart of the residuals were also considered.

In general it is concluded that the performance of the *EWMA* and the *CUSUM* charts are very similar. The Shewhart chart of the observations performs better than the one of the residuals when the shift to be detected is small but when the shift is large the two charts have similar behaviour. For low to moderate levels of correlation though, a Shewhart chart of the observations performs much better than a Shewhart chart of the residuals When the level of the autocorrelation is moderate to low, the *EWMA* and the *CUSUM* charts perform much better than the *X*-chart but when the autocorrelation is high, all charts perform similarly.

The above *ARL* results are presented at TABLE A.11 for $\delta=0,5,\ 1,0,\ 2,0$, for $\psi=0,5,\ 0,9$ and $\varphi=0,4,\ 0,8$. The parameters of the charts were chosen to ensure their optimal performance.

CHAPTER 9

Performance of Control Charts for Monitoring the Variance

9.1 Introduction

At this chapter the performance of the control charts designed for monitoring the variance of autocorrelated processes is investigated. In section *9.2* the R and S^2 charts are evaluated in terms of their *ARL*, while in section *9.3* the *EWMA* of the Logs of Sqrt Residuals.

9.2 The *ARL* performance of the R and S^2 Charts

Amin et al.(1997), evaluated the effects of autocorrelation on the in control average run length of the R and S^2 Shewhart type charts for the case when the variance is correctly estimated but constants for independent data are used, and for the case when modified control limits which account for autocorrelation are used. The implementation of the R and S^2 charts for these cases is presented in section *6.2*.

An *AR(1)* time series model has been considered to describe the correlation structure, while Laguarre polynomials and simulation were used to derive numerical results for the in control average run length.

The graphs in *Figure 9.1* show the effect of correlation of an *AR(1)* model for different levels of correlation, for correlation within samples of length $n=5$ *(model I)* and between samples of length $n=5$ *(model II)* to S^2 and R charts as discussed in *6.2*.

The main conclusion could be, that for both charts the effect of the autoregressive parameter φ on the values of the in-control *ARL* is less dramatic in the case of correlation between samples than its effect in the case of correlation within samples.

S^2-Chart **R-Chart**

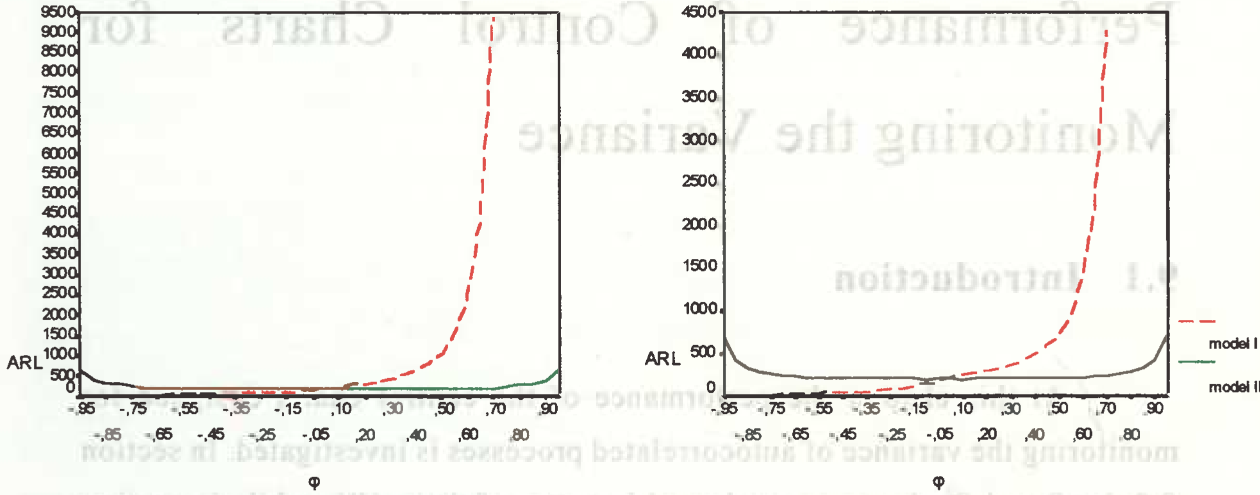

Figure 9.1: **The effect of autocorrelation on the in control *ARL* for models I and II.**

Specifically, for both charts the *ARL* values for *model I* reveal an increase (from *ARL=200*) for $\varphi>0$ and a decrease for $\varphi<0$, whereas *model II* results in increased values for the *ARL* as $|\varphi|$ increases.

When there is a strong correlation such as $-0.5 \leq \varphi \leq 0.5$ the *ARL* values for the S^2–chart range between *44.25* and *720.98* for *model I* and fall from *211.38* to *209.93* for *model II* . For the *R*-chart, they range between *60.95* and *1114.23* for *model I* and fall from *212.23* to 209.48 for *model II*.

Amin et al. (1997) further investigated the effect of correlation within samples and evaluated the *ARL* of the S^2 and *R* charts for different sample sizes and levels of autocorrelation. A graphical representation of their studies is shown in *Figure 9.2* for the case of positive autocorrelation *($\varphi>0$)* and in *Figure 9.3* for the case of negative autocorrelation *($\varphi<0$)*.

It is obvious that for *n* fixed the *ARL* decreases in $\varphi<0$ from *200* monotonically, and the amount of decrease gets larger as *n* increases. For any fixed value of $\varphi>0$, the *ARL* decreases as *n* increases.

S^2-Chart **R-Chart**

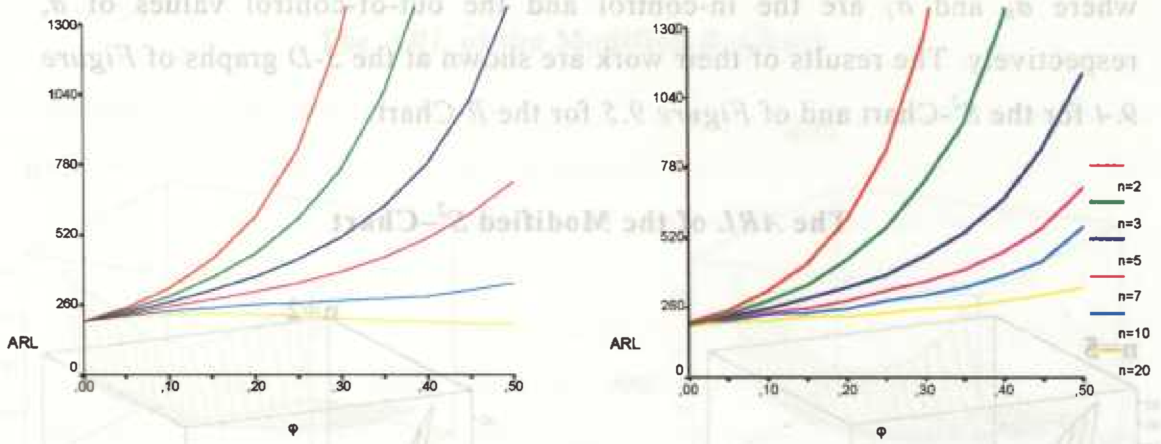

Figure 9.2: **The in-control *ARL* of the S^2 and *R* charts for different values of *n* and *φ>0.***

For $3 \leq n \leq 7$ fixed, the *ARL* increases when $\varphi > 0$ from *200* monotically and *ARL* values can reach very large numbers for big values of the autoregressive parameter. For $n \geq 8$ the *ARL* first increases and then decreases for small φ values, but then increases again as φ gets larger. Besides the *ARL* is relative stable for $0 \leq \varphi \leq 0.4$ when the sample size $n=20$.

S^2-Chart **R-Chart**

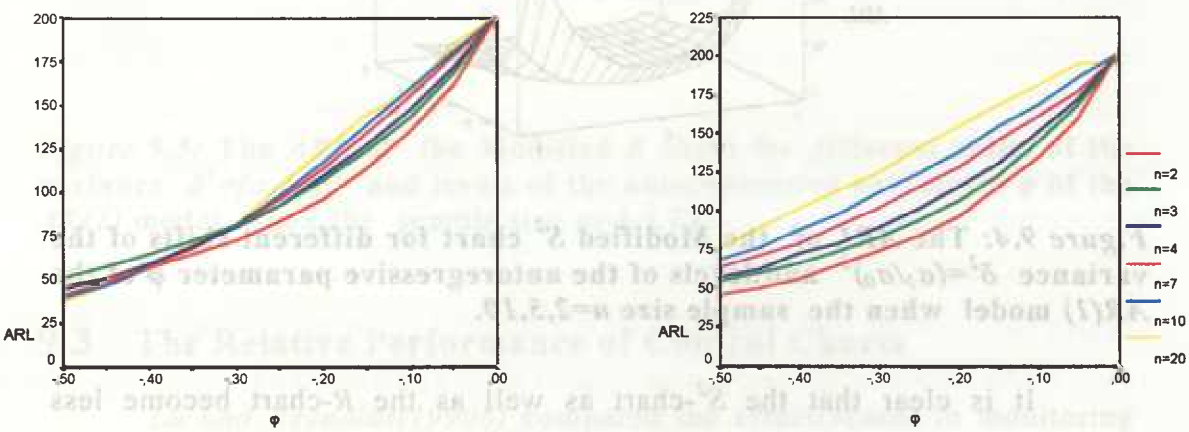

Figure 9.3: **The *ARL* of the S^2 and *R* charts for different values of *n* and *φ<0.***

Amin et al.(1997) also compared the *R*-chart and S^2-chart by their *ARL's*, in the case when modified control limits are used. The *ARL* of the *R*-chart was computed by simulation and that of the S^2-chart was computed numerically. The in-control *ARL* equals around *200* and the out-of-control *ARL*'s are calculated for shifts of magnitude $\delta^2=(\sigma_1/\sigma_0)^2=1.0,\ 1.5,\ldots,\ 5.0$ where σ_0 and σ_1 are the in-control and the out-of-control values of σ, respectively. The results of their work are shown at the *3-D* graphs of *Figure 9.4* for the S^2-Chart and of *Figure 9.5* for the *R*-Chart.

The *ARL* of the Modified S^2–Chart

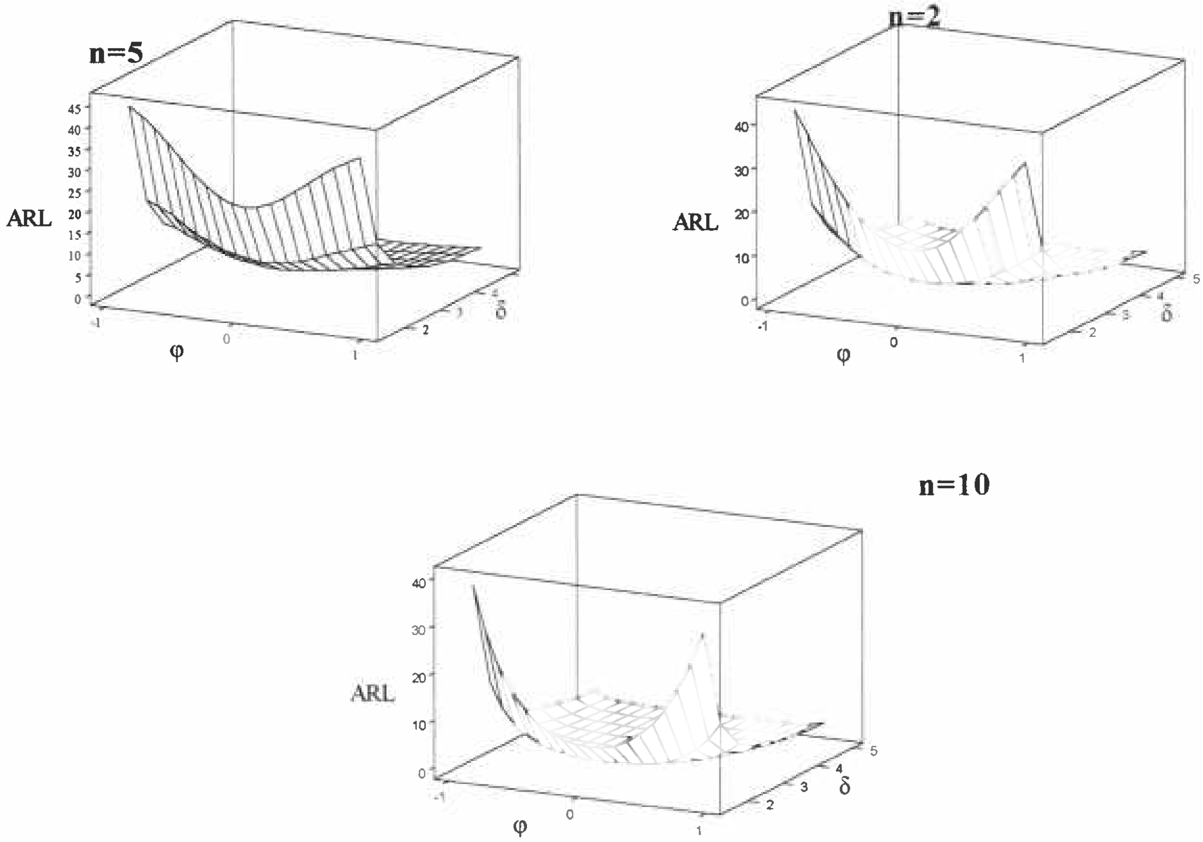

Figure 9.4: **The *ARL* of the Modified S^2 chart for different shifts of the variance $\delta^2=(\sigma_1/\sigma_0)^2$ and levels of the autoregressive parameter φ of the *AR(1)* model when the sample size $n=2,5,10$.**

It is clear that the S^2-chart as well as the *R*-chart become less sensitive to increases in the process variability as $|\varphi|$ increases. Besides, the out-of-control *ARL* values for both charts get smaller as the sample size *n* increases. For large values of $|\varphi|$, the *R*-chart has smaller values for the ARL_0 than the corresponding S^2-chart. Both modified charts are slightly more sensitive in detecting increases in the process variability when there is positive autocorrelation compared to negative autocorrelation .

The *ARL* of the Modified *R*–Chart

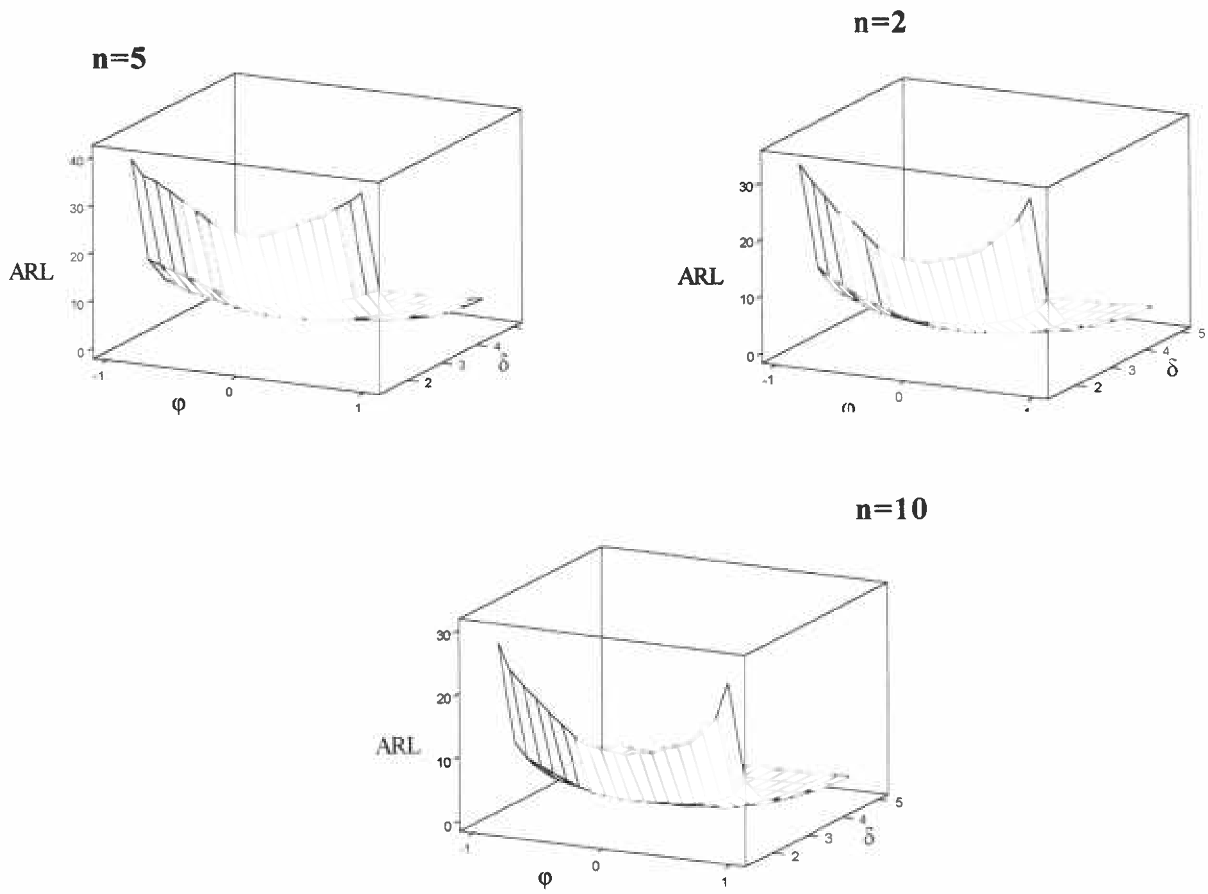

Figure 9.5: **The *ARL* of the Modified *R* chart for different shifts of the variance $\delta^2=(\sigma_1/\sigma_0)^2$ and levels of the autoregressive parameter φ of the *AR(1)* model when the sample size $n=2,5,10$.**

9.3 The Relative Performance of Control Charts

Lu and Reynolds(1999b) compared the effectiveness in monitoring the process variance of four control charts for various parameter combinations of the *AR(1)* model plus a random error. The *EWMA* chart of the logs of the squared residuals, a Shewhart chart of the squared residuals which is equivalent to a X-chart of the residuals, the traditional moving range R-chart, and the $EWMA$ chart of the residuals were considered.

None of the four charts seem to perform well for all shifts and parameter combinations. The $EWMA$ chart of logs of squared residuals and the Shewhart chart of residuals though, show a satisfying performance in almost all cases. Moreover, the Shewhart chart of residuals is good for detecting large shifts while the $EWMA$ of logs of squared residuals for detecting small shifts. Both charts are more sensitive in an increase in the process variance caused by σ^2_ε, than to the same increase caused by σ^2_γ (see section *6.3*).

The R-chart performs poorly for small shifts when the variance is increased through σ^2_ε. This is more intense when the levels of autocorrelation are high and the percentage of the in-control variance of the process due to the variance of the mean is small.

Finally, the $EWMA$ chart of residuals is, in most cases, not as good as the charts designed specifically for monitoring the variance, especially for detecting small shifts when the increase is caused by σ^2_γ.

The steady-state ARL results for the four charts under consideration are shown at TABLE A.12 for shifts in σ^2_γ and TABLE A.13 for shifts in σ^2_ε.

The control limits for all charts were adjusted so that the in-control ARL equals to 370.4 and the out-of-control ARL's are calculated for shifts of magnitude $(\sigma_X/\sigma_{X0})^2 = 1.5,\ 2,\ 3,\ 5,\ 7,\ 9$ and 11, where σ_{X0} and σ_X are the in-control and the out-of-control values of σ, respectively. The percentage of the in-control variance of the process due to the variance of the mean, $\psi_0=\sigma^2_{\mu 0}/\sigma^2_{X0}$, considered at both tables was $0,1$ or $0,9$, and the levels of the autoregressive parameter φ were taken to be $0,\ 0{,}2,\ 0{,}4,\ 0{,}6$ and $0{,}8$. The $EWMA$ smoothing parameter λ was chosen to be $0{,}2$ in both tables. The in-control ARL results were obtained by the integral equation approach for the two $EWMA$ charts, while the out-of-control ARL's by simulation.

CHAPTER 10

Miscellaneous Topics for Autocorrelated Processes

10.1 Introduction

This chapter is about other techniques and issues concerning data for which correlation is an inherent part of its generation's mechanism. In such situations several issues involved with classical *SPC* procedures are differentiated.

Section *10.2* considers a different aspect of estimating the parameters of the model that describes the correlated structure of the process. In section *10.3* and in section *10.4* the problem of short-run data and the economic design of the Shewhart $\overline{X}$ control chart in the presence of autocorrelation are discussed. In section *10.5* some policies of the integrated process control that integrates the engineering and the statistical process control are applied and evaluated. Section *10.6* considers a variable sampling interval methodology and its contribution to the performance of the averages chart in the presence of correlation. The use of neural networks to recognize shifts in correlated processes parameters is discussed in section *10.7*, while in section *10.8* another model free approach.

10.2 Phase I Analysis

A common representation of the correlation structure of a process is by the *AR(1)* time series model and in particular the cases of positive autocorrelation ($\varphi>0$). Usually, the parameters of the model *i.e.*, the autoregressive parameter φ and the standard deviation of the process σ_ε are estimated from some in-control baseline data or are considered to be known. In real applications though, these parameters are estimated from data which contains trends, level shifts or outliers due to assignable causes. Under these circumstances, the standard maximum likelihood estimators of the parameters can be severely biased.

Kramer and Schmid (1997), *Adams and Tseng(1998)*, *Lu and Reynolds (1999a)* and others, have shown that even small errors in parameter estimates can significantly hurt the performance of control charts.

Boyles (1997) designed, compared and contrasted several estimators suitable for data with assignable causes for processes that are independent and identically distributed .

Boyles (2000) presented a new estimation method for processes whose system of common causes produces autocorrelated data. This method referred to as Phase I analysis, tries to overcome the problem of estimating parameters from baseline data that may be corrupted by assignable causes. He assumed the stationary *AR(1)* model for the common cause variation, implying variation about a fixed level μ within fixed limits.

For autocorrelated processes, assignable causes not only tend to increase the variation, but may also change process dynamics. In particular, trends or level shifts will produce non stationary behavior in observations from an otherwise stationary process. When assignable causes are present, a non stationary model which allow the process level to wander freely, represents better the baseline data.

The choice though, between a stationary and a non stationary model in terms of goodness of fit is difficult. For example the *AR(1)* and the *ARIMA(0,1,1)* model are empirically indistinguishable. In such cases, the choice should rest on the purpose for which the model is being used. Thus when the objective is to establish control limits it is preferable to represent the data by a stationary model which has some sense of statistical control.

Boyles(2000) addressed situations where it makes sense to assume a stationary common cause model even though the baseline data may look non stationary due to assignable causes. He established a method which fits an

ARIMA(1,1,1) model and then derives *AR(1)* as a stationary submodel. This method gives smaller estimates for φ and σ_ε and hence smaller estimate for the standard deviation of the process σ_X. The new method filters out the effects of the mean shift and does not require prior knowledge of the shift. He showed through simulation that the new method offers substantial improvements in mean squared error (*MSE*) over the standard method for small to moderate φ, but not for large φ.

Boyles(2000) recommends the use of the phase I analysis when process owners do not want to accept the baseline data as the reference or template for future data because they suspect that it is corrupted by assignable causes. When on the other hand the baseline data reflect a high level of process capability and the assignable causes are not likely to occur the standard method should be preferred.

10.3 Short-Run *SPC*

Traditional *SPC* techniques are based on the assumption that the process data are *i.i.d.* and that there is sufficient historical data for proper analysis. Some or all of these assumptions are often violated in real applications. In chapter *4* there are some examples of autocorrelated processes. The short-run data can be result of situations that do not allow for frequent data measurements such as short production runs, process start ups, new equipment, major tool changes, different raw materials, and different production processes. A review of *SPC* techniques suitable for short-run data can be found in *Del Castillo et al. (1996)*.

Throughout this thesis, methods for dealing with situations where only the problem of autocorrelation is present in the data are discussed. Sometimes though, a situation where both the independent and the long run data assumption are violated simultaneously arises.

Wright et al. (2001) investigated the use of the joint estimation (*JE*) outlier detection method of *Chen and Liu (1993a,1993b)* as an *SPC* technique for short run autocorrelated data for time series with length ranging from *n=9* to *n=25*. An outlier is considered equivalent with an out of control observation.

There are four types of outliers. The first is the additive outlier (AO) which is a one-time event in the series and its only effect is at the time when it occurs. The second type is the level shift (LS) which occurs through a step function and permanently changes the series altering the observations. The third is the innovational outlier (IO) which occurs through a pulse function and affects the time series by altering the random shock. The last outlier, the temporary change (TC), also occurs through a pulse function but has an initial impact that decays exponentially according to some dampening factor.

All the outliers are not equivalently significant and false alarms are very costly, therefore in *SPC* monitoring one would like to now both the type and the time of an out of control observation. Joint Estimation (*JE*) is able to detect process shifts (*i.e.*, an LS) which are the most significant in terms of statistical process control. *Wright et al.(2001)* presented a brief description of the *JE* method and investigated its effectiveness for short run autocorrelated data.

The *JE* method is conducted in three stages. At the first stage the maximum likelihood estimates of the parameters for the proper *ARMA* model are derived and the outliers are detected. At stage two the procedure jointly estimates the outliers effect using multiple regression, computes the estimated t-values of the estimated weights and compares their absolute value with a critical value. For any outlier detected, if the absolute value of its t-value is less than the critical value, the outlier is determined to be not significant and is removed from the set of identified outliers. Then the procedure obtains the adjusted series by removing significant outliers effects and derives new maximum likelihood estimates of the model parameters based on the adjusted series. At stage three the procedures seeks to detect outliers based on final parameter estimates.

The implementation of the joint estimation method includes the selection of the appropriate time series model to describe the available data and the determination of the critical value. The time series model can be selected using the methods described in chapter 3. *Wright et al. (2001)* provided tables for choosing the critical value taking into consideration the acceptable levels of outlier location detection, the type of the outlier to be identified and the false alarm rates. They recommended though a choice of a critical value at least equal to *2*.

10.4 The Economic Design of Averages Control Charts

The economic design of a control chart is a way of making decisions concerning the sample size *n*, the frequency of sampling *h*, and the coefficient *k* of the control limits of a chart by minimizing the average overall cost.

In *1956*, Duncan presented the first cost model to determine the three parameters for the $\overline{X}$ -charts. Later many other models were developed.

Chou et al. (2001) used Duncan's cost model as the objective function to be minimized for the economic design application of average control charts for correlated data.

The components of Duncan's model include:

- The cost of an out of control condition
- The cost of false alarms
- The cost of finding an assignable cause
- The cost of sampling, inspection, evaluation and plotting.

Duncan(1956) assumes that the process starts in an in-control situation, shifts to an out-of-control condition, has the out of control condition detected, and results in the assignable cause being identified. The total time for these four states of monitoring a process via *SPC* to happen, defines the cycle length.

Considering the time parameters and the associated cost as given, the optimal values for the three decision factors of the average chart can be determined by using optimization techniques.

The expected length of a cycle *E(T)* is

$$E(T)=\frac{1}{\lambda}+\frac{h}{1-\beta}-\tau+gn+D$$

where

λ: reciprocal of the average process in control time

β: Type II error probability of the chart

g: the average sampling, inspecting, evaluating, and plotting time per sample

D: the time required to find the assignable cause

Besides, τ is the average time the process goes out of control within an interval between the jth and the $(j+1)$th samples and equals to

$$\tau = \frac{\int_{jh}^{(j+1)h} e^{-\lambda t}\lambda(t-jh)dt}{\int_{jh}^{(j+1)h} e^{-\lambda t}dt} = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})} \approx \frac{h}{2} - \frac{\lambda h^2}{12}$$

The expected cost per hour , denoted by $E(C)$ is

$$E(C) = \frac{a_1 + a_2 n}{h} + \frac{a_4[E(T)-(1/\lambda)] + a_3 + \alpha a_5 e^{\lambda h}/(1-e^{\lambda h})}{E(T)}$$

where

a_1: the fixed sampling cost

a_2 : variable sampling cost

a_3: cost of finding an assignable cause,

a_4: hourly penalty cost for operating out-of –control

a_5: cost of investigating a false alarm

It is obvious from the above equations that the Type I error probability of the chart namely the a risk, and the Type II error probability of the chart β must be determined to obtain the value of the expected cost.

Chou et al. (2001) provided these probabilities for correlated observations after assuming that each sample is a realization of a random vector $\boldsymbol{X}=\{X_1,\dots,X_n\}$ which has a multivariate normal distribution $N(\boldsymbol{\mu},\boldsymbol{V})$ where $\boldsymbol{\mu}$ is the mean vector and $\boldsymbol{V}$ is the covariance matrix which equals to $\sigma^2\boldsymbol{R}$, where $\boldsymbol{R}=\{r_{i,j}\}$ with $i,j=1,\dots,n$ is the correlation matrix. Based on these assumptions the sample mean is normally distributed with mean $E(\bar{x}) = \mu$ and variance $V(\bar{x}) = \frac{\sigma^2}{n}[1+(n-1)\rho]$

where $\rho = \frac{\sum_{i\neq j} r_{ij}}{n(n-1)}$.

The error probabilities for correlated samples are

$$\alpha = 2P\left(Z > \frac{k}{\sqrt{1+(n-1)\rho}}\right)$$

$$\beta = P\left(\frac{-k-\delta\sqrt{n}}{\sqrt{1+(n-1)\rho}} < Z < \frac{k-\delta\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right)$$

where Z is the standard normal distribution and δ is the amount of the shift in the mean in terms of standard deviation.

Chou et al. (2001) implemented the economical design of the averages chart to correlated samples and concluded that highly positive correlated data result in a smaller sample size, a frequent sampling interval and narrower control limits. They also mentioned that the power of the chart becomes worse as the correlation coefficient increases. Highly negatively correlated data on the other side, yields a smaller sample size and narrower control limits, not significantly affecting the sampling interval (this model could be questioned see *Woodall(1986)*).

10.5 The Integrated Process Control

The Integrated Process Control (*IPC*) is a procedure which simultaneously applies techniques of the Engineering Process Control (*EPC*) and the Statistical Process Control (*SPC*) aiming to the reduction of the variation of the process.

EPC techniques can be applied to processes where there is a maniputable variable that affects their output. During the *EPC* procedure the process whose level wanders about, is manipulated by some compensators to bring the process level back to target. The adjustment is usually employed to the maniputable variable of the process using information about its current level or deviation from a desired target. This procedure is called feedback control. For issues involving the *EPC* procedures see *Ogata(1990)* or *Box and Luceno(1997)* and *Montgomery(2001)* .

A simple adjustment scheme is the **Integral Control** . To see how this procedure is applied suppose that the process output characteristic of interest at time t is X_t and also that the level of the process which we want to maintain is T. This process presents a drifting behavior away from the target level caused by unknown and uncontrollable disturbances but it can be compensated by making adjustments to the set point of a maniputable variable v_t. The output of such a process which is usually autocorrelated might be the molecular weight of a polymer and the maniputable variable the catalyst feed rate (see *Montgomery(2001)*). Considering the constant g as a regression coefficient that relates the magnitude of a change in v_t to a change in X_t within one period , the process can be described by

$$X_{t+1}-T = gv_t$$

and if no adjustment is made the process drifts away from the target according to

$$X_{t+1}-T = N_{t+1}$$

where N_{t+1} is a disturbance which is unknown but can be predicted using an *EWMA* with parameter λ

$$\hat{N}_{t+1} = \hat{N}_t + \lambda(N_t - \hat{N}_t) = \hat{N}_t + \lambda e_t$$

where e_t is the prediction error at time period t. As shown in *3.5.2* this is equivalent with the prediction for an *ARIMA(0,1,1)* with $\theta=1-\lambda$.

Under these assumptions the adjustment to be made to the maniputable variable at time t can be shown equal to

$$v_t - v_{t-1} = -\frac{\lambda}{g}(X_t - T) = -\frac{\lambda}{g} e_t$$

by summing up this equation the actual setpoint of the maniputable variable at time t is

$$v_t = -\frac{\lambda}{g}\sum_{i=1}^{t} e_i$$

The integral control is a feedback control scheme that sets the level of the maniputable variable equal to a weighted sum of all current and previous deviations from the target. When there is a reason to consider the last two errors e_t and e_{t-1} the adjustment equation can be written in terms of two constants as $g(v_t-v_{t-1})=c_1e_t+c_2e_{t-1}$ and by summing up this expression the setpoint becomes

$$v_t = k_p e_t + k_I \sum_{i=1}^{t} e_i$$

where $k_p=-(c_2/g)$ and $k_I=(c_1+c_2)/g$. This adjustment procedure is called **Proportional Integral (PI)**.

When a process level has a wandering behavior as well as level shift , which is the case of correlated data, an *IPC* procedure which along with the *EPC* control applies also an *SPC* activity is to detect the occurrence of a shift in the process level due to a special cause might be used. For details in the *IPC* techniques see *Nembhard and Mastrangelo(1998)* .

Nembhard(1998) investigated the use and the effectiveness of *IPC* policies in noisy dynamic systems which resemble the dynamics that can produce correlated data. The output of noisy dynamic systems is the result of combining a dynamic plant process and a noise process which can be represented by an *ARMA(p,q)* model.

Nembhard(1998) considered a proportional-integral (*PI*) controller as the *EPC* policy and a Shewhart *X*-chart as the *SPC* policy. He evaluated the control policies based on the average squared error of the output from target, the number of adjustments, the average magnitude of adjustments and the number of alarms. He concluded that for simple noisy dynamic systems *i.e.*, first order processes with *ARMA(1,1)* noise it is preferable to use only *PI* control. For complex noisy dynamic systems *i.e.*, second order processes with *ARMA(2,2)* noise he recommended the implementation of a Shewhart chart to indicate a special cause and only when the special cause is detected the use of the *PI* controller.

Park(2001) developed an *IPC* procedure to adjust the process level close to target and in the meantime to monitor the occurrence of special causes. This procedure can be applied when correlation is present in the data. He considered a process noise which follows an *ARIMA(0,1,1)* model in the absence of a special cause, and an *ARIMA(0,1,1)* with a step shift in its level under the presence of a special cause. The *EPC* part of the *IPC* scheme performs an adjustment procedure to the process whenever the predicted deviation exceeds some bounds. The monitoring part of the scheme is performed by using the *EWMA* chart with forecast errors .

The optimal control procedures and the effectiveness of the *IPC* policy are derived through the economic design of the charts. *Park(2001)* concluded that the *IPC* procedure is more efficient than the *EPC* procedure alone.

10.6 Variable Sample Interval

Through all this thesis we have assumed that the samples or the individual observations for use with all the charts, were taken from the various processes at evenly spaced or fixed sampling intervals (*FSI*). For data that exhibits some serial dependence though, it is natural to consider that shorter sample intervals will increase the autocorrelation. It is useful to consider sampling intervals whose length of time between samples varies as long as this is done with caution.

Reynolds et al. (1988) proposed a sampling technique for use with the averages Shewhart chart. Later, *Reynolds (1995)* studied the properties of such charts and *Reynolds et al.(1996)* used these techniques with correlated data.

The variable sampling interval (*VSI*) technique introduced by *Reynolds et al (1988)* determines the sampling rate according to whether a sample point approaches the control limits. If a sample point falls well within the control limits, the next sample is delayed but if it falls within but close to the control limits, the sampling rate is increased.

Reynolds et al. (1988) showed that the *ATS* (average time to signal) for the *VSI* chart is lowest when two sampling intervals are chosen as *0.1* and *1.9* times the sampling interval used for the *FSI* case. To determine how much the current sample mean varies from the target mean two pairs of guidelines I_1 and I_2 each centered at the sample mean μ_0, defined as $I_2 = (\mu_0 - k\sigma_X, \mu_0 + k\sigma_X)$ and $I_1 = (\mu_0 - 3\sigma_X, \mu_0 + k\sigma_X) \cup (\mu_0 - k\sigma_X, \mu_0 + 3\sigma_X)$. The value of k should fall within I_1 and I_2 with equal probability when the process is in control.

When taking into consideration autocorrelation, σ_X is calculated following the methods of *Box and Luceno (1997)* for the time series model that is used to describe the process.

If the current measurement falls relatively far from the target mean *i.e.*, falls in I_1 the shorter sampling interval is used; whereas, if it falls in I_2' the longer interval is used. The structure of the averages control chart with variable control limits is shown in *Figure 10.1*.

Prybutok et al. (1997) investigated the differences in terms of simulated *ATS* performance for *VSI* and *FSI* sampling when the data is positively autocorrelated. They used fixed sampling interval length of $L=10,20,50$ and consequently variable sampling interval at $d_1=0.1L$ and $d_2=1.9L$. They compared two *VSI* and two *FSI* charts. One *VSI* and one *FSI* chart were assigned preset control limits at $\pm 3\sigma_X$ and the other charts were assigned limits calculated from estimates obtained from the process observations using *25* initial samples. To model the correlation structure of the process they used the *AR(1)* stationary time series model.

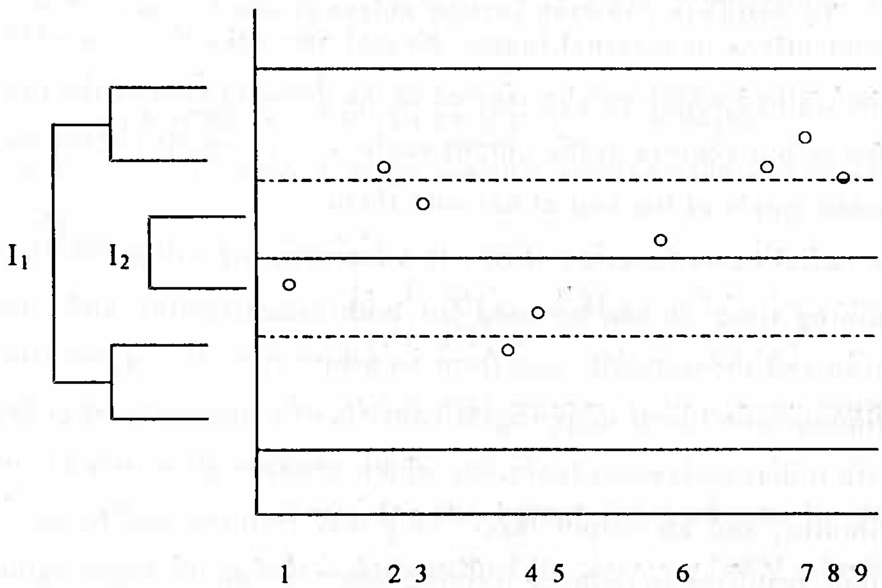

Figure 10.1: **Shewhart averages control chart with variable sampling interval**

They compared via simulation the rate of response for *VSI* control charts with that for *FSI* charts after adjusting them for the amount of autocorrelation and false alarms. They found that for all variable combinations the *ATS* was smaller for the *VSI* than for the corresponding *FSI* and that this difference was more pronounced for moderate levels of correlation. They also compared the performance of charts with preset and calculated limits and found that for moderately correlated processes preset limits are more effective than calculated limits in detecting shifts in the mean but tend to increase the false alarm rate. For highly correlated processes though, the preset limits are not causing more false alarms than the calculated but are not detecting more effectively the shifts either.

10.7 Neural Networks

A neural network consists of a number of simple, highly interconnected processing elements or nodes and is a computational algorithm that processes information by a dynamic response of its processing elements and their connections to external inputs. Neural networks have the ability to learn or to be trained which can be viewed as the development of the function that maps the in put vectors to the output vectors. In Caudill (*1989*) one can find the fundamentals of the neural network theory.

The radial basis function (*RBF*) is a learning algorithm which needs reduced training time. It can be used for both classification and function approximation and theoretically can form an arbitrarily close approximation to any continuous non linear mapping. It consists of a hidden layer composed of nodes with radial activation functions which produce a localized response to input stimulus, and an output layer. This way training can be done one layer at a time, resulting in reduced training times. When the input pattern is close to its center the output of a hidden node produces a greater output. The *RBF* neural network architecture is shown in *Figure 10.2*. Detailed descriptions of the *RBF* network can be found in *Moody and Darken(1989)* and *Renals and Rohwer(1989)* .

The most common radial basis function is a Gaussian kernel function and for every kernel function the center and the width must be determined.

The Gaussian kernel function is defined as follows

$$b_j = exp\left[-h_j\left(X_p - W_j\right)'\left(X_p - W_j\right)\right], j = 1,2,...N$$

where b_j is the output of the jth node in the hidden layer, X_p is the input pattern, W_j is the center of the Gaussian function for node j, h_j is the inverse of the width associated with the kernel function of node j, and N is the number of nodes in the hidden layer.

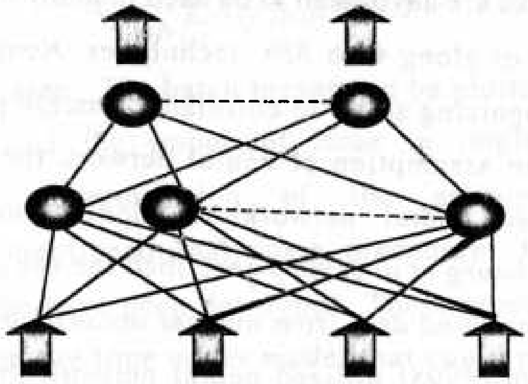

Figure 10.2: **Radial basis function neural network architecture**

The h_j parameters are obtained after the termination of the clustering algorithm by equalizing the average distance between the cluster centers and the training patterns, thus

$$h_j = inverse\left[\left(\frac{1}{M_j}\right)\sum\left(X - W_j\right)\left(X - W_j\right)\right]$$

where θ_j is the set of training parameters grouped with cluster center W_j, and M_j is the number of parameters in θ_j.

The appropriate center for the kernel function can be determined through the conscience function described by *Desieno(1988)* which is added to the *RBF* clustering algorithm to eliminate the problems caused by the absence of equal probability and ordered cluster centers. The number of hidden nodes N is determined by experimentation and rules of thumb.

After the learning in the hidden layer, a supervised learning algorithm is applied to train the weights between the hidden and output nodes. The learning in the output layer is conducted after the hidden layer training is complete.

The values of the output layer nodes can be calculated as

$$c_k = \sum_{j=1}^{N} w_{jk} b_j$$

where c_k is the output of the kth node in the output layer, w_{jk} is the weight from, the jth hidden layer neuron to the kth output layer neuron, and b_j is the output of the jth node in the hidden layer.

Neural networks are advocated to be used in many different areas, one of those is instead of or along with *SPC* techniques. Neural networks are a potential tool for recognizing shifts in correlated process parameters as data independence is not an assumption of neural network theory. *Nieckula and Hryniewicz(1997)* used neural networks in conjunction with averages Shewhart charts. *Hambourg et al.(1996)* described the use of neural networks for monitoring autocorrelated data from nuclear material balances.

Cook and Chiu (1998) utilized neural network theory to develop a method for identifying process shifts and compared that method to *SPC* techniques suggested for use with correlated data. It should be mentioned that no modeling effort is needed to implement the method of neural networks.

The viscosity data set from *Box and Luceno (1997)* and the papermaking data set from *Pandit and Wu(1983)* were used for network development and training. They compared the performance of the neural network model with the performance of the residuals chart. They concluded that the capability of the *RBF* neural network to identify shifts in process mean is a substantial improvement over the *SPC* control charts for correlated data.

Although the use of neural networks has become very popular because of their model free and non parametric nature *Ryan(2000)* points out that their superiority over other techniques has not yet been clearly established. He also claims that they have many negative features, one of those being that when neural networks are repeatedly run on the same data set, different answers are produced . Therefore, he suggests that they should be applied with caution at least until more research is conducted and all such matters are resolved.

10.8 The Batch Means Control Chart

Another approach for monitoring correlated data could be breaking successive groups of sequential observations into batches. *Runger and Willemain (1996)* proposed a control chart based on unweighted batch means (*UBM*) for monitoring autocorrelated process data. The *UBM* chart assigns weights to every point in the batch letting the jth unweighted batch mean be

$$\overline{X}_j = \frac{1}{b}\sum_{i=1}^{b} X_{(j-1)b+i},\, j = 1,2,...$$

where b is the batch size. The batch means can be plotted and analyzed on a Shewhart X-chart, and the important issue in implementing the *UBM* procedure is the determination of the appropriate batch size. *Montgomery(2001)* mentioned that some procedures for determining the appropriate batch size have been developed. These procedures are empirical and do not depend on the time series model that could possibly describe the correlative structure of the process. The time series model though can be used as a guidance. For the *AR(1)* models *Runger and Willemain (1996)* recommended the use of a batch size that reduces the lag *1* autocorrelation of the batch means to approximately *0.10*. This can be done experimentally by starting with $b=1$ and doubling b until the lag *1* autocorrelation of the batch means approaches *0.1*.

The *UBM* control chart is used to break up the autocorrelation of the data without any time series modeling retaining the basic simplicity of just averaging observations to form a point in a control chart. This method is suitable for highly autocorrelated process data such as the output from computer simulation models.

Neural networks [illegible]

[illegible]

data.

Although the [illegible]

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10.8 The Batch Means Control Chart

Another approach for monitoring [illegible]

successive groups of sequential [illegible]

Willemain (1992) proposed a [illegible]

(UBM) for monitoring autocorrelated [illegible]

weights to every point in the batch [illegible]

Appendix A

TABLES

			λ=0,1					λ=0,2				
φ	ψ	c	δ=0	0,5	1,0	2,0	4,0	0	0,5	1,0	2,0	4,0
		3,00	838,6	36,8	11,2	4,6	2,3	559,5	43,5	10,6	3,7	1,8
		3,25	1787,1	48,2	12,8	5,0	2,5	1228,2	64,5	12,9	4,1	2,0
0	-	3,50	4071,6	65,0	14,7	5,5	2,7	2880,4	100,6	15,8	4,6	2,1
		3,75	9864,5	90,6	16,8	6	2,8	7223,3	166,4	19,8	5,0	2,3
		4,00	25273,3	135,7	19,4	6,5	3,0	19361,6	293,7	25,4	5,6	2,4
		3,00	711,2	36,6	11,5	4,9	2,6	481,5	42,5	10,9	4,0	2,1
		3,25	1465,2	47,4	13,1	5,4	2,8	1024,6	62,1	13,1	4,4	2,2
,2	,1	3,50	3209,7	63,2	14,9	5,9	2,9	2323,3	95,4	16,0	4,8	2,4
		3,75	7474,7	87,7	17,1	6,3	3,1	5615,4	154,9	19,9	5,3	2,5
		4,00	18478,1	128,4	19,7	6,7	3,3	14456,6	267,6	25,4	5,8	2,6
		3,00	413,8	34,8	11,5	4,9	2,6	292,6	38,,4	10,9	4,1	2,1
		3,25	767,5	44,3	13,1	5,3	2,7	563,6	54,1	13,0	4,5	2,2
,2	,5	3,50	1498,0	57,6	14,,9	5,8	2,9	1145,9	79,2	15,8	4,9	2,4
		3,75	3079,0	77,6	17,0	6,3	3,1	2461,9	121,4	19,4	5,4	2,5
		4,00	6652,4	110,1	19,5	6,8	3,3	5589,6	195,7	24,3	5,9	2,6
		3,00	276,5	33,4	11,6	4,9	2,6	200,4	35,5	10,9	4,0	2,1
		3,25	474,8	42,0	13,2	5,4	2,7	358,2	48,6	13,0	4,4	2,2
,2	,9	3,50	850,4	53,8	15,0	5,8	2,9	670,3	68,6	15,6	5,0	2,4
		3,75	1590,7	70,1	17,2	6,3	3,1	1315,7	100,4	19,0	5,4	2,5
		4,00	3108,3	93,3	19,7	6,8	3,3	2709,6	153,4	23,5	6,0	2,6
		3,00	566,3	36,4	11,9	5,4	3,0	395,5	41,5	11,4	4,5	2,4
		3,25	1114,8	46,7	13,6	5,8	3,1	807,9	59,6	13,6	4,9	2,5
,4	,1	3,50	2323,5	61,5	15,4	6,3	3,3	1752,7	89,5	16,5	5,4	2,7
		3,75	5129,7	84,0	17,6	6,8	3,5	4039,3	141,6	20,3	5,9	2,8
		4,00	11985,2	121,6	20,2	7,3	3,7	9885,6	237,3	25,6	6,4	3,0
		3,00	209,1	33,0	12,1	5,4	3,0	158,2	34,5	11,5	4,6	2,4
		3,25	340,1	40,9	13,7	5,8	3,1	268,9	46,2	13,5	5,0	2,6
,4	,5	3,50	573,5	51,6	15,5	6,3	3,3	475,8	63,7	16,1	5,5	2,7
		3,75	1005,1	66,5	17,6	6,8	3,5	879,1	90,7	19,4	6,0	2,9
		4,00	1833,2	88,6	19,9	7,3	3,7	1697,3	133,9	23,8	6,5	3,0
		3,00	120,2	30,7	12,2	5,5	3,0	93,9	30,5	11,5	4,7	2,4
		3,25	177,3	37,5	13,8	5,9	3,1	143,2	39,4	13,5	5,1	2,5
,4	,9	3,50	267,9	46,1	15,6	6,4	3,3	225,7	51,7	15,8	5,5	2,7
		3,75	415,8	57,0	17,7	6,9	3,5	359,3	69,4	18,8	6,1	3,0
		4,00	664,2	71,5	20,1	7,4	3,7	621,9	95,7	22,6	6,6	3,1
		3,00	407,1	36,5	13,0	6,2	3,6	302,7	40,8	12,5	5,4	3,0
		3,25	748,6	46,2	14,6	6,6	3,8	583,8	57,1	14,8	5,9	3,2
,6	,1	3,50	1447,4	59,8	16,5	7,1	4,0	1188,8	83,0	17,6	6,4	3,4
		3,75	2946,7	79,8	18,7	7,6	4,2	2559,8	126,5	21,4	7,0	3,6
		4,00	6317,2	112,3	21,2	8,1	4,4	5827,2	203,1	26,5	7,6	3,7
		3,00	109,0	31,7	13,3	6,3	3,6	89,9	31,8	12,7	5,5	3,0
		3,25	156,8	38,3	14,9	6,7	3,8	135,6	40,7	14,8	6,0	3,2
,6	,5	3,50	230,4	46,7	16,7	7,3	4,0	210,2	53,1	17,3	6,5	3,4
		3,75	346,6	58,0	18,3	7,8	4,2	335,5	70,6	20,3	7,0	3,6
		4,00	535,0	74,9	21,2	8,4	4,4	552,2	96,0	24,2	7,6	3,7
		3,00	62,9	28,4	13,5	6,4	3,6	52,8	27,1	12,8	5,6	3,0
		3,25	83,0	33,9	15,1	6,9	3,8	72,4	33,8	14,7	6,1	3,1
,6	,9	3,50	110,7	40,4	16,9	7,4	4,0	100,6	42,1	17,0	6,5	3,4
		3,75	149,6	48,0	18,8	7,9	4,2	140,5	53,1	19,6	7,1	3,6
		4,00	205,1	57,3	21,0	8,5	4,4	193,2	67,9	23,0	7,7	3,7
		3,00	244,5	39,0	15,8	8,2	5,0	208,8	43,8	15,7	7,5	4,4
		3,25	401,5	48,6	17,7	8,8	5,3	374,0	59,4	18,2	8,1	4,6
,8	,1	3,50	684,3	61,0	19,7	9,3	5,5	699,6	83,7	21,4	8,8	4,9
		3,75	1213,3	79,1	22,1	10,0	5,7	1392,8	123,7	25,3	9,5	5,1
		4,00	2240,3	100,0	25,0	10,6	6,0	3026,3	194,8	30,6	10,2	5,3
		3,00	60,8	32,3	16,5	8,4	5,0	56,6	32,1	16,3	7,,8	4,4
		3,25	78,2	38,0	18,4	9,1	5,2	76,7	39,8	18,6	8,5	4,6
,8	,6	3,50	101,4	45,5	20,4	9,7	5,5	105,3	49,8	21,4	9,2	4,9
		3,75	132,8	55,4	22,1	10,3	5,8	146,9	62,3	24,7	9,9	5,2
		4,00	175,9	61,9	24,5	11,0	6,1	208,6	80,2	28,7	10,7	5,4
		3,00	38,9	26,4	16,4	8,7	5,0	36,5	26,1	16,2	7,9	4,4
		3,25	47,1	30,9	18,3	9,4	5,3	46,1	31,8	18,5	8,6	4,6
,8	,9	3,50	57,2	35,9	20,2	10,0	5,6	58,3	38,2	21,0	9,3	5,0
		3,75	69,7	41,2	22,1	10,7	5,8	73,8	45,7	23,7	10,1	5,3
		4,00	85,4	47,1	24,2	11,4	6,1	93,9	55,2	26,9	10,9	5,5

TABLE A.1: *ARL* for the *EWMA* chart of the observations for the *AR(1)* plus a random error model where φ is the autoregressive parameter, $\psi=\sigma^2_\mu/\sigma^2_X$ and δ is the magnitude of the shift.

φ_1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=20	
-0,9	12,61	16,59	23,79	28,04	33,86	37,96	42,98	46,86	51,31	84,67	ARL=100
-0,8	11,94	15,53	21,37	24,82	29,16	32,28	35,80	38,62	41,64	64,72	
-0,7	11,28	14,51	19,24	22,13	25,47	28,02	37,71	33,02	35,36	54,61	
-0,6	10,62	13,55	17,39	19,92	22,61	24,85	27,09	29,13	31,14	48,47	
-0,5	09,95	12,66	15,83	18,13	20,43	22,47	24,45	26,33	28,15	44,34	
-0,4	09,29	11,85	14,53	16,69	18,74	20,65	22,48	24,24	25,95	41,40	
-0,3	08,63	11,12	13,48	15,52	17,44	19,25	20,99	22,67	24,31	39,28	
-0,2	07,96	10,45	12,64	14,60	16,43	18,18	19,87	21,50	23,10	37,77	
-0,1	07,30	09,82	11,94	13,86	15,67	17,39	19,05	20,66	22,23	16,76	
0,0	06,63	09,21	11,34	13,28	15,09	16,81	18,48	20,09	21,67	36,19	
0,1	05,97	08,59	10,79	12,78	14,64	16,41	18,11	19,75	21,35	36,02	
0,2	05,31	07,94	10,23	12,32	14,27	16,12	17,88	19,58	21,24	36,23	
0,3	04,64	07,24	09,62	11,83	13,90	15,86	17,73	19,52	21,26	36,77	
0,4	03,98	06,49	08,91	11,23	13,44	15,54	17,54	19,46	21,32	37,62	
0,5	03,32	05,65	08,05	10,44	12,74	15,01	17,17	19,25	21,26	38,73	
0,6	02,65	04,73	07,00	09,36	11,74	14,09	16,40	18,65	20,84	39,96	
0,7	01,99	03,71	05,72	07,91	10,20	12,55	14,92	17,28	19,61	40,86	
0,8	01,33	02,59	04,15	05,95	07,92	10,03	12,23	14,50	16,82	40,00	
0,9	00,66	01,35	02,26	03,36	04,63	06,06	07,62	09,30	11,09	32,42	
-0,9	14,67	19,66	28,18	33,17	40,02	44,82	50,70	55,20	60,40	98,60	ARL=200
-0,8	14,18	18,36	25,23	29,22	34,26	37,82	41,87	45,04	48,46	73,62	
-0,7	13,40	17,12	22,62	25,90	29,70	32,53	35,52	38,03	40,59	61,00	
-0,6	12,61	15,93	20,35	23,13	26,13	28,54	30,96	33,14	35,27	53,42	
-0,5	11,82	14,83	18,40	20,87	23,37	25,54	27,64	29,62	31,53	48,37	
-0,4	11,03	13,81	16,77	19,05	21,24	23,25	25,17	27,01	28,80	44,80	
-0,3	10,24	12,89	15,44	17,59	19,60	21,50	23,31	25,07	26,77	42,24	
-0,2	09,46	12,07	14,38	16,43	18,35	20,17	21,93	23,62	25,28	40,42	
-0,1	08,67	11,31	13,53	15,54	17,42	19,21	20,93	22,61	24,24	39,23	
0,0	07,88	10,60	12,84	14,86	16,75	18,55	20,28	21,96	23,59	38,58	
0,1	07,09	09,89	12,23	14,33	16,28	18,13	19,90	21,61	23,28	38,44	
0,2	06,30	09,16	11,63	13,86	15,93	17,88	19,73	21,52	23,24	38,78	
0,3	05,52	08,38	10,98	13,38	15,61	17,70	19,68	21,58	23,41	39,55	
0,4	04,73	07,53	10,22	12,78	15,19	17,46	19,62	21,68	23,65	40,73	
0,5	03,94	06,58	09,28	11,95	14,53	17,00	19,36	21,63	23,80	42,28	
0,6	03,15	05,52	08,11	10,79	13,46	16,09	18,65	21,14	23,54	44,06	
0,7	02,36	04,34	06,65	09,15	11,77	14,44	17,11	19,76	22,37	45,62	
0,8	01,58	03,03	04,84	06,92	09,19	11,62	14,14	16,74	19,37	45,32	
0,9	00,79	01,59	02,64	03,92	05,40	07,06	08,87	10,82	12,88	37,36	
-0,9	18,14	23,79	34,07	40,05	48,29	54,01	61,06	66,41	72,60	117,32	ARL=500
-0,8	17,19	22,17	30,41	35,12	41,11	45,27	50,01	53,67	57,62	85,55	
-0,7	16,23	20,62	27,17	30,96	35,39	38,58	42,00	44,77	47,62	69,48	
-0,6	15,28	19,14	24,33	27,46	30,87	33,50	36,17	38,50	40,81	59,94	
-0,5	14,32	17,74	21,86	24,57	27,32	29,64	31,91	34,00	36,03	53,63	
-0,4	13,37	16,44	19,77	22,21	24,58	26,70	28,74	30,67	32,55	49,21	
-0,3	12,41	15,26	18,05	20,33	22,46	24,46	26,36	28,20	29,98	46,04	
-0,2	11,46	14,21	16,68	18,85	20,87	22,78	24,61	26,38	28,10	43,81	
-0,1	10,50	13,28	15,62	17,73	19,70	21,57	23,38	25,12	26,82	42,36	
0,0	09,55	12,43	14,80	16,92	18,91	20,79	22,60	24,35	26,06	41,61	
0,1	08,59	11,61	14,11	16,35	18,41	20,36	22,22	24,02	25,76	41,51	
0,2	07,64	10,78	13,47	15,89	18,11	20,18	22,15	24,03	25,85	42,03	
0,3	06,68	09,89	12,79	15,43	17,87	20,13	22,26	24,28	26,22	43,12	
0,4	05,73	08,92	11,97	14,84	17,52	20,03	22,38	24,61	26,73	44,76	
0,5	04,77	07,82	10,93	13,98	16,90	19,67	22,30	24,79	27,17	46,92	
0,6	03,82	06,58	09,60	12,70	15,77	18,78	21,68	24,47	27,16	49,48	
0,7	02,86	05,19	07,90	10,84	13,89	16,98	20,06	23,10	26,08	51,95	
0,8	01,91	03,63	05,77	08,22	10,91	13,75	16,71	19,74	22,81	52,46	
0,9	00,95	01,90	03,16	04,68	06,44	08,40	10,55	12,85	15,29	44,00	

TABLE A.4: Modified Control Limits for the S^2 –chart with n: the sample size φ:the autoregressive parameter for the $AR(1)$ model.

φ_1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=20	
-0,9	5,02	5,23	5,39	5,51	5,62	5,71	5,79	5,84	5,91	6,32	ARL=1005,895,68
-0,8	4,88	5,15	5,35	5,50	5,61	5,72	5,79	5,87	5,93	6,33	
-0,7	4,75	5,04	5,26	5,40	5,53	5,62	5,72	5,78	5,85	6,26	
-0,6	4,61	4,92	5,14	5,31	5,42	5,52	5,60	5,76	5,74	6,30	
-0,5	4,46	4,79	5,01	5,80	5,29	5,40	5,47	5,55	5,61	6,01	
-0,4	4,31	4,65	4,89	5,04	5,60	5,27	5,35	5,42	5,49	5,89	
-0,3	4,60	4,52	4,75	4,92	5,04	5,50	5,24	5,32	5,38	5,80	
-0,2	3,99	4,38	4,62	4,80	4,93	5,04	5,14	5,22	5,28	5,72	
-0,1	3,82	4,25	4,51	4,70	4,84	4,96	5,05	5,14	5,21	5,68	
0,0	3,64	4,12	4,40	4,61	4,76	4,88	4,98	5,08	5,16	5,65	
0,1	3,46	3,98	4,29	4,51	4,68	4,82	4,93	5,02	5,11	5,62	
0,2	3,25	3,83	4,17	4,42	4,60	4,74	4,87	4,97	5,06	5,60	
0,3	3,05	3,65	4,04	4,30	4,51	4,66	4,79	4,91	5,00	5,56	
0,4	2,83	3,46	3,88	4,17	4,39	4,55	4,69	4,82	4,92	5,52	
0,5	2,58	3,24	3,67	3,99	4,23	4,42	4,57	4,70	4,81	5,45	
0,6	2,30	2,96	3,42	3,75	4,01	4,22	4,38	4,53	4,65	5,36	
0,7	1,99	2,63	3,08	3,43	3,70	3,91	4,11	4,26	4,40	4,19	
0,8	1,63	2,20	2,63	2,95	3,23	3,46	3,66	3,83	3,98	4,87	
0,9	1,15	1,59	1,94	2,22	2,46	2,66	2,85	3,02	3,17	4,16	
-0,9	5,47	5,67	5,83	5,95	6,06	6,14	6,21	6,27	6,33	6,72	ARL=200
-0,8	5,33	5,58	5,77	5,91	6,02	6,11	6,19	6,26	6,32	6,70	
-0,7	5,18	5,47	5,67	5,81	5,92	6,02	6,09	6,16	6,23	6,60	
-0,6	5,02	5,33	5,54	5,68	5,80	5,89	5,96	6,04	6,08	6,47	
-0,5	4,86	5,18	5,39	5,53	5,65	5,74	5,82	5,88	5,95	6,33	
-0,4	4,70	5,02	5,24	5,38	5,50	5,59	5,68	5,74	5,81	6,18	
-0,3	4,53	4,87	5,08	5,24	5,36	5,45	5,53	5,60	5,67	6,07	
-0,2	4,35	4,72	4,94	5,10	5,23	5,33	5,41	5,50	5,56	5,98	
-0,1	4,16	4,57	4,81	4,99	5,13	5,23	5,33	5,41	5,48	5,93	
0,0	3,97	4,43	4,69	4,88	5,03	5,15	5,26	5,34	5,42	5,89	
0,1	3,77	4,28	4,58	4,79	4,96	5,09	5,19	5,29	5,37	5,86	
0,2	3,55	4,12	4,46	4,69	4,88	5,01	5,13	5,24	5,32	5,84	
0,3	3,32	3,94	4,31	4,59	4,78	4,94	5,06	5,17	5,26	5,81	
0,4	3,08	3,74	4,16	4,44	4,66	4,83	4,97	5,09	5,19	5,77	
0,5	2,81	3,50	3,95	4,26	4,50	4,69	4,84	4,96	5,08	5,71	
0,6	2,51	3,20	3,68	4,02	4,28	4,49	4,66	4,80	4,92	5,61	
0,7	2,18	2,85	3,33	3,68	3,95	4,18	4,38	4,53	4,67	5,45	
0,8	1,78	2,39	2,84	3,19	3,47	3,70	3,91	4,08	4,24	5,15	
0,9	1,26	1,73	2,10	2,40	2,65	2,87	3,07	3,24	3,39	4,42	
-0,9	6,03	6,21	6,39	6,50	6,60	6,67	6,74	6,81	6,85	7,22	ARL=500
-0,8	5,87	6,12	6,29	6,42	6,53	6,61	6,69	6,76	6,81	7,17	
-0,7	5,70	5,97	6,17	6,31	6,41	6,49	6,57	6,63	6,69	7,05	
-0,6	5,53	5,82	6,01	6,15	6,26	6,34	6,42	6,48	6,53	6,88	
-0,5	5,35	5,65	5,85	5,98	6,09	6,18	6,25	6,31	6,37	6,71	
-0,4	5,17	5,48	5,67	5,81	5,92	6,01	6,07	6,14	6,19	6,54	
-0,3	4,98	5,30	5,49	5,63	5,75	5,84	5,91	5,98	6,04	6,40	
-0,2	4,78	5,13	5,33	5,48	5,60	5,69	5,78	5,85	5,91	6,30	
-0,1	4,58	4,96	5,19	5,36	5,48	5,58	5,67	5,74	5,81	6,24	
0,0	4,37	4,80	5,05	5,23	5,38	5,49	5,59	5,67	5,74	5,20	
0,1	4,15	4,64	4,93	5,13	5,29	5,42	5,53	5,62	5,69	6,17	
0,2	3,91	4,47	4,80	5,04	5,21	5,35	5,46	5,56	5,65	6,14	
0,3	3,66	4,29	4,67	4,93	5,12	5,27	5,40	5,50	5,59	6,12	
0,4	3,39	4,08	4,50	4,79	5,00	5,17	5,31	5,42	5,52	6,08	
0,5	3,09	3,83	4,28	4,60	4,84	5,03	5,18	5,30	5,42	6,02	
0,6	2,76	3,52	4,01	4,35	4,61	4,82	4,99	5,13	5,26	5,92	
0,7	2,39	3,13	3,63	3,99	4,28	4,51	4,70	4,86	5,00	5,77	
0,8	1,96	2,63	3,10	3,47	3,77	4,01	4,22	4,40	4,56	5,47	
0,9	1,38	1,91	2,30	2,62	2,88	3,12	3,32	3,51	3,67	4,74	

TABLE A.5: Modified Control Limits for the ***R*** –chart with *n*: the sample size, φ:the autoregressive parameter for the *AR(1)* model

σ^2_X/σ^2_{X0}			1		2		3		10	
ξ_1-ξ_0/σ_{X0}	ψ_0	φ								
0,0	,1	,4	**365,8**	370,3	25,82	26,07	12,00	**11,96**	3,34	3,32
			370,0	363,3	32,17	**25,03**	13,02	12,03	**3,14**	3,91
		,8	**367,0**	370,6	**24,90**	26,28	**13,91**	14,80	**5,00**	5,09
			369,5	377,3	44,58	25,84	20,06	14,64	5,17	5,39
	,9	,4	370,9	363,4	31,16	31,88	12,87	12,62	3,15	3,19
			369,9	371,3	30,55	**29,40**	**12,40**	13,09	**2,99**	3,84
		,8	375,5	366,0	**38,51**	42,88	**16,12**	18,33	3,66	3,88
			370,4	367,8	40,64	38,73	16,56	16,97	**3,54**	4,57
1,0	,1	,4	12,08	**11,81**	**9,31**	9,44	7,36	7,35	3,09	3,12
			50,09	12,35	14,40	9,46	8,86	7,52	3,03	3,47
		,8	16,87	**15,63**	**13,28**	13,42	**10,38**	10,78	**4,60**	4,64
			78,30	15,89	25,09	13,40	15,19	10,39	4,86	4,84
	,9	,4	22,78	**21,63**	**12,47**	12,64	**8,32**	8,66	2,96	2,97
			96,58	22,41	18,01	12,77	9,42	8,79	**2,82**	3,44
		,8	83,40	**59,03**	**22,40**	24,69	**12,08**	13,49	3,36	3,55
			207,5	62,63	30,35	23,57	13,66	13,07	**3,26**	4,03
2,0	,1	,4	3,75	**3,74**	**3,77**	**3,77**	**3,68**	3,70	2,64	2,69
			7,31	3,98	5,11	3,95	4,34	4,01	**2,54**	2,95
		,8	4,11	**4,08**	**4,87**	4,96	**5,03**	5,23	**3,72**	3,79
			11,76	4,18	8,58	5,07	7,16	5,33	3,93	4,07
	,9	,4	5,43	**5,37**	**4,78**	**4,81**	**4,23**	4,31	2,58	2,58
			17,38	5,94	7,22	5,16	4,95	4,90	**2,50**	3,04
		,8	12,05	**9,62**	**7,55**	8,23	**5,83**	6,39	2,71	2,88
			50,48	13,23	12,64	9,43	7,05	7,33	**2,64**	3,48
3,0	,1	,4	**1,95**	1,96	2,08	**2,07**	2,18	**2,17**	2,15	2,14
			2,14	2,43	2,26	2,36	2,27	2,50	2,12	2,47
		,8	2,18	**2,03**	**2,22**	2,29	**2,45**	2,54	**2,73**	2,75
			2,79	2,50	2,86	2,57	3,04	2,81	2,82	3,16
	,9	,4	**2,25**	2,34	**2,32**	2,38	**2,30**	2,38	2,11	2,08
			3,44	3,07	2,79	2,88	2,59	2,93	**2,06**	2,55
		,8	**1,75**	1,83	**2,13**	2,26	**2,28**	2,48	2,06	2,17
			4,49	4,55	2,98	3,90	2,70	3,75	**2,01**	2,66

Upper le ft entries are the steady state *ARL* for shceme *a*
Upper right entries are the steady state *ARL* for shceme *b*
Lower left entries are the steady state *ARL* for shceme *c*
Lower right entries are the steady state *ARL* for shceme *d*

TABLE A.6: Steady –State *ARL* of Four Charts for Monitoring the Process Mean and Variance when the increase in σ^2_X is caused by an increase in $\boldsymbol{\sigma^2_\gamma}$ where φ is the autoregressive parameter for the *AR(1)* plus a random error model and $\psi_0=\sigma^2_{\mu 0}/\sigma^2_{X0}$ ($\lambda=0,2$ for both *EWMA* Charts).

	σ^2_X/σ^2_{X0}		1		2		3		10	
ξ_1-ξ_0/σ_{X0}	ψ_0	φ								
0,0	,1	,4	367,9	365,8	30,06	30,72	12,61	12,53	3,08	3,05
			317,7	365,5	29,14	**27,76**	**12,01**	12,31	**2,94**	3,65
		,8	**364,4**	366,3	30,73	31,75	12,26	12,61	2,99	2,97
			370,2	375,0	28,08	**27,12**	**11,28**	12,16	**2,82**	3,61
	,9	,4	368,9	371,8	25,96	27,24	10,57	11,09	2,82	2,82
			370,3	**364,2**	21,90	**21,77**	**9,23**	10,13	**2,62**	3,47
		,8	374,4	376,2	10,09	10,29	5,03	5,24	2,07	2,10
			369,9	371,5	**8,33**	9,34	4,51	5,50	**1,97**	2,60
1,0	,1	,4	11,97	**11,88**	8,65	**8,63**	**6,57**	6,77	2,79	2,82
			49,57	12,39	13,27	8,80	7,62	6,94	**2,72**	3,18
		,8	16,78	**15,49**	**10,18**	10,27	7,24	**7,21**	2,84	2,83
			78,52	16,06	15,13	10,42	8,08	7,51	**2,71**	3,25
	,9	,4	23,25	**21,31**	11,16	11,16	**6,97**	7,17	**2,60**	2,61
			96,46	22,49	13,69	**11,10**	7,23	7,19	2,47	3,03
		,8	82,07	**59,02**	7,56	8,08	4,19	4,43	1,95	1,99
			217,0	63,50	**7,08**	7,69	**3,96**	4,80	**1,89**	2,37
2,0	,1	,4	**3,74**	**3,74**	**3,42**	3,48	3,29	**3,27**	2,39	**2,34**
			7,16	3,97	4,49	3,69	3,66	3,58	2,35	2,71
		,8	4,36	4,03	**3,68**	3,81	**3,37**	3,47	2,38	2,41
			11,56	4,18	5,24	4,09	3,94	3,86	**2,31**	2,77
	,9	,4	**5,44**	5,46	**4,22**	4,27	**3,63**	3,70	2,25	2,31
			17,24	5,93	5,67	4,82	4,07	4,26	**2,23**	2,67
		,8	12,07	9,50	**3,74**	4,17	**2,67**	2,91	1,79	1,82
			51,14	13,21	3,91	5,02	2,72	3,63	**1,74**	2,16
3,0	,1	,4	**1,97**	1,98	**1,95**	1,96	**1,99**	**1,99**	1,95	**1,93**
			2,16	2,43	2,06	2,29	2,05	2,35	1,89	2,25
		,8	2,19	**2,04**	**1,99**	2,03	**2,02**	2,07	1,93	1,96
			2,80	2,48	2,24	2,40	2,13	2,44	**1,91**	2,29
	,9	,4	**2,25**	2,33	**2,14**	2,23	**2,12**	2,16	1,89	1,92
			3,41	3,12	2,44	2,78	2,24	2,71	**1,84**	2,28
		,8	**1,76**	1,83	**1,71**	1,86	**1,65**	1,79	**1,54**	1,60
			4,53	4,48	1,78	2,72	1,67	2,42	**1,54**	1,90

Upper le ft entries are the steady state *ARL* for shceme *a*
Upper right entries are the steady state *ARL* for shceme *b*
Lower left entries are the steady state *ARL* for shceme *c*
Lower right entries are the steady state *ARL* for shceme *d*

TABLE A.7: Steady –State *ARL* of Four Charts for Monitoring the Process Mean and Variance when the increase in σ^2_X is caused by an increase in σ^2_ε where φ is the autoregressive parameter for the *AR(1)* plus a random error model and $\psi_0=\sigma^2_{\mu 0}/\sigma^2_{X0}$ (λ=0,2 for both *EWMA* Charts).

δ	θ	ARL	SRL	ARL	SRL	ARL	SRL	ARL	SRL	ARL	SRL
		φ=,950		φ=,475		φ=,000		φ=-,450		φ=-,450	
0,0	,900	370,38	369,88	370,38	369,88	370,38	369,88	370,38	369,88	370,38	369,88
0,5		272,90	278,58	10,53	4,43	5,00	1,55	3,06	0,80	1,50	0,50
1,0		135,35	150,34	4,74	1,77	2,76	0,80	1,94	0,44	1,00	0,04
1,5		54,98	72,95	3,01	1,13	2,01	0,58	1,59	0,49	1,00	0,00
2,0		18,53	31,26	2,18	0,83	1,63	0,50	1,24	0,43	1,00	0,00
2,5		5,76	10,77	1,69	0,65	1,36	0,48	1,05	0,22	1,00	0,00
3,0		2,38	3,12	1,39	0,52	1,15	0,36	1,01	0,07	1,00	0,00
0,0	,450	370,38	369,88	370,38	369,88	370,38	369,88	370,38	369,88	370,38	369,88
0,5		349,69	356,77	164,40	164,00	45,64	43,74	8,95	6,49	1,76	0,44
1,0		274,69	318,63	48,67	48,36	8,04	6,00	2,74	0,98	1,06	0,23
1,5		147,60	244,01	16,85	16,57	3,45	1,78	1,89	0,54	1,00	0,01
2,0		43,51	132,79	7,01	6,69	2,24	0,94	1,54	0,50	1,00	0,00
2,5		6,61	45,22	3,49	3,09	1,71	0,66	1,27	0,44	1,00	0,00
3,0		1,30	9,44	2,08	1,58	1,40	0,52	1,09	0,28	1,00	0,00
0,0	,000	370,38	369,88	370,38	369,88	370,38	369,88	370,38	369,88	370,38	369,88
0,5		330,95	357,37	253,13	253,58	155,22	154,72	65,54	64,54	2,67	1,27
1,0		138,84	267,19	117,96	120,19	43,89	43,39	11,44	10,28	1,42	0,49
1,5		11,21	75,75	52,01	55,80	14,90	14,46	3,88	2,68	1,04	0,19
2,0		1,08	6,21	22,64	27,06	6,30	5,78	2,20	1,06	1,00	0,02
2,5		1,00	0,15	9,57	13,37	3,24	2,70	1,64	0,63	1,00	0,00
3,0		1,00	0,00	4,02	6,42	2,00	1,41	1,35	0,49	1,00	0,00
0,0	-,450	370,38	0,999	370,38	369,88	370,38	369,88	370,38	369,88	370,38	369,88
0,5		268,11	1,080	271,96	273,55	210,64	210,58	151,73	151,22	24,19	23,43
1,0		16,69	1,924	137,62	144,63	78,83	73,41	42,13	41,60	3,44	2,51
1,5		1,01	6,757	59,61	71,77	30,57	31,62	14,26	13,71	1,65	0,66
2,0		1,00	5,755	21,92	34,22	12,74	13,91	6,01	5,45	1,22	0,42
2,5		1,00	0,154	6,70	14,49	5,67	6,60	3,12	2,54	1,04	0,20
3,0		1,00	0,00	2,11	5,11	2,77	3,23	1,95	1,34	1,00	0,06
0,0	-,900	370,38	0,999	370,38	369,8	370,38	369,88	370,38	369,88	370,38	369,88
0,5		42,75	3,878	265,34	276,67	221,02	223,09	184,67	184,64	147,52	147,00
1,0		1,00	0,038	108,52	142,84	82,95	89,30	60,11	60,70	40,04	39,51
1,5		1,00	0,00	22,85	57,79	27,42	34,99	21,19	22,00	13,40	12,86
2,0		1,00	0,00	2,79	9,56	8,11	12,59	8,37	8,91	5,64	5,08
2,5		1,00	0,00	1,13	0,89	2,71	3,91	3,82	3,94	2,95	2,35
3,0		1,00	0,00	1,01	0,14	1,42	1,26	2,10	1,91	1,87	1,23

TABLE A.8: The *ARL* and *SRL* Values for the Residuals *X*-chart for the *ARMA(1,1)* model where φ is the autoregressive parameter, θ is the moving average parameter and δ is the magnitude of the shift.

	δ	0,0	0,5	1,0	1,5	2,0	2,5	3,0	4,0	5,0
c	φ									
					ψ=0,1					
2,859	0,0	370,4	35,56	9,60	5,13	3,54	2,75	2,27	1,75	1,45
2,859		370,4	35,51	9,52	5,15	3,54	2,72	2,27	1,75	1,45
2,859	0,2	370,4	37,26	9,97	5,28	3,61	2,78	2,30	1,76	1,45
2,904		370,7	37,59	9,91	5,26	3,63	2,80	2,31	1,77	1,47
2,859	0,4	370,4	40,06	10,57	5,49	3,71	2,83	2,32	1,76	1,45
2,973		370,2	39,37	10,44	5,47	3,74	2,86	2,36	1,80	1,50
2,859	0,6	370,4	45,48	11,67	5,83	3,84	2,89	2,35	1,77	1,44
3,094		370,2	44,55	11,55	5,81	3,94	3,00	2,47	1,88	1,56
2,859	0,8	370,4	60,41	14,47	6,48	4,02	2,95	2,37	1,77	1,44
3,297		369,6	55,96	14,04	6,73	4,37	3,27	2,64	1,98	1,63
					ψ=0,5					
2,859	0,0	370,4	35,56	9,60	5,13	3,54	2,74	2,27	1,75	1,45
2,859		370,7	35,44	9,57	5,10	3,54	2,77	2,26	1,74	1,45
2,859	0,2	370,4	44,02	11,50	5,87	3,91	2,95	2,39	1,78	1,45
3,095		368,8	42,96	11,48	5,87	3,92	3,01	2,46	1,87	1,55
2,859	0,4	370,4	57,49	14,69	7,02	4,44	3,22	2,53	1,81	1,44
3,391		370,2	53,68	14,09	6,91	4,49	3,35	2,70	2,02	1,67
2,859	0,6	370,4	81,78	21,12	9,21	5,33	3,61	2,69	1,81	1,40
3,797		369,5	73,43	19,10	8,75	5,48	3,94	3,10	2,28	1,84
2,859	0,8	370,4	136,30	39,72	15,31	7,33	4,18	2,79	1,73	1,32
4,375		370,1	107,41	30,77	13,18	7,39	5,02	3,80	2,64	2,11
					ψ=0,9					
2,859	0,0	370,4	35,56	9,60	5,13	3,54	2,74	2,27	1,75	1,45
2,859		369,0	35,58	9,64	5,12	3,54	2,74	2,27	1,75	1,45
2,859	0,2	370,4	50,63	13,07	6,47	4,21	3,12	2,49	1,81	1,44
3,266		370,4	49,87	12,78	6,50	4,26	3,19	2,63	1,96	1,62
2,859	0,4	370,4	73,66	19,05	8,68	5,24	3,65	2,77	1,84	1,39
3,750		369,9	68,42	18,04	8,40	5,28	3,88	3,07	2,25	1,83
2,859	0,6	370,4	111,62	31,20	13,15	7,15	4,51	3,12	1,80	1,28
4,375		370,2	97,39	27,46	12,23	7,21	4,92	3,76	2,64	2,10
2,859	0,8	370,4	182,82	64,43	26,28	12,20	6,20	3,42	1,49	1,08
5,203		370,9	148,30	48,13	21,54	11,41	7,25	5,12	3,29	2,52

Upper entries are the ARL for the EWMA Chart of the residuals
Lower entries are the ARL for the EWMA Chart of the observations

TABLE A.9: Steady –State *ARL* for the *EWMA* Chart of Residuals and of Observations for the *AR(1)* plus a random error model where φ is the autoregressive parameter, $\psi = \sigma^2_{\mu}/\sigma^2_X$ and δ is the magnitude of the shift (λ=0,2 for both *EWMA* Charts)

δ	θ	ρ	SCC	Xsp	EWMA λ=0.1	EWMAST λ=0.1	EWMAST λ=0.2	X
					φ=0,950			
0,0	,900	,073	370,38	370,82	366,37	1379,47	763,08	368,62
0,5			272,90	163,31	32,81	111,81	102,78	169,99
1,0			135,35	48,53	15,94	22,40	22,19	50,19
2,0			18,53	6,91	5,54	6,82	5,01	7,36
3,0			2,38	2,02	3,49	4,16	2,89	2,05
0,0	,450	,824	370,38	392,89	362,87	3227,38	2424,48	877,87
0,5			349,69	262,72	232,22	1460,09	1133,17	488,69
1,0			274,69	108,71	93,81	413,09	354,05	184,66
2,0			43,51	20,79	23,61	63,65	55,94	31,56
3,0			1,30	2,23	8,79	19,28	14,55	
0,0	,000	,950	370,38	369,15	365,16			
0,5			330,95	259,73	245,67			
1,0			138,84	118,92	107,83			
2,0			1,08	22,44	27,79			
3,0			1,00	1,43	10,01			
0,0	-,450	,971	370,38	390,33	366,87	3383,52	2691,28	1575,05
0,5			268,11	268,04	247,44	1701,00	1433,31	850,97
1,0			16,69	121,17	109,73	508,81	459,38	323,97
2,0			1,00	24,82	28,71	84,73	76,85	51,95
3,0			1,00	1,01	10,05	23,66	19,28	10,49
0,0	-,900	,975	370,38	385,13	366,44	3390,75	2708,74	1623,09
0,5			42,75	267,86	248,74	1703,72	1440,28	877,18
1,0			1,00	123,08	11,62	511,32	464,28	332,13
2,0			1,00	25,68	29,13	85,18	77,67	53,25
3,0			1,00	1,00	10,28	23,78	19,49	10,58
					φ=0,475			
0,0	,900	-,255	370,38	394,29	377,50	388,06	368,41	395,04
0,5			10,53	166,97	7,33	7,50	9,37	163,57
1,0			4,74	47,31	3,79	3,82	4,00	43,06
2,0			2,18	5,83	2,06	2,07	2,08	5,44
3,0			1,39	1,80	1,55	1,52	1,45	,176
0,0	,450	,025	370,38	390,05	383,78	873,56	593,67	362,37
0,5			164,40	169,16	31,42	40,13	48,16	157,14
1,0			48,67	43,76	10,52	12,25	11,69	45,99
2,0			7,01	6,71	4,40	4,74	3,86	6,20
3,0			2,08	2,05	2,87	3,07	2,43	1,98
0,0	,000	,475	370,38	365,34	376,53			
0,5			253,13	166,77	70,05			
1,0			117,96	51,05	20,69			
2,0			22,64	8,69	7,16			
3,0			4,02	2,50	4,28			
0,0	-,450	,689	370,38	383,21	370,17	1301,46	898,88	441,13
0,5			271,96	188,41	81,10	160,00	187,33	196,45
1,0			137,62	59,99	25,06	36,53	41,98	63,40
2,0			21,92	11,19	8,39	10,03	9,19	10,83
3,0			2,11	2,83	4,80	5,67	4,40	2,77
0,0	-,900	,737	370,38	382,60	362,78	1298,24	944,76	455,18
0,5			265,34	190,65	85,90	166,81	198,65	211,91
1,0			108,52	60,64	25,49	38,62	44,86	66,35
2,0			2,79	11,26	8,56	10,48	9,65	11,56
3,0			1,01	3,01	4,97	5,85	4,56	2,93

TABLE A.10 : The *ARL* Values for the Residuals *X*-chart (*SCC*), the *X*-chart and the *EWMA* chart of the observations using the correct variance , the *EWMAST* and the X-chart of the observations for the *ARMA(1,1)* model

δ	θ	ρ	SCC	Xsp	EWMA λ=0.1	EWMAST λ=0.1	EWMAST λ=0.2	X
φ=0,000								
0,0	,900	-,497	370,38	377,69	372,92	354,86	359,61	397,40
0,5			5,00	143,98	5,01	5,04	6,60	151,91
1,0			2,76	44,20	2,76	2,76	2,96	43,73
2,0			1,63	5,64	1,60	1,63	1,67	5,51
3,0			1,15	1,64	1,15	1,14	1,15	1,69
0,0	,450	-,374	370,38	375,04	390,56	426,22	426,22	400,45
0,5			45,64	161,72	11,15	12,79	12,79	153,56
1,0			8,04	44,85	4,89	4,65	4,65	44,09
2,0			2,24	5,82	2,54	2,19	2,19	5,72
3,0			1,40	1,72	1,81	1,55	1,55	1,71
0,0	,000	,000	370,38	370,38	369,00			
0,5			152,22	152,22	28,19			
1,0			43,89	43,89	9,73			
2,0			6,30	6,30	4,18			
3,0			2,00	2,00	2,76			
0,0	-,450	,374	370,38	381,31	383,02	1013,49	890,53	384,01
0,5			210,64	170,85	45,67	67,49	84,55	161,19
1,0			18,83	49,01	14,53	18,15	18,85	49,07
2,0			12,74	8,12	5,59	6,21	5,13	7,70
3,0			2,77	2,26	3,58	3,85	3,01	2,20
0,0	-,900	,497	370,38	381,94	382,09	1037,22	720,56	404,17
0,5			221,02	171,01	51,95	78,36	100,21	169,85
1,0			82,95	52,82	16,04	20,38	21,82	51,47
2,0			8,11	8,38	5,97	6,76	5,72	8,49
3,0			1,42	2,58	3,78	4,15	3,23	2,32
φ=-0,475								
0,0	,900	-,737	370,38	378,60	378,17	411,85	399,53	445,84
0,5			3,06	144,60	4,11	4,07	4,86	160,80
1,0			1,94	41,85	2,36	2,30	2,45	47,36
2,0			1,24	6,63	1,39	1,42	1,47	6,03
3,0			1,01	1,55	1,01	1,03	1,05	1,63
0,0	,450	-,689	370,38	392,45	392,76	438,43	383,62	435,71
0,5			8,95	153,45	5,96	6,03	6,47	158,76
1,0			2,74	44,14	3,10	3,13	3,00	46,25
2,0			1,54	6,53	1,76	1,75	1,65	6,06
3,0			1,09	1,61	1,22	1,24	1,15	1,64
0,0	,000	-,475	370,3	389,84	399,42			
0,5			65,54	159,86	13,71			
1,0			11,44	45,72	5,56			
2,0			2,20	5,94	2,70			
3,0			1,35	1,79	1,97			
0,0	-,450	-,025	370,38	365,65	375,57	840,61	576,52	357,41
0,5			151,73	154,38	26,75	35,18	41,99	152,73
1,0			42,13	44,18	9,89	11,29	10,56	43,13
2,0			6,01	6,16	4,16	4,46	3,65	7,01
3,0			1,95	1,91	2,70	2,92	2,33	2,95
0,0	-,900	,255	370,38	373,96	392,93	905,32	629,58	375,37
0,5			184,67	164,29	36,54	50,32	64,04	159,42
1,0			60,11	45,92	12,036	14,61	14,36	46,26
2,0			8,37	7,00	4,83	5,35	4,41	6,90
3,0			2,10	2,13	3,13	3,41	2,71	2,11

TABLE A.10 continued: The *ARL* Values for the Residuals *X*-chart (*SCC*), the *X*-chart and the *EWMA* chart of the observations using the correct variance , the *EWMAST* and the X-chart of the observations for the *ARMA(1,1)* model .

δ	θ	ρ	SCC	X_σ	$EWMA_\sigma$ $\lambda=0.1$	EWMAST $\lambda=0.1$	EWMAST $\lambda=0.2$	X
				φ=-0.950				
0,0	,900	-,975	370,38	366,86	369,74	1330,82	1301,31	1582,92
0,5			1,50	138,59	3,36	3,56	4,23	517,36
1,0			1,00	53,16	1,98	2,04	2,17	145,27
2,0			1,00	6,58	1,00	1,24	1,31	15,12
3,0			1,00	1,00	1,00	1,00	1,00	1,53
0,0	,450	-,971	370,38	382,39	387,92	1005,58	1065,87	1537,97
0,5			1,76	141,13	3,59	3,71	4,37	497,90
1,0			1,06	53,50	2,01	2,11	2,22	146,76
2,0			1,00	6,83	1,00	1,28	1,33	15,20
3,0			1,00	1,00	1,00	1,00	1,00	1,53
0,0	,000	-,950	370,38	365,38	361,76			
0,5			2,67	141,76	4,42			
1,0			1,42	53,97	2,32			
2,0			1,00	6,11	1,24			
3,0			1,00	1,03	1,00			
0,0	-,450	-,824	370,38	388,63	390,63	649,75	521,67	821,18
0,5			24,19	170,76	8,34	8,60	9,05	326,87
1,0			3,44	58,52	3,81	3,98	3,43	101,01
2,0			1,22	7,68	2,04	2,06	1,80	9,99
3,0			1,00	1,34	1,44	1,50	1,27	1,65
0,0	-,900	-,073	370,38	382,56	364,04	833,08	569,25	360,31
0,5			147,52	158,10	25,90	34,21	39,98	158,18
1,0			40,04	47,00	9,15	10,95	10,22	44,95
2,0			5,64	6,50	4,03	4,35	3,55	6,32
3,0			1,87	2,02	2,66	2,88	2,28	1,94

TABLE A.10 continued: The *ARL* Values for the Residuals *X*-chart (*SCC*), the *X*-chart and the *EWMA* chart of the observations using the correct variance , the *EWMAST* and the X-chart of the observations for the *ARMA(1,1)* model with φ: the autoregressive parameter, θ: the moving average parameter δ: magnitude of the shift ρ: the first lag autocorrelation

φ	Optimal for	Chart	ψ=0.5			ψ=0.9		
			δ					
			0.5	**1.0**	**2.0**	**0.5**	**1.0.**	**2.0**
0.4	0.5	**CUSUM-Obs**	36,05	14,68	6,76	43,14	17,71	8,01
		EWMA-Obs	36,98	14,06	6,21	44,90	16,48	7,16
		CUSUM-Res	36,44	13,76	5,92	43,58	18,74	8,43
		EWMA-Res	36,69	13,97	6,02	44,92	16,55	6,85
	1.0	**CUSUM-Obs**	40,33	12,90	5,21	49,15	15,95	6,38
		EWMA-Obs	40,48	13,27	5,18	51,63	16,16	6,00
		CUSUM-Res	40,48	13,01	5,11	51,33	15,80	5,81
		EWMA-Res	42,08	13,27	5,06	53,17	16,28	5,82
	2.0	**CUSUM-Obs**	77,63	17,13	4,07	101,10	25,10	4,95
		EWMA-Obs	75,72	17,59	4,24	89,75	22,31	5,06
		CUSUM-Res	86,45	18,78	4,05	96,72	22,57	4,83
		EWMA-Res	74,69	17,70	4,33	84,24	21,22	5,21
		Shewhart-Obs	160,89	46,95	7,22	162,47	48,17	7,75
		Shwehart-Res	205,36	74,35	11,95	232,20	98,91	17,21
0.4	0.5	**CUSUM-Obs**	71,31	29,44	13,00	96,28	41,30	18,36
		EWMA-Obs	78,13	27,06	10,09	108,40	38,58	13,55
		CUSUM-Res	71,39	28,02	10,87	97,68	38,22	14,35
		EWMA-Res	80,28	26,86	8,84	113,20	39,20	12,22
	1.0	**CUSUM-Obs**	83,30	26,77	8,27	103,44	37,96	14,57
		EWMA-Obs	78,13	27,06	10,09	108,40	38,58	13,55
		CUSUM-Res	81,47	26,16	8,39	97,68	38,22	14,35
		EWMA-Res	80,28	26,86	8,84	113,20	39,20	12,22
	2.0	**CUSUM-Obs**	135,28	38,63	6,77	152,25	49,48	10,90
		EWMA-Obs	127,23	36,24	7,31	148,30	48,13	11,41
		CUSUM-Res	135,76	37,07	6,35	161,97	51,87	9,82
		EWMA-Res	119,09	34,21	7,27	140,61	46,23	11,28
		Shewhart-Obs	176,27	56,35	9,49	187,08	65,77	12,26
		Shwehart-Res	291,82	173,91	46,60	317,17	209,41	50,37

TABLE A.11: Steady State ARL values for the *CUSUM* and the *EWMA* charts for specific shifts and for various values of the autoregressive parameter φ, ψ: σ^2_μ/σ^2_X and δ : the magnitude of the shift

σ^2_X/σ^2_{X0}			1,5		2		3		5		7		9		11	
c		φ														
									ψ_0=0,1							
0,944	2,859	0,0	**62,60**	91,68	**26,05**	44,77	**11,41**	21,12	5,93	10,46	4,33	7,23	3,58	5,56	3,15	4,79
3,000	4,219		70,75	75,21	29,57	33,24	11,87	14,53	**5,53**	6,98	**3,94**	5,00	**3,14**	4,14	**2,72**	3,62
0,944	2,859	0,2	**60,50**	72,50	**26,02**	35,42	**11,45**	17,33	5,98	9,12	4,35	6,56	3,64	5,20	3,18	4,49
3,000	4,181		71,56	92,47	29,31	42,07	12,24	17,81	**5,63**	8,30	**3,97**	5,69	**3,19**	4,49	**2,82**	3,79
0,944	2,859	0,4	59,79	**59,32**	**26,03**	29,36	**12,03**	14,84	6,34	8,30	4,66	6,16	3,86	5,21	3,38	4,53
3,000	4,135		72,39	115,0	31,61	57,11	13,34	23,98	**6,11**	1033	**4,31**	6,91	**3,46**	5,27	**3,01**	4,39
0,944	2,859	0,6	62,51	**48,88**	27,32	**24,98**	**12,76**	13,82	**7,12**	8,34	5,20	6,42	4,39	5,36	3,81	4,72
3,000	4,085		77,97	153,0	35,12	83,37	15,27	34,45	7,36	15,65	**5,16**	9,93	**4,04**	7,14	**3,45**	5,70
0,944	2,859	0,8	69,31	**44,04**	32,73	**24,64**	16,21	**14,41**	**9,14**	9,29	**6,80**	7,23	**5,63**	6,19	4,92	5,55
3,000	4,047		94,32	229,1	45,70	153,8	20,71	82,36	10,37	36,69	7,16	21,41	5,73	14,89	**4,79**	11,28
									ψ_0=0,9							
0,944	2,859	0,0	**60,12**	92,57	**25,70**	45,42	**11,27**	21,22	5,84	10,33	4,31	7,12	3,53	5,67	3,14	4,70
3,000	4,219		71,06	76,35	29,62	33,29	12,04	14,78	**5,54**	7,11	**3,88**	5,09	**3,17**	4,07	**2,71**	3,58
0,944	2,859	0,2	**60,89**	91,00	**25,76**	44,79	**11,53**	20,62	5,87	10,18	4,34	7,10	3,59	5,58	3,10	4,75
3,000	3,833		70,76	78,35	29,52	33,63	12,10	14,20	**5,52**	6,96	**3,87**	4,91	**3,18**	4,00	**2,76**	3,45
0,944	2,859	0,4	**61,83**	89,03	**26,84**	44,04	11,85	20,94	5,93	10,26	4,43	7,14	3,65	5,60	3,20	4,67
3,000	3,388		72,84	77,60	30,95	34,37	**11,62**	14,49	**5,78**	6,82	**4,04**	4,80	**3,21**	3,83	**2,77**	3,36
0,944	2,859	0,6	**68,04**	90,45	**28,84**	44,76	**12,55**	21,27	6,38	10,63	4,60	7,43	3,80	5,82	3,33	4,89
3,000	2,872		76,83	82,21	32,89	36,99	13,36	15,17	**6,11**	7,05	**4,22**	4,90	**3,39**	3,90	**2,87**	3,35
0,944	2,859	0,8	**78,80**	92,74	35,18	47,22	**14,86**	22,52	7,39	11,13	5,18	8,02	4,23	6,25	3,70	5,38
3,000	2,242		90,93	102,1	40,66	**16,25**	16,66	18,90	**7,21**	8,35	**4,93**	5,52	**3,83**	4,36	**3,23**	3,64

Upper le ft entries are the ARL for the EWMA Chart of Logs of Squared residuals
Upper right entries are the ARL for the EWMA Chart of Residuals
Lower left entries are the ARL for the Shewhart Chart of Residuals
Lower right entries are the ARL for the Moving Range Chart

TABLE A.12: Steady –State *ARL* of Four Charts for Monitoring the Process Variance when the increase in σ^2_X is caused by an increase in σ^2_Y where φ is autoregressive parameter for the *AR(1)* plus a random error model, $\psi_0=\sigma^2_{\mu 0}/\sigma^2_{X0}$ ($\lambda=0,2$ for both *EWMA* Charts ; the In-control $ARL=370,4$ for all Charts)

σ^2_X/σ^2_{X0}			1,5		2		3		5		7		9		11	
c		φ														
							ψ_0=0,1									
0,944	2,859	0,0	**61,57**	92,40	25,97	45,12	**11,29**	20,93	5,89	10,43	4,36	7,17	3,60	5,66	3,17	4,76
3,000	4,219		69,92	75,07	**29,09**	33,80	12,15	14,49	**5,54**	7,06	**3,88**	5,03	**3,15**	4,13	**2,73**	3,59
0,944	2,859	0,2	**59,32**	95,23	**25,79**	46,35	**11,54**	21,20	5,74	10,64	4,33	7,31	3,58	5,68	3,17	4,74
3,000	4,172		70,12	73,70	29,46	31,71	12,14	13,97	**5,56**	6,91	**3,89**	5,00	**3,18**	4,05	**2,72**	3,49
0,944	2,859	0,4	**60,77**	99,48	**25,33**	48,55	**11,43**	22,11	5,73	10,74	4,30	7,41	3,61	5,77	3,13	4,83
3,000	4,137		70,63	72,21	29,72	31,72	11,74	13,85	**5,54**	6,84	**3,91**	4,90	**3,20**	4,01	**2,73**	3,45
0,944	2,859	0,6	**60,85**	104,3	**25,73**	51,47	**11,33**	23,27	5,76	10,97	4,28	7,55	3,58	5,80	3,12	4,81
3,000	4,086		69,67	68,74	28,97	30,89	11,81	12,30	**5,57**	6,77	**3,88**	4,82	**3,15**	3,94	**2,72**	3,43
0,944	2,859	0,8	**59,86**	108,9	**24,69**	54,43	**11,07**	24,39	5,80	11,34	4,30	7,56	3,51	5,85	3,12	4,82
3,000	4,044		67,99	68,60	28,55	29,75	11,65	13,21	**5,45**	6,54	**3,82**	4,74	**3,10**	3,84	**2,71**	3,41
							ψ_0=0,9									
0,944	2,859	0,0	**60,84**	93,63	**26,23**	45,30	**11,41**	21,12	5,87	10,24	4,30	7,17	3,62	5,59	3,19	4,76
3,000	4,213		70,79	76,09	29,35	33,16	11,98	14,46	**5,55**	7,00	**3,88**	5,00	**3,13**	4,09	**2,74**	3,54
0,944	2,859	0,2	**54,70**	112,1	**23,59**	56,69	**10,63**	25,75	5,60	11,70	4,21	7,81	3,50	5,89	3,07	4,96
3,000	3,828		65,36	59,69	26,82	26,01	11,25	11,48	**5,39**	5,95	**3,76**	4,33	**3,05**	3,61	**2,67**	3,20
0,944	2,859	0,4	**43,96**	124,2	**18,65**	63,70	8,82	28,34	4,95	12,10	3,86	7,85	3,26	5,91	2,86	4,84
3,000	3,388		52,18	43,59	21,49	18,73	9,17	**8,79**	**4,71**	4,87	**3,46**	3,67	**2,85**	3,13	**2,51**	2,82
0,944	2,859	0,6	**28,69**	119,3	**12,90**	60,10	6,83	25,65	4,20	10,66	3,28	6,72	2,89	5,02	2,63	4,18
3,000	2,859		34,83	29,29	14,55	12,72	6,84	**6,55**	**3,85**	**3,85**	**2,88**	3,02	**2,52**	2,61	**2,22**	3,40
0,944	2,859	0,8	**15,44**	86,83	**7,91**	39,93	4,72	15,45	6,84	6,84	2,67	4,52	2,40	**3,57**	2,19	3,08
3,000	4,047		18,38	15,98	8,50	7,62	**4,51**	4,38	**2,90**	**2,90**	**2,28**	2,40	2,05	2,12	**1,87**	1,97

Upper le ft entries are the ARL for the EWMA Chart of Logs of Squared residuals
Upper right entries are the ARL for the EWMA Chart of Residuals
Lower left entries are the ARL for the Shewhart Chart of Residuals
Lower right entries are the ARL for the Moving Range Chart

TABLE A.13: Steady –State *ARL* of Four Charts for Monitoring the Process Variance when the increase in σ^2_X is caused by an increase in σ^2_ε where φ is autoregressive parameter for the *AR(1)* plus a random error model, $\psi_0=\sigma^2_{\mu 0}/\sigma^2_{X0}$ ($\lambda=0,2$ for both *EWMA* Charts ; the In-control $ARL=370,4$ for all Charts)

Appendix B

FIGURES

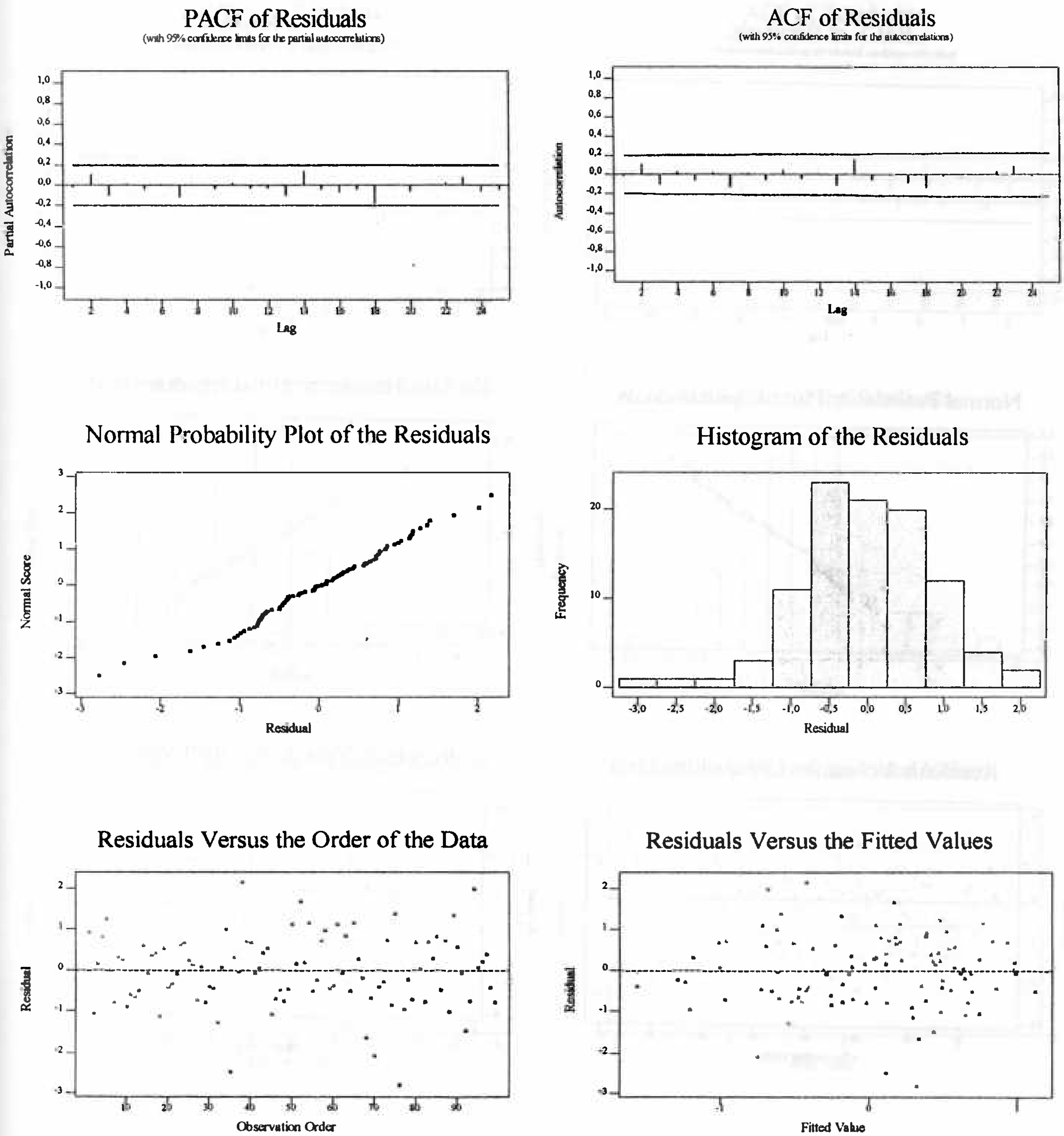

Figure B.1 : Checking Plots for the *ARMA(1,1)* time series fit for *100* simulated observations .

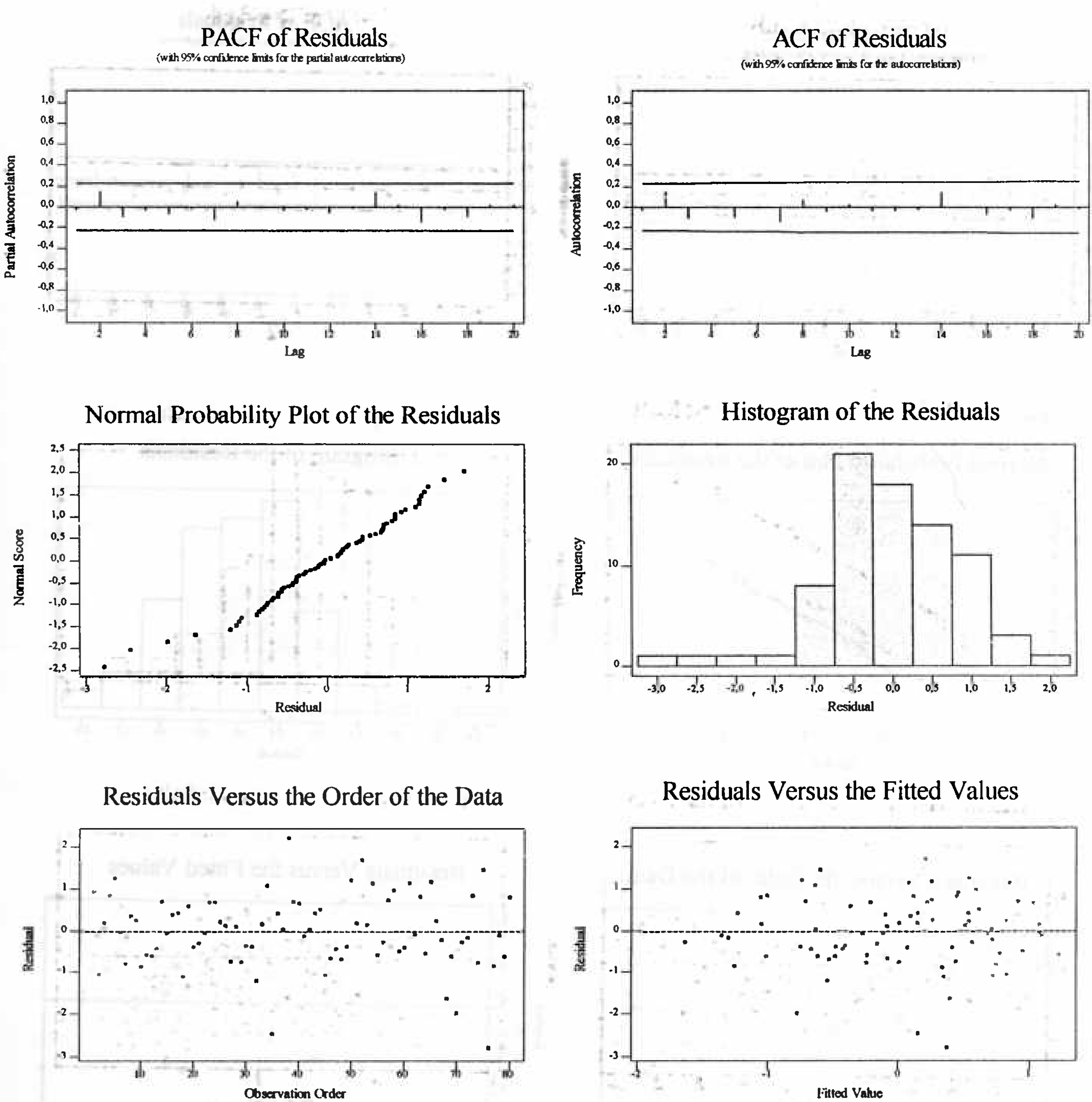

Figure B.2 : Checking Plots for the *ARMA(1,1)* time series fit for *80* simulated observations.

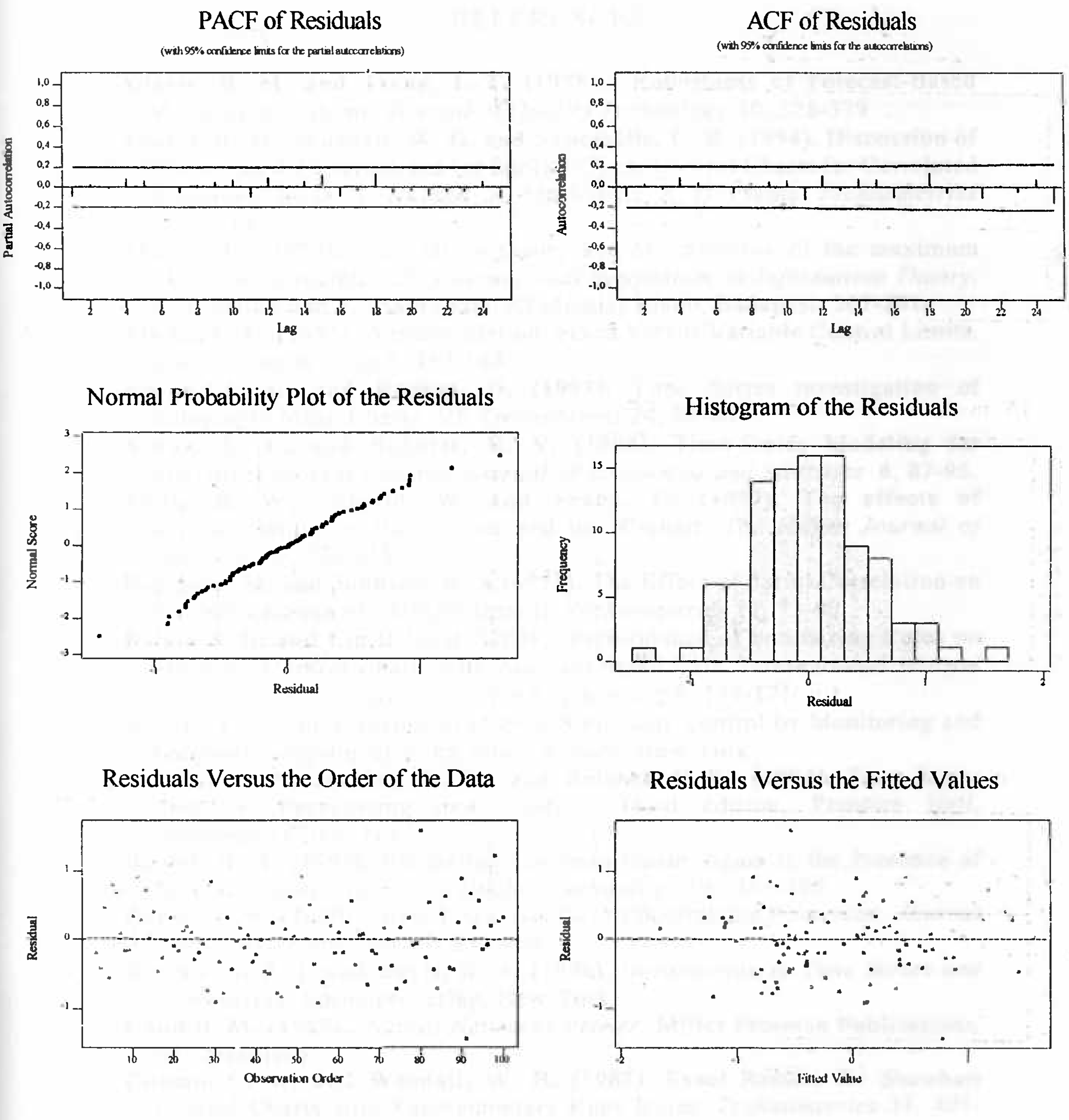

Figure B.3 : Checking Plots for the *AR(1)* time series fit for *100* simulated observations.

observations.

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