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Optimal Portfolio Management with Hedging

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Abstract

In the present thesis we deal with optimal portfolio management with hedging. Hedging aims at reducing the risk an investor faces, usually by means of taking the opposite position in the corresponding – or a suitably selected– futures market. Needless to say the goal is to achieve considerable risk reduction without sacrificing expected returns.

After giving the basic definitions of hedging and the measures of hedging performance, we present all types of hedging techniques, from the most usual ones such as perfect one-to-one hedging and minimum variance hedging ratio strategy to more complicated such as the conditional hedging rules to hedge against fluctuations of exchange rates and the dynamic hedging strategies of maintaining the portfolio delta-neutral and gamma-neutral.

Apart from the presentation of the main results and seminal publications of the field, we ourselves examined the hedging effectiveness of hedging strategies with index futures. We used six large capitalization indices and the corresponding futures contracts to measure the risk reduction achieved and the implications of hedging in the expected returns. It is important to note that we conducted both in-sample and out-of-sample analysis. Our results support those of the current literature.

PROLOGUE

Optimal portfolio management and asset pricing are the two central problems with which modern Finance Theory attempts to cope. In this thesis we deal·with the former and present various types of hedging strategies that can be applied to eliminate the risk exposure an investor faces, without necessarily sacrificing returns.

The problem an investor faces is essentially a maximization problem. Every investor's goal is to maximize expected utility gained from his wealth. The problem would be trivial if we lived in a world with no uncertainty. Alas, this is not the case, so the investor seeks to maximize his expected utility, given a certain level of risk he is willing to take. This trade-off between expected returns and risk has given rise to a voluminous literature aiming at minimizing the risk an investor faces, without ending up with a portfolio yielding nothing more than the risk-free rate.

In the pursuit of less risky portfolios, investors have employed diversification in many assets. Nonetheless this may not be enough and this is where hedging plays its role. Derivatives, from plain vanilla to more sophisticated exotic ones, may be used to offer the reduction of risk sought. Roughly speaking, hedging involves taking the opposite to the initial investment position in a suitably chosen futures market. With this position, the investor is immunized against unpleasant surprises. Needless to say, hedging is not as easy as we have just presented it. The choice of the underlying asset, as well as the amount of opposite investment to the futures market is crucial.

Hedging should be applied with caution and one should never forget that it is impossible to consistently beat an efficient market.

Except for a hedge against the whole risk exposure, one may wish to hedge against only one factor of risk. The case of an international portfolio depicts such a situation. International diversification has long been recognized as a means of reducing the overall risk of a portfolio without simultaneously sacrificing expected returns. On the other hand, international diversification induces another kind of risk, namely currency risk. Therefore the investor is susceptible to fluctuations of exchange rates. To eliminate this newly introduced type of risk the investor may hedge only against currency risk, by selling a forward contract with underlying asset the foreign currency. When and how much to hedge is an interesting open issue and gives rise to interesting conditional hedging strategies.

The thesis is organized in chapters as follows:

In the first chapter we present basic notions of modern portfolio theory. More specifically we mention the derivative products: forward contracts, futures, options and swaps. Next we make a short note on market efficiency and we present the main aspects of modern portfolio theory: utility theory, CAPM and the beta coefficients as well as APT. Next we present the most widely used measures of performance of a portfolio or a hedge fund. Finally we refer to the famous Black-Scholes model and the corresponding equation, as well as the Greeks-parameters appearing in the Black-Scholes equation and widely used in the evaluation of a portfolio and in dynamic hedging. This chapter is introductory and gives the framework that is necessary for the next chapters.

Having established the background needed, we proceed in the second chapter to deal with hedging. We give the basic definitions and present the hedging strategies, from the dummy hedging strategies, which are merely used as benchmarks, to the most sophisticated ones entailing dynamically adjusting the portfolio to ensure it is delta and gamma neutral.

In the third chapter we refer to the basic results of the literature. As the field is huge, we have concentrated on issues concerning hedging a portfolio with index futures, which we have tested.

In the fourth and final chapter we ourselves examine the hedging effectiveness of the Minimum Variance Hedging Ratio and the Least Trimmed Squares Hedging Ratio, when the portfolios consist of several large capitalization indices and their counterparts in the futures markets. Our analysis is in the spirit of Butterworth and Holmes 2001 paper and resulted in similar conclusions.

Before finishing this short prologue, a note on acknowledgements must be made. Given last year's special circumstances, the fulfillment of the requirements of MSc. in Economics would have been impossible without the aid of certain people. I feel that this is the time and the place to thank them all. First and foremost, I wish to express my gratitude to Alexandros Kontogiannis, for his aid has been inestimable. In addition, I would like to thank my brother-in-law, Giorgos Patsis; his contribution was equally significant. I could never neglect mentioning my family and friendly environment for the continuous, long-term support that covered all aspects, be that academic, intellectual or personal, of my life. A special tribute must be paid to my undergraduate thesis instructor, Paul Spirakis, for introducing me to Game theory and consulting me in the early stages of my academic life. In the same spirit I would like to thank my professors at Athens University of Economics & Business for the high-quality education they provided me. I consider them my mentors and appreciate their opening a window to the realm of economics, thus revealing to me the beauty of the field. Especially I would like to thank the supervisor of the present thesis, Professor Elias Tzavalis. His aid ranging from lecturing Finance theory and Econometrics and suggesting the topic to the enlightening discussions pertaining (but definitely not restricted) to the subject of the thesis has been immense. Last but not least I would like to mention my dear friend Takis Panagiotou for helping me in the collection of the data, and Anastasia Roumelioti, for with her encouragement and manifold support she has contributed significant value.

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Constantinos Georgios V. Kalfarentzos

Chapter 1

Introduction

In this chapter we provide with the basic notions of modern portfolio theory, thus establishing the necessary background to be able to proceed with the hedging strategies that will follow in the next chapters. In section 1.1 we present derivatives, financial instruments widely used in all hedging techniques. In section 1.2 we make a brief note on the efficiency of markets, while in section 1.3 we define the utility function and the coefficients of absolute and relative risk-aversion. In the next section we depict the basic ingredients of modern portfolio theory and present the models CAPM and APT. In section 1.5 we give one of the most important and elegant equations in finance theory- the Black-Scholes equation and define its parameters, namely the Greek letters. Finally in section 1.6 we state the measures of evaluation of portfolio performance, including the most widely used: the Sharpe ratio.

1.1 Derivatives

Derivatives are financial instruments, whose value is contingent upon the value of primary assets such as commodities or securities. They generally take the form of contracts under which the parties agree to payments between them based upon the value of an underlying asset or other data at a particular point in time.

The main use of derivatives is to reduce risk for one party while offering the potential for a high return (at increased risk) to another. The diverse range of potential underlying assets and payoff alternatives leads to a huge range of derivatives contracts available to be traded in the market. Derivatives can be based on different types of assets such as commodities, equities (stocks), bonds, interest rates, exchange rates, or indices (such as a stock market index, consumer price index or even an index of weather conditions, or other derivatives). Their performance can determine both the amount and the timing of the payoffs. The main types of derivatives are forwards, futures, options, and swaps.

Forward Contracts

A forward contract is an agreement between two parties to buy or sell an asset (which can be of any kind) at a pre-agreed future point in time. Therefore, the trade date and delivery date are separated. It is used to control and hedge risk, for example currency exposure risk (e.g. forward contracts on USD or EUR) or commodity prices (e.g. forward contracts on oil).

The forward price of such a contract is commonly contrasted with the spot price, which is the price at which the asset changes hands (on the spot date, usually two business days). The difference between the spot and the forward price is the forward premium or forward discount, depending on the sign of the difference.

Futures

A standardized forward contract that is traded on an exchange is called a futures contract. The future date is called the delivery date or final settlement date. The pre-set price is called the futures price. The price of the underlying asset on the delivery date is called the settlement price.

A futures contract gives the holder the obligation to buy or sell. In other words both parties of a "futures contract" must fulfill the contract on the settlement date. The seller delivers the commodity to the buyer, or, if it is a cash-settled future, then cash is transferred from the futures trader who sustained a loss to the one who made a profit. To exit the commitment prior to the settlement date, the holder of a futures position has to offset their position by either selling a long position or buying back a short position, effectively closing out the futures position and its contract obligations.

Options

Options are financial instruments that convey the right, but not the obligation, to engage in a future transaction on some underlying security. Options are traded both on exchanges and in the over-the-country market. We can distinguish between two basic types of options. A call option gives the holder the right to buy the underlying asset by a certain date for a certain price. A put option gives the holder the right to sell the underlying asset by a certain date for a certain price. The price in the contract is known as the expiration date or maturity. American options can be exercised at any time up to the expiration date. European options cannot be exercised at any time but to the expiration date itself. Most of the options that are traded are American.

It should be emphasized that an option gives the right to do something. The holder does not have to exercise this right. This is what distinguishes options from forwards and futures, where the holder is obligated to buy or sell the underlying asset. Note that whereas it costs nothing to enter into a forward or futures contract, there is a cost to acquiring an option.

Swaps

A swap is a derivative in which two counterparties agree to exchange one stream of cash flows against another stream. These streams are called the *legs* of the swap. The cash flows are calculated over a notional principal amount, which is usually not exchanged between counterparties. Consequently, swaps can be used to create unfunded exposures to an underlying asset, since counterparties can earn the profit or loss from movements in price without having to post the notional amount in cash or collateral.

Swaps can be used to hedge certain risks such as interest rate risk, or to speculate on changes in the underlying prices. Most swaps are traded over the counter, tailor-made for the counterparties, while some types of swaps are also exchanged on futures markets. The five generic types of swaps, in order of their quantitative importance, are: interest rate swaps, currency swaps, credit swaps, commodity swaps and equity swaps.

The most common type of swap is a "plain vanilla" interest rate swap. With this swap a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principle for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time.

Another popular type of swap is known as a currency swap. In its simplest form, this involves exchanging principal and interest payments in one currency for principal and interest payments in another. A currency swap agreement requires the principal to be specified in each of the two currencies. The principal amounts in each currency are usually exchanged at the beginning and at the end of the lifetime of the swap. Usually the principal amounts are chosen to be approximately equivalent using the exchange rate at the swap's initialization. On the contrary, when they are exchanged at the end of the life of the swap, their values may be quite different.

An equity swap is an agreement to exchange the total return (dividends and capital gains) realized on an equity index for either a fixed or a floating rate of interest. Equity swaps can be used by portfolio managers to convert returns from a fixed or floating investment to the returns from investing in an equity index, and vice versa.

Commodity swaps are in essence a series of forward contracts on a commodity with different maturity dates and the same delivery prices. In a volatility swap there are a series of time period. At the end of each period, one side pays a preagreed volatility, while the other side pays the historical volatility realized during the period. Both volatilities are multiplied by the same notional principal in calculating payments.

1.2 EFFICIENCY OF MARKETS

When a market is identified as an efficient market, then the price in the market is an unbiased estimator of the true value of the investment.

Several key concepts are implicit to this derivation:

a) Market efficiency does not require that the market price be equal to true value at every point in time. All it requires is that errors in the market price be unbiased, i.e., that prices can be greater than or less than true value, as long as these deviations are random.
b) The fact that the deviations from true value are random implies, in a rough sense, that there is an equal chance that stocks are under or over valued at any point in time, and that these deviations are uncorrelated with any observable variable.
c) If the deviations of market price from true value are random, it follows that no group of investors should be able to consistently find under or over valued stocks using any investment strategy.

Some basic remarks:

It is extremely unlikely that all markets are efficient to all investors, but it is entirely possible that a particular market is efficient as far as the average investor is concerned. It is also possible that some markets are efficient while others are not, and that a market is efficient with respect to some investors and not to others. This is a direct consequence of differential tax rates and transactions costs, which confer advantages to some investors relative to others.

Definitions of market efficiency are also linked up with assumptions on the information that is available to investors and reflected in the price. For instance, a strict definition of market efficiency that assumes that all information, public as well as private, is reflected in market prices would imply that even investors with precise inside information will be unable to beat the market.

We thus distinguish between three forms of market efficiency:

- Under weak form efficiency, the current price reflects the information contained in all past prices, suggesting that charts and technical analyses that use past prices alone would not be useful in finding undervalued stocks.
- Under semi-strong form efficiency, the current price reflects the information contained not only in past prices but all public information (including financial statements and news reports) and no approach that was predicated on using and managing this information would be useful in finding undervalued stocks.
- Under strong form efficiency, the current price reflects all information, public as well as private, and no investors will be able to consistently find undervalued stocks.

An immediate implication of an efficient market is that no group of investors should be able to consistently beat the market using a common investment strategy. An efficient market would also carry very negative implications for many investment strategies and actions that are taken for granted.

(a) In an efficient market, equity research and valuation would be a costly task that provided no benefits. At best, the benefits from information collection and equity research would cover the costs of doing the research.

(b) In an efficient market, a strategy of randomly diversifying across stocks or indexing to the market, carrying little or no information cost and minimal execution costs, would be superior to any other strategy that created larger information and execution costs. There would be no value added by portfolio managers and investment strategists.

(c) In an efficient market, a strategy of minimizing trading, i.e., creating a portfolio and not trading unless cash was needed would be superior to a strategy that required frequent trading.

An efficient market does not imply that stock prices cannot deviate from true value; in fact, there can be large deviations from true value. The only requirement is that the deviations be random. It also does not imply that no investor will 'beat' the market in any time period. To the contrary, approximately half of all investors, assuming zero transactions costs, should beat the market in any period. Furthermore, no group of investors will beat the market in the long term. Given the number of investors in financial markets, the laws of probability would suggest that a fairly large number are going to beat the market consistently over long periods, not because of their investment strategies but because they are lucky. It would not, however, be consistent if a disproportionately large number of these investors used the same investment strategy.

In an efficient market, the expected returns from any investment will be closely related to the risk of that investment in the long run, though there may be deviations from these expected returns in the short term.

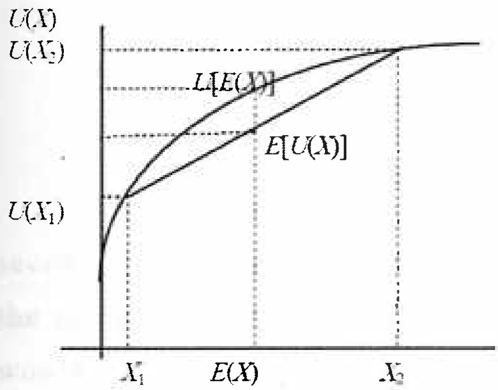
Proposition 1: The probability of finding inefficiencies in an asset market decreases as the ease of trading on the asset increases. To the extent that investors have difficulty trading on a stock, either because open markets do not exist or there are significant barriers to trading, inefficiencies in pricing can continue for long periods.

Proposition 2: The probability of finding an inefficiency in an asset market increases as the transactions and information cost of exploiting the inefficiency increases. The cost of collecting information and trading varies widely across markets and even across investments in the same markets. As these costs increase, it pays less to try to exploit these inefficiencies.

1.3 UTILITY THEORY

The utility theory is a systematic way to rank investors' preferences relating to alternative investment projects. This is achieved by means of the definition of a utility function and the introduction of the concept of risk aversion and measures of it. In what follows we shall give the technicalities of the utility function which provide with the basic framework for portfolio theory.

We define the utility function U(X) to be an increasing, concave function of X where X denotes a random variable that represents the investors' wealth.



For two possible levels of wealth X_1 and X_2 it is clear that investors derive a higher level of utility by choosing the average level of wealth $\overline{X} = E(X) = \frac{X_1 + X_2}{2}$, rather than choosing a linear combination of the two wealth levels.

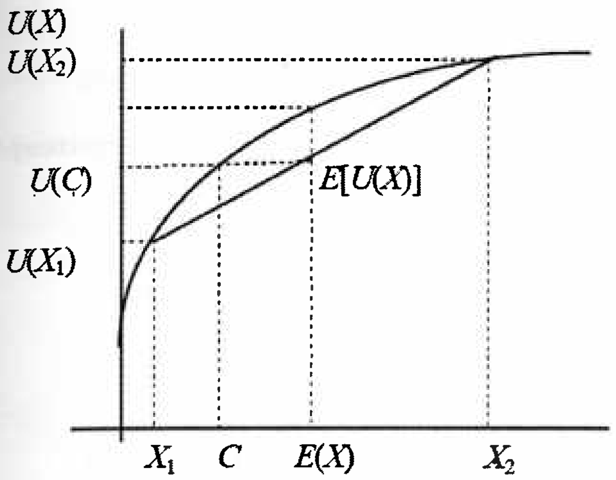
This means that $U[\mathrm{E}(X)] > \mathrm{E}[U(X)]$

The latter inequality is nothing but Jensen's inequality for concave functions and holds on the premise that the utility function is indeed concave. Concavity implies that investors' utility increases at a diminishing rate as the level of wealth increases. We call the concave utility function as risk-averse, since the implications of the above inequality can be exploited to model consumers' risk-averse attitudes against uncertainty. If the utility function were a straight line, i.e. U(X) = aX + b then by linearity of expected value, the inequality would become an equality. In this case we would call the corresponding utility function risk neutral and that would model the behaviour of investors that do not take any account of uncertainty when choosing a level of wealth. The increasingness and concavity assumptions may be expressed in calculus terms as a positive first derivative and negative second derivative, i.e

$$U' = \frac{dU(X)}{dX} > 0$$

$$U'' = \frac{d^2U(X)}{dX^2} < 0$$

The degree of concavity of the utility function, captured by the second derivative, determines the degree of risk aversion exhibited by the utility function. To avoid a present gamble a risk-averse individual would be willing to pay an insurance premium. This premium can be thought of as a casualty or liability insurance premium. The amount of a certain level of wealth, defined by C, that has a utility level equal to the expected utility of X, i.e. $U(C) = \mathrm{E}[U(X)]$ is known as certainty equivalent.



Note that for a concave utility curve C<E(X). The difference Π=E(X)-C (or Π = X − C) is the insurance premium that investors would be willing to pay as an insurance to obtain a level of utility E[U(X)]. This can be approximately determined as follows.

Expand both terms E[U(X)] and U(C) around X=E[X] using a second-order Taylor series expansion. This yields U[C]=E[U(X)]. Applying Taylor's series expansion, the RHS of the above, certainty equivalent equation yields

$$\{E[U(X)] \approx E[U(\overline{X})+U'(\overline{X})(X-\overline{X})+\frac{1}{2}U''(\overline{X})(X-\overline{X})^2\} = U(\overline{X})+\frac{1}{2}U''(\overline{X})E(X-\overline{X})^2$$

$$= U(\overline{X})+\frac{1}{2}U''(\overline{X})Var(X)^2$$

Since $E[U(X)] = U(\overline{X}), E[X-\overline{X}] = 0, Var(X) = E(X-\overline{X})^2$.

Again applying Taylor's expansion, the LHS yields

$$\{U(C) \approx U(\overline{X})+U'(\overline{X})(C-\overline{X})+\frac{1}{2}U''(\overline{X})(C-\overline{X})^2\}$$

Assuming that $(\overline{X})(C-\overline{X})^2 \approx 0$. So, the certainty equivalent equation now yields:

$$\frac{1}{2}U''(\overline{X})E(X-\overline{X})^2 \approx U'(\overline{X})(C-\overline{X}) \Rightarrow$$

$$(\overline{X}-C) \approx -\frac{1}{2}\frac{U''(\overline{X})}{U''(\overline{X})}V(X) \Rightarrow$$

$$(\overline{X}-C) \approx \frac{1}{2}\left[-\frac{U''(\overline{X})}{U''(\overline{X})}\right]V(X)$$

or equivalently we can write $(\overline{X}-C) \approx \frac{1}{2}\left[A(\overline{X})\right]V(X)$

where,

$A(\overline{X}) = \left[-\frac{U''(\overline{X})}{U''(\overline{X})}\right]$ is the Arrow-Pratt absolute risk aversion coefficient, a measure of risk aversion.

Since U(.) is concave, this coefficient is a function of wealth level and implies that risk aversion decreases as the level of wealth increases.

This may reflect individuals' attitude to take more risk when they are more financially secure. The greater the concavity the greater the risk aversion is. The term in the denominator is used to normalise the coefficient. With this normalisation A(X) is the same for all equivalent utility functions.

The absolute coefficient of risk aversion determines how much wealth an investor will put in a risky investment in absolute terms. Another measure of risk aversion which is closely related to the absolute risk aversion is the relative coefficient of risk aversion. This coefficient refers to the percentage investment in risky assets as wealth changes.

The coefficient of relative risk aversion is given by

$$R(X) = \left[-\frac{X \cdot U''(X)}{U''(X)} \right] = X \cdot A(X)$$

Notice that by multiplying A(X), which is inversely related to wealth, by the level of wealth, X,.we render R(X) invariant of X.

1.4 PORTFOLIO THEORY

Modern portfolio theory proposes how rational investors will use diversification to optimize their portfolios, and how a risky asset should be priced. The basic concepts of the theory are Markowitz diversification, the efficient frontier, Capital Asset Pricing Model, the alpha and beta coefficients, the Capital Market Line and the Securities Market Line. In Modern Portfolio Theory, an asset's return is represented as a random variable, and a portfolio as a weighted combination of assets; the return of a portfolio is the weighted combination of the assets' returns. Moreover, a portfolio's return is a random variable, and consequently has an expected value and a variance. Risk, in this model, is the standard deviation of the portfolio's return

CAPM

The Capital Asset Pricing Model is a model that describes the relationship between risk and expected return and is widely used in the pricing of risky securities. The general idea behind CAPM is that investors need to be compensated in two ways: time value of money and risk. The time value of money is represented by the risk-free (R_f) rate in the formula and compensates the investors for placing money in any investment over a period of time. The other half of the formula represents risk and calculates the amount of compensation the investor needs for taking on additional risk. This is calculated by taking a risk measure (beta) that compares the returns of the asset to the market over a period of time and to the market premium (R_m-R_f).

$$\overline{R_a} = R_f + \beta_a(\overline{R_m} - R_f),$$

where:

R_f is the risk free rate

β_a is the beta of the security

$\overline{R_m}$ is the expected market return

According to CAPM the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium. If this expected return does not meet or beat the required return, then the investment should not be undertaken. The security market line plots the results of the CAPM for all different risks (betas). Using the CAPM model and the following assumptions, we can compute the expected return of a stock.

The betas

The beta coefficient measures the riskiness of a portfolio, i.e. the portfolio's volatility in relation to the rest of the market. In CAPM, the beta measures the part of the asset's statistical variance that cannot be mitigated by the diversification provided by the portfolio of many risky assets, because it is correlated with the return of the other assets that are in the portfolio.

The CAPM returns the asset-appropriate required return or discount rate - i.e. the rate at which future cash flows produced by the asset should be discounted given that asset's relative riskiness. Betas exceeding one signify more than average "riskiness"; betas below one indicate lower than average. Thus, a more risky stock will have a higher beta and will be discounted at a higher rate; less sensitive stocks will have lower betas and will be discounted at a lower rate. The CAPM is consistent with intuition and concavity of investors' utility function- investors should require a higher return for holding a more risky asset.

Since beta reflects asset-specific sensitivity to non-diversifiable, i.e. market risk, the entire market's beta should be one. Stock market indices are frequently used as local proxies for the market - and in that case (by definition) have a beta of one. An investor in a large, diversified portfolio (such as a mutual fund) therefore expects performance in line with the market.

The risk of a portfolio comprises systematic risk and unsystematic risk-also known as idiosyncratic risk or diversifiable risk. Systematic risk refers to the risk common to all securities - i.e. market risk. Unsystematic risk is the risk associated with individual assets. Unsystematic risk can be diversified away to smaller levels by including a greater number of assets in the portfolio.

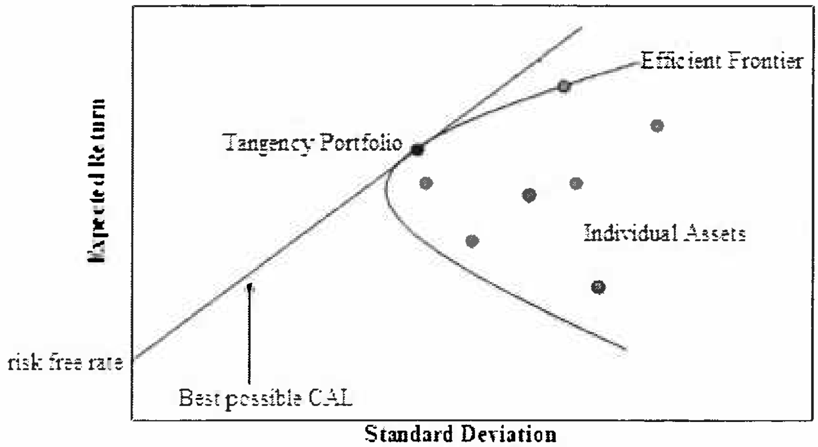
In the CAPM context, portfolio risk is represented by higher variance i.e. less predictability. In other words, the beta of the portfolio is the defining factor in rewarding the systematic exposure to risk taken by an investor.

The (Markowitz) efficient frontier

An optimal portfolio displays the lowest possible level of risk for its level of return. Each time an asset is introduced into a portfolio, it further diversifies it; so the optimal portfolio must comprise every asset, (assuming no trading costs) with each asset value-weighted to achieve the above (assuming further that all assets are infinitely divisible). All such optimal portfolios, i.e., one for each level of return, comprise the efficient frontier. Because the unsystematic risk is diversifiable, the total risk of a portfolio can be viewed as its beta.

The optimal portfolio

For a given level of return, only one portfolio will be optimal (in the sense of lowest risk). Since the risk free asset is, by definition, uncorrelated with any other asset, any other alternative will generally have the lower variance and hence be the more efficient of the two. This relationship also holds for portfolios along the efficient frontier: a higher return portfolio plus cash is more efficient than a lower return portfolio alone for that lower level of return. For a given risk free rate, there is only one optimal portfolio which can be combined with cash to achieve the lowest level of risk for any possible return. This is the market portfolio.



Arbitrage Pricing Theory

Arbitrage Pricing Theory (APT) was a model devised by Ross, approximately a decade after the CAPM, as an alternative asset pricing model. The premise upon which APT is based is that the risk exposure of a portfolio is not solely attributed to the market risk (that is the covariance with the market portfolio). Certainly this contradicts the CAPM's fundamental assumption that the sole risk exposure of a portfolio is the market risk. In what follows we shall give the basic ingredients of APT; the interested reader is referred to Ross's 1976 and 1977 papers for in-depth treatment of the subject.

According to APT, at any time t, the assets' returns may be determined by a linear factor model. APT is fairly general and does not determine which are the factors that influence the asset's returns, or how many are these factors. Nevertheless modern literature has distinguished five macroeconomic risk sources, which we shall present shortly. Before that we shall give the basic ingredients of the model. The mathematical relation of the APT is:

$$R_{it} - R_{ft} = \sum_{k=1}^{K} \beta_{ik}\left(P_k + f_{kt}\right) + \varepsilon_{it},$$

where

R_{it} is the asset i's return at time t,

R_{ft} is the risk-free rate at time t,

P_k is the expected return of the k^{th} factor,

β_{ik} is the sensitivity of the asset i to changes of the k^{th} factor,

ε_{it} is the idiosyncratic(diversifiable) part of asset i's returns at time t.

All terms are determined by means of the basic assumption that at equilibrium no profitable arbitrage is feasible. To complete the model's description we must refer to the basic sources of risk or in other words the factors. As we have mentioned before there exist five fundamental sources of risk that affect the assets' returns. These are:

The credit risk: Credit risk is induced by the probability that the firms may default the bonds they have issued. Credit risk may be estimated by the difference between the internal return of risky bonds issued by corporations and the corresponding internal rate of return of bonds issued by countries, which are considered risk-free.

Time horizon risk: Time Horizon risk is induced by the fact that next period's returns cannot be accurately predicted, as they are random variables. A measure of time horizon risk is the difference between the internal rate of return of long-term bonds and the spot interest rate.

Inflation risk: Inflation risk is induced by the fact that inflation is a random variable as well.

Business cycle risk: This type of risk is induced by the unexpected occurrence of business cycles.

Market timing risk: This type of risk common in both APT and CAPM is trying to capture all risk not attributed to the aforementioned factors.

Other types of risk are exchange risk that is induced by fluctuations of exchange rates and is present in internationally diversified portfolios, liquidity risk that is induced by the fact that markets are not perfectly liquid and so on.

One last point should be made regarding the relation between CAPM and APT. Those two models are equivalent provided the following relation holds:

$$P_k = \frac{\text{cov}(f_{kt}, R_{mt})}{\text{var}(R_{mt})}\left(E[R_{mt}] - R_{ft}\right),$$

where:

R_{mt} is the return of the market portfolio and the rest terms are defined as before.

It is noteworthy that the two models are highly unlikely to be equivalent. Econometric tests by Roll and Ross imposed on APT, with the market portfolio simulated by the general stock exchange index produced results that reject the above condition. In the light of these results we conclude that under the assumption that the stock exchange index is a suitable proxy of the market portfolio, the CAPM is less accurate than the APT.

1.5 Black Scholes and the Greeks

The Black–Scholes model is a mathematical model of the market for an equity, in which the equity's price is a stochastic process.

The key assumptions of the Black–Scholes model are:

- The price of the underlying instrument S_t follows a geometric Brownian motion with constant drift μ and volatility σ:

$$dS = \mu S dt + \sigma S dW$$

- It is possible to short sell the underlying stock.
- There are no arbitrage opportunities.
- Trading in the stock is continuous.
- There are no transaction costs or taxes.
- All securities are perfectly divisible
- It is possible to borrow and lend cash at a constant risk-free interest rate.
- The stock does not pay a dividend (see below for extensions to handle dividend payments).

By invoking impossibility profitable arbitrage conditions, Black and Scholes derived the following famous equation of pricing an option:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2 = rV,$$

Where,

V is the option's price

S is the underlying asset's price

r is the risk-free rate

t denotes time

Before giving the Black-Scholes equation in terms of parameters we must first define the Greek letters or simply the Greeks as they are widely called. The Greeks are the quantities representing the market sensitivities of options or other derivatives. Each "Greek" measures a different aspect of the risk in an option position, and corresponds to a parameter on which the value of an instrument or portfolio of financial instruments is dependent. The name is used because the parameters are often denoted by Greek letters.

The Greeks are vital tools in risk management. Each Greek (with the exception of *theta*) represents a specific measure of risk in owning an option, and option portfolios can be adjusted accordingly to achieve a desired exposure. As a result, a desirable property of a model of a financial market is that it allows for easy computation of the Greeks. The Greeks in the Black-Scholes model are very easy to calculate and this is one reason for the model's continued popularity in the market.

After this short introduction, we shall now define the Greek letters:

- The delta measures the sensitivity to changes in the price of the underlying asset. The Δ of an instrument is the mathematical derivative of the value function with respect to the underlying asset's price,

$$\Delta = \frac{\partial V}{\partial S}$$

- The gamma measures the rate of change in the delta. The Γ is the second derivative of the value function with respect to the underlying price,

$$\Gamma = \frac{\partial^2 V}{\partial S^2}.$$

Gamma is important because it indicates how a portfolio will react to relatively large shifts in price.

- **The vega, which actually** is **not a Greek letter, measures sensitivity to volatility.** The vega is **the derivative of the option value with respect to the** volatility **of the underlying,**

$$\nu = \frac{\partial V}{\partial \sigma}$$

- The speed **measures third order sensitivity to price.** The speed is the third **derivative of the value function with** respect to the underlying price,

$$\frac{\partial^3 V}{\partial S^3}$$

- **The theta measures sensitivity to the passage of time. Θ is the negative of the derivative of the option value with respect to the amount of time to expiry of the option,**

$$\Theta = -\frac{\partial V}{\partial T}$$

- **The rho measures sensitivity to the applicable interest rate. The ρ is the derivative of the option value with respect to the risk free rate,**

$$\rho = \frac{\partial V}{\partial r}$$

We are now in position to give the Black Scholes equation in terms of the Greeks.

$$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = rV$$

1.6 Measures of Portfolio Performance

In this section we will define the metrics used in the evaluation of the performance of a portfolio. The most widely used metric is definitely Sharpe's ratio, but we mention others as well for concreteness.

Sharpe ratio

Sharpe's ratio was proposed by Sharpe in his seminal 1964 paper Mutual Fund Performance and is widely used by both academics and practitioners as well. The Sharpe ratio for an asset (or a portfolio of assets) is defined as the ratio of mean excess returns over the standard deviation of the asset. In mathematical notation:

(Sharpe ratio) $= \frac{R_p - R_F}{\sigma_p}$, where:

R_p is the asset's expected return in time interval T

R_f is the risk-free interest rate in the same time interval T

σ_p is the standard deviation of the excess expected return in the time interval T.

Assuming that no arbitrage conditions hold and further that the conditions ensuring the validity of CAPM hold also, it can be shown that the greatest feasible Sharpe ratio is the market's Sharpe ratio. Sharpe ratio takes into consideration merely the first and second moments of the returns of the portfolio. This means that it neglects possible kurtosis or the presence of symmetries or asymmetries in the distribution of the returns. One should therefore use the Sharpe ratio with extreme caution, as it systematically overestimates the assets.

Sortino ratio

Sortino ratio was proposed in 1994 by Sortino and Price as a refinement of Sharpe Ratio. Close observation of the terms comprising the Sharpe ratio reveals that the ratio decreases as the volatility of the returns of the portfolio increases, irrespective of the direction of the deviations from the mean. Sortino ratio on the other hand, takes into account only the deviations that are smaller than the mean. In other words, Sortino ratio does not count the returns that are in favor of the investor and captures asymmetries that may be characteristic of the distribution of the returns. .

The computation of Sortino Ratio is similar to that of Sharpe Ratio. The sole difference lies in the computation of the denominator. The volatility is replaced by the downside deviation (DD), which neglects the values that are greater than a certain minimum acceptable return (MAR).

The downward deviation is given by the following formula:

$$DD = \sqrt{\frac{1}{n-1} \sum_{R_p < MAR} \left(R_p - MAR\right)^2}$$

As we said earlier Sortino ratio is defined as:

$$\text{(Sortino Ratio)} = \frac{R_p - R_F}{DD},$$

where

MAR is the minimum acceptable return

DD is the downward deviation and

n the number of observations

Sortino ratio has the obvious merits that captures certain asymmetries of the distribution of returns but has been criticized because it is not robust with respect to the selection of the minimum acceptable return, and in addition this selection is rather arbitrary.

Jensen's alpha

Jensen in his 1968 paper considered a simple linear model, in the spirit of CAPM and introduced the alpha coefficient bearing his name, which serves as a measure of an asset's performance. According to Jensen an asset's performance may be broken down to two terms: one that depends on the sensitivity of the asset's returns with the movements of the market and another one reflecting all other interactions. Jensen's linear model of asset pricing is:

$$R_p-R_f=\alpha+\beta(R_m-R_f)+e_p,$$

where:

R_p is the asset's return

R_m is the market's return

R_f is the risk-free rate

β is the sensitivity of the asset's return with respect to market movements

α is the intercept in the model-Jensen's alpha

e_p is a zero mean disturbance term

By a simple comparison to CAPM, one may deduce that according to CAPM α is predicted to be zero. That is the intuition behind using Jensen's alpha as a performance measure.

Treynor ratio

The last measure of performance we are presenting is Treynor ratio. Treynor ratio is a refinement of the Sharpe ratio as well. The only difference is that the volatility of the portfolio in the denominator of the Sharpe ratio is replaced with the beta of the portfolio. The formula that defines the Treynor ratio is:

$$\text{(Treynor ratio)} = \frac{R_p - R_F}{\beta_p}$$

A closer look at the Treynor ratio reveals that this measure of performance considers as the sole source of risk the market risk and as a result, expresses the performance of the portfolio in comparison to the performance of the market.

Chapter 2

Hedging

This chapter is the core of the thesis. In section 2.1 we provide with basic definitions of hedging and introduce the idea of the basis. In the next section we describe three trivial hedging strategies, staying unhedged, fully hedged and half-hedged that are widely used as benchmarks, to test the performance of any proposed hedging strategy. In section 2.3 through 2.5 we define index futures strategies such as the minimum variance hedging ratio, the beta hedging and the least trimmed squares hedging ratio. In the next four sections we focus on the hedging against currency risks, inherent in any internationally diversified portfolio and describe the Forward Hedging Rule, the Real-Interest Rate Hedge rule and their combination the Real Forward Hedge Rule. Finally in the last section we describe dynamic hedging strategies that aim at maintaining the portfolio delta and gamma neutral.

2.1 HEDGING - BASIC DEFINITIONS

Hedging aims at reducing the risk an investor faces, usually by means of taking the opposite position in the corresponding –or a suitably selected– futures market. We distinguish between short and long hedges. A short hedge involves a short position in forward contracts. A short hedge is appropriate when the investor already owns or expects to purchase the asset or security and has the intention to sell it in the near future. On the other hand if the investor wishes to purchase an asset in the future he may take a long position in the futures market. By virtue of the aforementioned long hedge the investor locks the effective price of purchasing the asset and hence minimizes the uncertainty.

Perfect hedging completely eliminates risk, but, alas, perfect hedging instruments are extremely rare, mainly due to the following reasons:

- The asset to be hedged may not be exactly the same with the asset underlying the corresponding futures contract.
- The hedger may be uncertain as to the exact date when the asset will be purchased or sold.
- The hedge may require the futures contract to be closed well before its delivery month.

All of the above naturally give rise to what is known as the basis risk, ie. the uncertainty with respect to the basis.

In a hedging position the basis is defined as:

$\boxed{b_i = S_i - F_i}$ where b_i is the basis, S_i is the spot price of the asset and F_i the futures contract price, all at time i.

It is obvious that, as the futures contract reaches maturity, the basis tends to zero. On the other hand, well before the maturity of the contract the basis may be either positive or negative depending on the type of the asset, since the price is determined by the corresponding demand and supply. When the increase in the spot price is greater than the increase in the futures price, the basis increases. This is called strengthening the basis. The opposite scenario, namely a decreasing basis, is called weakening the basis respectively.

As we have already mentioned, the asset underlying the futures contract and the asset actually purchased may differ, giving rise to what is widely known as cross-hedging. In this case the basis may be decomposed into two fundamental components. Indeed, let S_i denote the stock price of the asset purchased, S_i^* denote the stock price of the underlying asset and F_i the futures contract price, i. Then by adding and subtracting S_i^* from the basis definition we obtain:

$$\boxed{b_i = (S_i - S_i^*) + (S_i^* - F_i)}$$

Observe that according to this formulation the basis is broken down to a term reflecting the difference in the spot prices of the two assets and a term that is actually the basis corresponding to the asset underlying the futures contract.

In a perfect hedge it is straightforward that the position in the hedge should equal the position in the asset, i.e. the exposure to risk. On the contrary in the case of cross-hedging, equating the two investment positions is generally suboptimal and the hedger must determine the optimal hedging ratio so as to minimize the variance of the total investment. The hedge ratio as implied by its name is the ratio of the amount invested in the hedge position to the amount invested in the exposure.

2.2 Trivial Hedging Strategies

In this section we will present the most common unconditional hedging strategies. These are also known as once-and-forget strategies, because the investor, after performing calculations about the optimal hedging position, takes the relevant position on futures and never re-optimizes.

First, we will describe three naive hedging strategies that are widely used in practice not as strategies per se (at least not in the long run) but as benchmarks, to measure the hedging performance of more sophisticated techniques. After the presentation of these trivial hedging strategies we shall proceed in the realm of more sophisticated hedging techniques.

The benchmarks

The most obvious hedging strategy is the trivial one, namely no hedging at all. Holding a naked position, as it is well-known, is suitable when the investor holds a portfolio with large beta but believes that the assets underlying it will outperform the market. Needless to say, this strategy is a degenerate one and exhibits no long term merits, apart from the lack of transaction costs.

Another trivial hedging strategy is the directly opposite, ie. the fully hedged strategy. Alternative names are one-to-one hedging and taking a covered position. As the latter name implies, in this strategy the investor invests an equal to the initial investment amount in the futures market. The reader might recall from the previous discussion that this short position completely eliminates risk in the (rare) special case of perfect hedging. When cross-hedging takes place, then this strategy is clearly inappropriate.

Half-hedged strategy combines the advantages of the aforementioned hedging strategies. Taking a short position in the futures market equal to half the value of the investment is a strategy that attempts to reduce the risk exposure of the investment without reducing the expected returns to the insufficient risk-free rate.

It is important to note that the three latter strategies should not be considered as a rule of thumb that can be used by an investor, as they are not even an approximation to an optimal strategy. None of these takes into account parameters such as the variance of the portfolio or the covariance of the portfolio and the hedging instrument. It is straightforward that all three provide bad hedges and they should be viewed as benchmarks, that any proposed hedging strategy must outperform. In the following we shall present more sophisticated hedging strategies and in the last section we shall actually use the aforementioned strategies as benchmarks.

2.3 MINIMUM VARIANCE HEDGING RATIO STRATEGY

The minimum variance hedging ratio, as the name implies, aims at holding an optimal hedging position with respect to the variance of the portfolio returns. Shortly we will derive the optimal hedge ratio, or the Minimum Variance Hedge Ratio. Suppose that at time t=1 the hedger owns N_A units of asset A, which he expects to sell at time t=2. To hedge against fluctuations of the spot price of the asset, the hedger takes a short position in the futures market by purchasing a put option on N_F units of underlying asset A*. Let ΔS be the difference in spot prices of asset A, ΔF be the difference in the futures contract prices, σ_S and σ_F be the corresponding volatilities, ρ the coefficient of correlation between ΔS and ΔF and $h=\frac{N_F}{N_A}$ the hedge ratio.

The profit (or loss) realized by the total investment is:

$$\Pi=S_2N_A-(F_2-F_1)N_F = S_1N_A+ N_A\Delta S-hN_A\Delta F=N_A(S_1+\Delta S-h\Delta F)$$

Recall that at time t=1 the only random variables are terms ΔS and ΔF, so the variance of the profit is:

$$Var(\Pi) = N_A^2\left(\sigma_S^2 + h^2\sigma_F^2 - 2\rho h\sigma_S\sigma_F\right)$$

Equating the first derivative with respect to the hedging ratio to zero yields:

$$\frac{dVar(\Pi)}{dh} = 0 \Rightarrow N_A^2 2h\sigma_F - 2\rho\sigma_S\sigma_F = 0 \Rightarrow h_{OPT} = \rho\frac{\sigma_S}{\sigma_F}.$$

Notice that the variance is a convex function of h, so the extremum point found is indeed a minimum.

Notice also that in case of $\rho=1$ and $\sigma_S=\sigma_F$, that is when the changes in the futures price mirror exactly the fluctuations in the spot price, the minimum variance hedge ratio is 1. The cautious reader must have spotted that in this case **perfect hedging is achieved.**

Another important property of the minimum variance hedge ratio is that it is the slope of the line of best fit, when ΔS is regressed on ΔF. To show this recall that

$$\Pi = N_A(S_1 + \Delta S - h\Delta F) \Rightarrow \Delta S = \underbrace{\frac{\Pi}{N_A} - S_1}_{k} + h\Delta F \Rightarrow \Delta S = \mathrm{k} + h\Delta F$$

Applying OLS in the latter relation yields the following estimator for the hedge ratio:

$$\hat{h}_{OLS} = \left(\frac{1}{N}(\Delta F)'(\Delta F)\right)^{-1} \frac{1}{N}(\Delta F)'(\Delta S) \to \frac{Cov(\Delta F, \Delta S)}{Var(\Delta F)} =$$

$$= \frac{\rho\sigma_S\sigma_F}{\sigma_F^2} = \frac{\rho\sigma_S}{\sigma_F} = h_{OPT}$$

This result is far from surprising, as we require the optimal hedge ratio to correspond to the ratio of changes in ΔS to changes in ΔF.

We define in the natural way the hedge effectiveness to be the proportion of the variance that is eliminated (per unit invested) by virtue of hedging. One may easily show that hedge effectiveness is the R^2 coefficient from the regression of ΔS against ΔF that we used earlier. In particular it holds that the hedge effectiveness equals the square of the coefficient of correlation between ΔS and ΔF. In mathematical notation:

$$\text{(Hedge effectiveness)} = \frac{\sigma_S^2 - \sigma_H^2}{\sigma_S^2} =$$

$$\frac{\sigma_S^2-\left(\sigma_S^2+h_{OPT}^2\sigma_F^2-2\rho h_{OPT}\sigma_S\sigma_F\right)}{\sigma_S^2}=\frac{-\left(\rho\frac{\sigma_S}{\sigma_F}\right)^2\sigma_F^2+2\rho\left(\rho\frac{\sigma_S}{\sigma_F}\right)\sigma_S\sigma_F}{\sigma_S^2}=\rho^2$$

.

An alternative notion is risk reduction and is defined as the proportion of the volatility (standard deviation) that is eliminated by virtue of hedging. Namely:

$$(\text{Risk reduction}) = \frac{\sigma_S-\sigma_H}{\sigma_S}.$$

R^2 coefficient is an adequate measure of hedge performance when the minimum variance hedging ratio is applied. Nevertheless, some researchers argue that when more complicated hedging strategies are concerned R^2 does not fully reflect the hedging performance and propose a variety of measures to better capture the beneficial effects of hedging.

2.4 BETA HEDGING

Beta Hedging is similar to the Minimum Variance Hedging Ratio and was proposed as a refinement of the latter strategy. The truth is that in the vast majority of papers, the minimum variance hedging strategy outperformed the beta hedging strategy. Nonetheless we define beta hedging strategy for concreteness.

The basic idea behind beta hedging is to neutralize against the risk imposed solely by the market, neglecting other sources of risk. If the assumptions of CAPM hold, naturally this means that beta hedging is the optimal hedging strategy.

The beta hedging sets the hedging ratio to be equal to the negative of beta in the following equation:

$$RS_t = \alpha + \beta RIND_t + \varepsilon_t,$$

Where,

RS_t is the vector of returns of the asset

$RIND_t$ is the vector of returns of the general index

Assuming that the General Index of the Stock Exchange to be a good approximation, a good proxy that is, of the market portfolio yields that the general index, or a forward contract with underlying asset the general index, is a suitable hedging instrument.

2.5 Least Trimmed Squares Hedging Ratio

Least Trimmed Squares Hedging Ratio is an enhancement to Minimum Variance Hedging Ratio strategy. As the name implies it is the Minimum Variance Hedging Ratio strategy, in a smaller (trimmed) set of data. From a computational point of view it is a two stage procedure that uses Minimum Variance Hedging Ratio procedure in each stage.

The procedure has as follows:

LTSHR algorithm

1. We estimate the standard Minimum Variance Hedging Ratio by applying Figlewski's regression.
2. We compute the residuals obtained by the regression and sort the sample in ascending order of absolute values of residuals. That is, the first data in the new list are associated with smaller residuals, while the last data exhibit the greatest residuals.
3. We drop a percentage of observations equal to the trimming coefficient. The trimmed data corresponding to the greatest residuals.
4. Finally we perform stage 1 once more, with the sole difference that the underlying dataset has the extreme outliers left over.

The intuition behind Least Trimmed Squares Hedging Ratio is that, although the OLS estimation of Minimum Variance Hedging Ratio has many desirable characteristics, it is nonetheless extremely sensitive to outliers, when the latter exist. In order to eliminate this sensitivity, that mostly derives from the kurtosis inherent in the futures prices time series one should generate hedge ratios that minimize the impact of the outliers. The trimming procedure between the two OLS stages succeeds in this goal.

One last issue remains to be solved concerning the application of the method. It is the answer to the question: What is the optimal percentage of data that should be trimmed? This question has been empirically answered, although not rigorously proven by Knez and Ready in their 1997 paper. By estimating the minimum variance hedging ratio and then the least trimmed squares hedging ratio for various trimming coefficients within the range of 5% to 50% of the sample, Knez and Ready showed that the LTS slopes are similar, whether they used 95% or 50% of the data. This implies that a tendency for extreme observations to be influential is explained by a small portion of the data (not greater that 5%). Following Butterworth and Holmes' approach, we shall adopt 10% as a suitable trimming coefficient.

2.6 CURRENCY HEDGING FOR INTERNATIONAL PORTFOLIOS

In this section we will deal with the hedging against the exchange risk inherent in any internationally diversified portfolio. International portfolio diversification has long been recognized as a means of reducing the overall risk of a portfolio without necessarily sacrificing returns. On the other hand, international diversification induces a new type of risk, namely the risk exposure to fluctuations of exchange rates. To hedge against currency risk various techniques have been proposed. In what follows we shall only present conditional hedging strategies.

Conditional hedging strategies

Once-and-forget strategies, with which we dealt in the previous section, are especially plausible when the moments of the time-series underlying the spot and futures prices are slowly changing, or when the investment takes place in a short time interval during which the moments do not change significantly. Of course, this assumption is far from reasonable and its relaxation gives rise to the devising of conditional hedging strategies.

Hazuka and Huberts provided a useful framework for analyzing the desirability of hedging through the use of forward contracts. They focused on the payoff a strategy receives at the end of the hedging period. If the foreign investment is fully hedged, then the investor receives the forward rate times the foreign currency value of the investment, since the exchange rate is locked at the forward contracts value. On the contrary, if the investor chose not to hedge at all, then at the end of the investment period he receives the exchange rate prevailing at the time times the the value of the investment.

The opportunity cost of hedging is defined in the natural way as the difference between the spot rate at the end of the hedging period t minus the forward exchange rate agreed upon at time t=0(the beginning of the investment). In mathematical notation:

$OCH=S_t-F_t^0$

It is straightforward to see that this formula implies that an investor should hedge only if the opportunity cost of hedging is expected to be negative or zero. Notice that in the present analysis we have implicitly assumed that transaction costs are zero.

Alternatively, the opportunity cost of hedging in a ratio format (OCR) can be defined as

$$OCR = \frac{S_t}{F_t^0}.$$

By this formula one should hedge when the expected ratio is less than or equal to 1. Taking the log of both sides yields an equation for what we shall call log opportunity cost ratio, locr.

$$locr = s_t - f_t^0$$

Hazuka and Huberts decomposed the opportunity cost of hedging by subtracting and adding the spot rate at time 0.

$$locr = (s_t - s_0) - (f_t^0 - s_0)$$

The equation above states essentially that the opportunity cost of hedging is the change of exchange rate less the forward premium. Thus

$$E[locr] = (E[s_t] - s_0) - (f_t^0 - s_0)$$

Because one should hedge whenever the expected opportunity cost of hedging is not positive, the formula above states that hedging is desirable when the expected percentage change in the exchange rate is less than the forward premium. Note that when E[locr] is zero, the expected change in the spot rate equals the forward premium or discount and therefore the forward rate becomes an unbiased predictor of the future spot rate.

The question of whether or not the forward rate is an unbiased predictor of the subsequent spot rate has sparked great controversy among academics. The consensus among practitioners, however, seems to be that the forward exchange rate is biased and that one may exploit this bias by using conditional hedging rules.

2.7 THE FORWARD HEDGING RULE

The simplest and arguably most widely used conditional hedging strategy is the Forward Hedging Rule (FHR). The Forward Hedging Rule assumes that the current spot exchange rate gives a better forecast of the subsequent spot rate than does the forward rate. A sufficient condition that ensures the previous statement is that exchange rates roughly follow a random walk.

Thus, the best forecast of tomorrow's exchange rate is today's spot rate $E\{S_t|\Omega_0\}=S_0$, where Ω_0 is the information set available at time t=0. Applying this assumption to the basic equation $E\{locr\}= [E\{S_t\}-S_0] - (F_t^0- S_0)$, nullifies the first term and yields the forward rule equation:

$$E\{locr\}= - (F_t^0- S_0)$$

According to this rule, it is recommended to hedge against currency risk, when the foreign currency is trading at a forward premium (or at most is at par) and not hedge when the foreign currency is selling at a forward discount.

This simple rule is well-known among academics and practitioners alike and has been surprisingly successful in empirical tests. Indeed, up to date there are no results showing that the rule failed to outperform any standard benchmark. As a result the acceptance of the Forward Hedging Rule among practitioners is widespread and the rule's efficacy is more or less taken for granted by most authors.

Despite its recognition for its efficacy and simplicity, the Forward Hedging rule is subject to enhancements.

2.8 The Real-Interest-Rate Hedge Rule

Rather than assuming that exchange rates approximate at random walk, Hazuka and Huberts proposed that the exchange rate tends to revert to a mean based on purchasing power parity. Purchasing Power Parity predicts that the expected exchange rate will offset differences in anticipated inflation:

$$E(s_t) = s_o + \Delta p - \Delta p^*,$$

where

Δp is the anticipated change in the domestic log price index and

Δp^* is its foreign counterpart.

In their derivation, Hazuka and Huberts invoked interest rate parity, which states that the forward rate is determined by differences in (domestic vs. foreign) interest rates. Specifically, interest rate parity proposes that today's forward rate should equal the current spot rate adjusted by interest rate differentials:

$$f_t^0 = s_0 + R - R^*,$$

where

R is the domestic nominal interest rate and

R* is the foreign nominal interest rate[1]

Interest rate parity is widely known to function well in countries with relatively unrestricted capital flows and little government credit risk (otherwise, arbitrage will occur). Trivial algebraic manipulations lead to:

$$E(locr) = (R^* - \Delta p^*) - (R - \Delta p),$$

[1] Both rates are continuously compounded

which is nothing but the Real Interest Rate rule. Under this rule, an investor should hedge when the real domestic interest rate equals or exceeds the foreign one.

Hazuka and Huberts reported that their proposed Real Interest Rate rule outperformed unhedged, half-hedged, and fully hedged alternatives in monthly tests of portfolios of currencies. These findings are particularly impressive because it is well known that the Purchasing Power Parity model is a poor short-term predictor of the future spot rate. The results do not indicate that Purchasing Power Parity forecasts the exchange rate well, however, but rather, that its prediction error is less than the forward premium or discount.

2.9 The Real Forward Hedge Rule

Both the FHR and the RIR find support in empirical tests. The question is whether combining them would improve hedging performance. The FHR relies on the assumption that the exchange rates roughly follow a random walk; the RIR depends on the presumed reversion to Purchasing Power Parity exchange rates. To prove effective as a hedging rule, either rule needs merely to predict the subsequent spot rate (adjusted for the forward premium or discount). The FHR relies on interest rate differences but does not attempt to distinguish among reasons for the differences. But some causes of higher interest rates are associated with subsequent appreciations of nation's currency whereas others are linked to depreciation. For example, on the one hand, when interest rates are high because of inflation expectations, the currency in a country tends to depreciate. On the other hand, the currency may appreciate if rates are high for other reasons, such as stronger economic growth. Unlike the FHR, the RIR, in essence, makes this distinction in its hedging recommendations because it calls for hedging only when the real (i.e. nominal less inflation) domestic interest rate exceeds the foreign one.

Therefore, there is some justification for combining these rules. When the foreign currency trades at a forward premium (which invokes hedging under the FHR), a strategy of hedging only when one expects the foreign currency to depreciate may be more profitable than an FHR-based strategy. The RIR predicts that the foreign currency will tend to depreciate when the real foreign interest rates are less than domestic rates. Thus, an investor could choose to hedge only when (1) the foreign currency sells at a forward premium (the FHR is invoked) and (2) the foreign real interest rate is less than the domestic rate (the RIR is invoked).

The RFHR incorporates information in both asset markets, as does the FHR, and goods markets, as does the RIR to form a forecast of the direction of the forward forecast error. By requiring the signals from the asset and goods markets to confirm each other, the RFHR improves the accuracy rate of hedge signals. Because the RFHR is more restrictive than either of the rules it comprises, it will recommend hedging less frequently than either of them. Nevertheless improved performance can be expected by RFHR since it uses what seem to be predictive components of both rules.

2.10 Dynamic Hedging and the Greeks

Dynamic hedging strategies by maintaining acceptable values of the Greeks in the Black-Scholes equation are beyond our concentration in the present thesis. Nevertheless we briefly outline the basic ideas behind delta and gamma hedging for completeness.

Recall from the first chapter that delta is the rate of change of the option value with respect to changes in the price of the underlying asset. A position with a delta of zero is known as a delta neutral position. Observe that a delta neutral position ensures a perfect hedge against small changes of the price of the underlying option.

It is important to realize that delta is not static and a delta neutral position may be achieved for a relatively small period of time. To maintain delta zero rebalancing must take place. This naturally gives rise to a dynamic hedging scheme, where the position of the investor in asset and option is suitably adjusted.

Recall that the gamma of a portfolio in the rate of change of the portfolio's delta with respect to the price of the underlying asset. When gamma is small, delta changes slowly and adjustments to keep the portfolio delta neutral need to be made relatively infrequently. However if the gamma is large, then delta is highly sensitive in small price changes and it is quite risky to leave a delta-neutral portfolio unchanged for any time interval. The problem that arises is that both a position in the underlying asset and the position in the forward contract have zero gamma and no combination of investment on these assets may alter the gamma of the portfolio. Thus, what is required is a position in an instrument, such as an option, that is not linearly dependent upon the underlying asset. Of course this alters delta and further rebalancing must take place to maintain a delta neutral position.

Making a delta-neutral portfolio gamma neutral can be regarded as a first correction for the fact that the position in the underlying cannot be changed continuously when delta hedging is used. Delta neutrality provides protection against relatively small stock price movements between rebalancing. Gamma neutrality provides protection against larger movements in this stock price between rebalancing.

A final note must be made concerning the transaction costs, which up to this point we have silently neglected. Maintaining a delta-neutral position in a single option and the underlying asset is liable to be prohibitively expensive because of the transaction costs. For a large portfolio of options, delta-neutrality is more feasible and less costly.

Chapter 3

Survey of literature

In this section we shall present briefly the seminal papers and the main results in the field. We shall focus on a specific type of hedging, which we shall analyze ourselves in our own dataset in the last section of the thesis. The aforementioned type of hedging is hedging against the whole risk inherent in the portfolio by taking an opposite position in the corresponding index futures market.

Hedging via index futures

First and foremost we must mention Johnson's 1960 paper, in which the minimum variance hedging ratio was defined. Johnson was the first to notice that since the asset's prices and the underlying forward contract's prices do not follow the same path, a hedge ratio different that one should be used.

The pioneer in the area of hedging portfolios by means of taking the opposite position in the corresponding futures index market is beyond doubt Figlewski. Figlewski, in his seminal 1984 and 1985 papers, was the first to prove that the minimum variance hedging ratio[2] may be derived by a regression of the returns of the spot prices of the assets on the prices of the futures contract. He also showed, although not proved, that the consideration of dividends is not significant in the estimation of the optimal minimum variance hedging ratio. In addition, based on ex post interpretation of the data, he deduced the result that minimum variance hedging exhibits greater hedging effectiveness than the beta hedging. In the second paper Figlewski showed that hedging effectiveness improved as the duration of the hedge increased from days to weeks. If we are to include transaction costs in our analysis, this provides with strong evidence that hedging on a weakly basis is by far superior to day by day hedging. Another outcome of Figlewski's work is that portfolios of small stocks were hedged less effectively than those comprising large stocks.

Junkus and Lee in their 1985 paper tested hedging effectiveness of three USA stock index futures exchanges using four commodity hedging models. Their findings support earlier results that the Minimum Variance Hedging Ratio is most effective at reducing the risk of a cash portfolio comprising the index underlying the futures contract.

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[2] In in-sample analysis. When out-of-sample analysis takes place OLS yields merely an approximation.

Peters in his 1986 paper examined the hedging effectiveness of hedging strategies and provided with further evidence that the Minimum Variance Hedging Ratio outperformed Beta Hedging.

Graham and Jennings in 1987 were the first to examine hedging effectiveness for cash portfolios not matching an index. Random sampling techniques were used to form ninety equity portfolios each comprising ten stocks. Interestingly, this study found that stock index futures were less than half as effective at hedging non-index portfolios as they were at hedging cash indices.

Lindahl in his 1992 paper examined hedge duration and hedge expiration effects for the MMI and S&P 500 futures contracts. Lindahl's results suggested that both hedge ratios and hedging effectiveness increase as hedging duration increases. However he did not manage to indicate a particular pattern in terms of risk reduction in relation to time to expiration.

Hedging effectiveness for the UK was first examined by Holmes in his 1995 paper, for the FTSE100 contract. Ex ante Minimum Variance Hedging Ratios for the period 1984-1992 were used and the cash portfolio hedged was that underlying the futures contract. His results showed that even using ex ante hedge ratios, the contract enabled risk reduction of more than 80%. In addition in his 1996 paper Holmes investigated ex post hedging effectiveness for the same contract and the same cash portfolio as n the earlier paper and showed that standard OLS provided Minimum Variance Hedging Ratios estimates superior to those estimated by GARCH or using an error correction method. His results suggested effectiveness increased with hedge duration, in line with Figlewski and Lindahl for the USA, but that there was no strong discernible pattern for expiration effects.

The impact of portfolio composition on systematic risk and hedging effectiveness was examined by Holmes and Amey in their work published in 1995. They constructed portfolios of UK stocks and considered the FTSE100 contract. As the number of stocks in portfolios increased from 1 through 5, 10, 15, 20 to 25 hedging effectiveness increased markedly. While previous studies suggested the FTSE100 contract removed approximately 80% of cash portfolio risk when the portfolio was the underlying index, risk reduction was only about 60% for portfolios comprising 25 stocks.

Park and Switzer in their 1995 paper were the first to apply dynamic hedging models to estimate the hedging ratio. Using daily observations from S&P 500 and Toronto 35 Futures Index, GARCH model yields superior results to those yielded by the standard OLS.

Butterworth and Holmes in their 2002 paper provided the first investigation of the hedging effectiveness of the FTSE Mid250 Stock index futures contract. They showed that, despite relatively thin trading, the FTSE Mid 250 contract is an important alternative hedging instrument. They also demonstrated that previous studies overestimate the hedging effectiveness of UK stock index futures.

There are a number of points to draw from the previous studies considered. First, the Minimum Variance Hedging Ratio provides superior performance in terms of risk reduction. Second a duration effect is evident, with longer hedges more effective. In contrast, there is no strong evidence of expiration effects. For instance, Figlewski found hedging effectiveness was less for portfolios comprising small stocks. Graham and Jennings found that hedging of portfolios comprising only ten stocks was much more effective than for any portfolios matching index and Holmes and Amey found similar results for the UK.

Chapter 4

A case study

In this chapter we test the hedging effectiveness of minimum variance hedging ratio and its refinement, least trimmed squares hedging ratio strategies. The underlying portfolios consist of 6 indices and the corresponding forward contracts. The chapter is organized as follows: In section 4.1 we briefly describe the model and the methodology used. In section 4.2 we present the dataset comprising indices and contracts. In section 4.3 we give the Matlab function that we programmed. Finally in section 4.4 we present the results, comment on them and reach to conclusions.

4.1 THE MODEL

Up till this point we have provided with basic notions of modern portfolio theory necessary in hedging against the risk and have depicted various types of hedging, including perfect hedging, hedging against individual risk factors and more sophisticated dynamic hedging techniques, related to the Greek letters. In this last section, as a case study, we examine the hedging effectiveness of various hedging strategies, aiming at completely eliminating the risk exposure.

In particular, we examine the risk reduction achieved, when Minimum Variance Hedging Ratio strategy and its enhancement, Least Trimmed Squares Hedging Ratio strategy are employed. We compare the performance of the aforementioned strategies to one another with respect to the risk reduction, as a relative decrease in variance. It should be noted that our analysis is in the spirit of that of Butterworth and Holmes in their 2001 paper.

As far as the methodology is concerned we apply both in sample and out of sample analysis. The former consists in using all the data available to determine the optimal hedging ratio, while the latter consists in using a large portion of the data to estimate the optimal hedging position and then testing the performance with the data left out. Needless to say, the latter approach is of greater interest to the investor.

A final note remains to be made here, regarding the platform used. We processed the data through scripts and functions programmed in the environment Matlab, version 6.1. The code may be found in subsection.

4.2 THE DATA

All data processed in the thesis, were gathered through Datastream. We collected the values for several general indices of the most important stock exchange markets and the corresponding forward contracts with underlying asset the respective index. More specifically, for the first part of the simulation, we used the prices of forward contracts on indices and on the values of indices themselves. We briefly describe the indices:

The Standard & Poor's (S&P) Index is based on a portfolio of 500 different stocks: 400 industrials, 40 utilities, 20 transportation companies and 40 financial institutions. The weights of stocks in the portfolio at any given time are proportional to market capitalizations. This index accounts for 80% of the market capitalization of all the stocks listed on the New York Stock Exchange.

DAX 30 (Deutsche Aktien Xchange 30 meaning German stock "Xchange") is a Blue Chip stock market index consisting of the 30 major German companies trading on the Frankfurt Stock Exchange. DAX measures the performance of the Prime Standard's 30 largest German companies in terms of order book volume and market capitalization.

The FTSE 100 Index is a share index of the 100 most highly capitalised companies listed on the London Stock Exchange. FTSE 100 companies represent about 80% of the market capitalisation of the whole London Stock Exchange. Even though the FTSE All-Share Index is more comprehensive, the FTSE 100 is by far the most widely used UK stock market indicator. The constituents of the index are determined quarterly; the largest companies in the FTSE 250 Index are promoted if their market capitalisation would place them in the top 90 firms of the FTSE 100 Index. As of 2006, the threshold for inclusion is about £2.9 billion.

Nikkei225 is a stock market index for the Tokyo Stock Exchange (TSE). The Nikkei average is the most watched index of Asian stocks. It has been calculated daily by the Nihon Keizai Shimbun (Nikkei) newspaper. It is a price-weighted average (the unit is Yen), and the components are reviewed once a year. Stocks are weighted on the Nikkei 225 by giving an equal weighting based on a par value of 50 yen per share. Events such as stock splits, removals and additions of constituents impact upon the effective weighting of individual stocks and the divisor. The Nikkei 225 is designed to reflect the overall market, so there is no specific weighting of industries.

The CAC 40, which takes its name from the Paris Bourse's early automation system Cotation Assistée en Continu (Continuous Assisted Quotation), is a benchmark French stock market index. The index represents a capitalization-weighted measure of the 40 most significant values among the 100 highest market caps on the Paris Bourse (now Euronext Paris).

The Swiss Market Index (SMI) is Switzerland's blue-chip stock market index, which makes it the most important in the country. It is made up of twenty of the largest and most liquid SPI large- and mid-cap stocks. As a price index, the SMI is not adjusted for dividends, but a performance index that takes account of such distributions is available (SMIC: SMI Cum Dividend). The securities contained in the SMI currently represent more than 90% of the entire market capitalization, as well as of 90% trading volume, of all Swiss and Liechtenstein equities listed on the SWX Swiss Exchange and is considered to be a mirror of the overall Swiss stock market.

The dataset used consists of daily observations upon closing of both values of indices and prices of the corresponding forward contracts. The values range from 2 January 2006 to 11 January 2008. This corresponds to 530 daily observations or equivalently 106 weekly observations[3]. In the in-sample analysis, all data were used to determine the optimal hedging ratios. In the out-of-sample analysis we used 90% of the data to determine the optimal hedging ratio and examined the hedging effectiveness using the last 10% of the data. That is, in the out-of-sample analysis we determined the 10-week performance of the hedger.

* * *

As it is customary in the bibliography we assumed zero transaction and taxation costs. In addition we omitted the possibility of dividends. To evaluate each index's and forward contract's returns per period we calculated the logarithm of the price changes.

$$RS_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

$$RF_t = \ln\left(\frac{F_t}{F_{t-1}}\right)$$

According to Figlewski the estimation of the minimum variance hedging ratio may be accomplished through regressing RS on RF in the standard linear model

$$RS_t = a + bRF_t + \varepsilon_t$$

[3] Especially for Nikkei225 we have values from 18 July 2006 to 14 January 2008. This corresponds to 390 daily observations or 78 weakly observations.

Following the previous work by Butterworth and Holmes we estimated the hedging efffectiveness as the relative decrease in standard deviations between the hedged and the unhedged portfolio.

(Hedging Effectiveness)= $\Delta\sigma = \frac{\sigma_S - \sigma_H}{\sigma_S}$

4.3 CODE

In this section we give the Matlab function myHedge() that we constructed to test the Minimum Variance Hedging Ratio and the Least Trimmed Squares Hedging Ratio hedging strategies. Function myHedge() is fairly generic and may be applied to perform perfect hedging with the hedging instrument the forward contract on the asset, as well as cross-hedging, using as hedging instrument any given asset or forward contract. To achieve that, myHedge() takes as arguments the time-series of the prices of any asset S, the time-series of the prices of the hedging instrument F as well as the trimming coefficient t and returns the optimal hedging ratios. Note that no calculation concerning hedging effectiveness is done, to allow for both in-sample and out-of sample analysis.

First, the function calculates the logarithmic changes of the prices, i.e. the returns of the asset. In what follows the function calculates the minimum variance hedging ratio by applying OLS and the least trimmed squares hedging ratio, by trimming the data appropriately and applying OLS in the trimmed dataset. Finally the function returns the hedging ratios. For full reference one may read the code below, which is carefully commented.

```
function [MV,LT]=myHedge(S,F,t)

%k: number of samples

k=length(S);

%Construct the vectors of returns

logF=log(F);
logS=log(S);
RF=logF(2:k)-logF(1:k-1);
RS=logS(2:k)-logS(1:k-1);
```

```
%----------------------MVHR----------------------------

%X:matrix of regressors

X=[ones(k-1,1) RF];

%Compute the MVHR
%via OLS in the Regression RS=a+bRF+e
MVHR=inv(X'*X)*(X'*RS);
MV=MVHR(2);

%----------------------LTSHR---------------------------

%Compute vector of absolute values of residuals

e=abs(RS-X*MVHR);

%Augment vectors of returns with vector of residuals

Aug=[RF RS e];

%Sort with respect to the vector of residuals e

sortrows(Aug,3);

%Calculate index of data to be trimmed
trim_ind=round((1-t)*(k-1));

%Trim data and drop residuals

RFT=Aug(1:trim_ind,1);
RST=Aug(1:trim_ind,2);

%Perform MVHR in the trimmed data,
%notice that the indices no longer
%indicate time

%XT:matrix of regressors

XT=[ones(trim_ind,1) RFT];

%Compute the LTSHR
%via OLS in the Regression RST=a+bRFT+v

LTSHR=inv(XT'*XT)*(XT'*RST);
LT=LTSHR(2);
```

4.4 RESULTS AND CONCLUSIONS

The purpose of this thesis was to examine the hedging effectiveness of the Minimum Variance Hedging Ratio strategy and the Least Trimmed Squares Hedging Ratio strategy. We tested the effectiveness of these hedging schemes, when these were applied on a daily and a weakly basis. We further examined the hedging effectiveness of the strategies, when some data were left out, to perform the out-of-sample analysis, which is of greater interest to the investors.

In the next table the mean return as well as the standard deviation of the returns may be found. We observe that the sole index yielding negative is the NIKKEI225 index. It should thus be interesting to measure the decrease in losses that will be achieved by means of hedging.

No hedging daily – in sample		
Index	Mean Return	Returns S.D.
S&P 500	$2.4*10^{-4}$	$7.5*10^{-3}$
FTSE100	$1.9*10^{-4}$	$9.4*10^{-3}$
NIKKEI225	$-5.8*10^{-5}$	$1.1*10^{-2}$
DAX30	$6.6*10^{-4}$	$9.6*10^{-3}$
CAC40	$3.4*10^{-4}$	$8.5*10^{-3}$
SMI	$1.4*10^{-4}$	$8.9*10^{-3}$

In the next two tables one may find the performance of the hedging strategies. A first remark is that both the Minimum Variance Hedging Ratio and the Least Trimmed Squares Hedging Ratio yield roughly the same results. Therefore, there were no extreme outliers to significantly affect the OLS estimate.

MVHR Hedging daily – in sample				
Index	MVHR	Mean Return	Returns S.D.	% Decrease in S.D.
S&P 500	$-7.5*10^{-2}$	$2.6*10^{-4}$	$7.5*10^{-3}$	35
FTSE100	1.00	$-1.1*10^{-6}$	$1.6*10^{-3}$	83.5
NIKKEI225	.95	$-1.8*10^{-5}$	$2.4*10^{-3}$	78.1
DAX30	1.00	$-7.5*10^{-6}$	$1.6*10^{-3}$	83.7
CAC40	$-7.4*10^{-3}$	$3.4*10^{-4}$	$8.5*10^{-3}$	37
SMI	.98	$-2.9*10^{-6}$	$1.3*10^{-3}$	85.4

LTSHR Hedging daily – in sample				
Index	LTSHR	Mean Return	Returns S.D.	% Decrease in S.D.
S&P 500	$-7.9*10^{-2}$	$2.6*10^{-4}$	$7.5*10^{-3}$	35
FTSE100	.99	$-4*10^{-9}$	$1.6*10^{-4}$	83.5
NIKKEI225	.96	$-1.7*10^{-5}$	$2.4*10^{-2}$	78.1
DAX30	1.00	$-7.9*10^{-6}$	$1.6*10^{-3}$	83.7
CAC40	$9.4*10^{-3}$	$3.4*10^{-4}$	$8.5*10^{-3}$	15
SMI	.97	$-2.0*10^{-6}$	$1.3*10^{-3}$	85.4

The performance of the two strategies, in terms of risk reduction is remarkable, as they reach 85.4%. It is a surprise though that in the cases of CAC40 and S&P 500 the performance is extremely poor in comparison to the other indices. This was not even cured with the employment of the Least Trimmed Squares method.

It is also noteworthy that in the case of the NIKKEI225, the losses were moderated by more than 320%. Of course hedging had a negative effect on the mean returns of those indices that yielded profit, but this is the price one pays to reduce the risk.

We also examined the hedging effectiveness when weakly observations were taken into account. The results showed that in all cases hedging on a weakly basis yields slightly superior results. These may be found in the following three tables.

No hedging weakly		
Index	Mean Return	Returns S.D.
S&P 500	$1.5*10^{-3}$	$1.7*10^{-2}$
FTSE100	$1.2*10^{-3}$	$2.0*10^{-2}$
NIKKEI225	$6.8*10^{-5}$	$2.5*10^{-2}$
DAX30	$3.9*10^{-3}$	$2.2*10^{-2}$
CAC40	$2.0*10^{-3}$	$2.1*10^{-2}$
SMI	$1.0*10^{-3}$	$2.1*10^{-2}$

MVHR Hedging weakly – in sample				
Index	MVHR	Mean Return	Returns S.D.	Decrease in S.D.
S&P 500	.27	$1.1*10^{-3}$	$1.6*10^{-2}$	4.4
FTSE100	1.0	$-4.6*10^{-5}$	$2.4*10^{-3}$	88.2
NIKKEI225	.96	$1.7*10^{-5}$	$2.7*10^{-3}$	89.0
DAX30	1.0	$-5.6*10^{-5}$	$3.1*10^{-3}$	86.0
CAC40	.27	$1.6*10^{-3}$	$2.0*10^{-2}$	3.78
SMI	.97	$-1.7*10^{-5}$	$2.4*10^{-3}$	88.6

LTSHR Hedging weakly – in sample				
Index	LTSHR	Mean Return	Returns S.D.	Decrease S.D.
S&P 500	.42	$8.6*10^{-4}$	$1.6*10^{-2}$	3.1
FTSE100	1.0	$-3.2*10^{-5}$	$2.4*10^{-3}$	88.1

NIKKEI225	.94	$1.7*10^{-5}$	$2.7*10^{-3}$	89.3
DAX30	1.0	$-9.4*10^{-5}$	$3.1*10^{-3}$	86.0
CAC40	.37	$1.5*10^{-3}$	$1.9*10^{-2}$	3.3
SMI	.95	$-2.8*10^{-6}$	$2.4*10^{-3}$	88.5

Finally we performed out-of-sample analysis. As expected, the results are worse than in the in-sample analysis, but not significantly, with the exception of S&P 500 and CAC40. We observe that hedging is beneficial and achieves considerable reduction in the standard deviation of returns. The results are listed in the following tables.

No hedging daily – out of sample		
Index	Mean Return	Returns S.D.
S&P 500	$2.4*10^{-4}$	$7.5*10^{-3}$
FTSE100	$1.9*10^{-4}$	$9.4*10^{-3}$
NIKKEI225	$-5.9*10^{-5}$	$1.1*10^{-2}$
DAX30	$6.6*10^{-4}$	$9.6*10^{-3}$
CAC40	$3.4*10^{-4}$	$8.5*10^{-3}$
SMI	$1.4*10^{-4}$	$8.9*10^{-3}$

MVHR Hedging daily – out of sample				
Index	MVHR	Mean Return	Returns S.D.	% Decrease in S.D.
S&P 500	-.08	$2.6*10^{-4}$	$7.5*10^{-3}$	.35
FTSE100	.99	$-3.2*10^{-9}$	$1.6*10^{-3}$	83.5
NIKKEI225	.96	$-1.7*10^{-5}$	$2.4*10^{-3}$	78.1
DAX30	1.0	$-7.9*10^{-6}$	$1.6*10^{-3}$	83.7
CAC40	.01	$3.4*10^{-4}$	$8.5*10^{-3}$	$-1.5*10^{-2}$
SMI	.97	$-2.0*10^{-6}$	$1.3*10^{-3}$	85.4

LTSHR Hedging daily – out of sample				
Index	LTSHR	Mean Return	Returns S.D.	% Decrease in S.D.
S&P 500	-.08	$2.6*10^{-4}$	$7.5*10^{-3}$	.35
FTSE100	.99	$-1.2*10^{-7}$	$1.6*10^{-3}$	83.5
NIKKEI225	.97	$-1.7*10^{-5}$	$2.4*10^{-3}$	78.0
DAX30	1.0	$-9.0*10^{-6}$	$1.6*10^{-3}$	83.7
CAC40	.01	$3.4*10^{-4}$	$8.5*10^{-3}$	-.02
SMI	.97	$-1.5*10^{-6}$	$1.3*10^{-3}$	85.4

Examining the results we conclude that hedging on a weakly basis is superior to daily hedging even when out-of-sample analysis takes place. If we combine the last statement with the consideration of the high transaction costs incurred by daily rebalancing we reach the safe conclusion that hedging should be conducted on a weakly basis. The results follow in the last three tables.

No hedging weakly		
Index	Mean Return	Returns S.D.
S&P 500	$1.5*10^{-3}$	$1.7*10^{-2}$
FTSE100	$1.2*10^{-3}$	$2.0*10^{-2}$
NIKKEI225	$6.8*10^{-5}$	$2.5*10^{-2}$
DAX30	$3.9*10^{-3}$	$2.2*10^{-2}$
CAC40	$2*10^{-3}$	$2.1*10^{-2}$
SMI	$1.0*10^{-3}$	$2.1*10^{-2}$

MVHR Hedging weakly – out of sample				
Index	MVHR	Mean Return	Returns S.D.	Decrease in S.D.
S&P 500	.42	$8.6*10^{-4}$	$1.6*10^{-3}$	3.1
FTSE100	1.0	$-3.2*10^{-5}$	$2.4*10^{-3}$	88.2
NIKKEI225	.94	$1.8*10^{-5}$	$2.7*10^{-3}$	89.2
DAX30	1.0	$-9.4*10^{-5}$	$3.1*10^{-3}$	86.0
CAC40	.37	$1.5*10^{-3}$	$2.0*10^{-2}$	3.33
SMI	.96	$-2.8*10^{-6}$	$2.4*10^{-3}$	88.5

LTSHR Hedging weakly – out of sample				
Index	LTSHR	Mean Return	Returns S.D.	Decrease S.D.
S&P 500	.39	$9.1*10^{-4}$	$1.6*10^{-2}$	3.7
FTSE100	1.0	$-4.3*10^{-5}$	$2.4*10^{-3}$	88.2
NIKKEI225	.92	$1.8*10^{-5}$	$2.8*10^{-3}$	88.8
DAX30	1.0	$-1.3*10^{-4}$	$3.1*10^{-3}$	85.9
CAC40	.34	$1.5*10^{-3}$	$2.0*10^{-2}$	3.5
SMI	.96	$-4.7*10^{-6}$	$2.4*10^{-3}$	88.5

A general remark to be made is that hedging should be applied with caution. On one hand, it results in the reduction of the volatilities of the returns, on the other hand it may result in diminishing and even negative expected excess returns. The obvious question arises: Why bother apply sophisticated trading techniques, if these merely end up yielding the risk-free rate? The answer is that hedging should be used to change the beta of a portfolio at times when the investor thinks that the market is extremely risky. In addition one may easily notice that the same techniques may be used to increase expected returns and relevant risk in order to achieve greater profit by speculation.

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